Quadratic Formula (U.B)

For a standard quadratic equation

$ax^2+bx+c=0$ where $a\neq 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Use of Quadratic Formula (U.B)

The quadratic formula is useful tool for solving all those equations which can or cannot be factorized.

Derivation of Quadratic Formula by using Completing Square Method

(K.B + U.B + A.P)

The quadratic equation in standard form is

$ax^2+bx+c=0,\ a\neq 0$

Shifting constant term c to the right, we have

$ax^2+bx=-c$

Dividing each term by a

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

Or $$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

Or $$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $a\neq 0$ **is known as "Quadratic Formula".**

Example 1: (Page # 6) **(A.B)**

Solve the quadratic equation $2+9x=5x^2$ by using quadratic formula.

Solution:

$2+9x=5x^2$

$\Rightarrow 5x^2-9x-2=0$

By comparing given equation with standard quadratic equation $ax^2+bx+c=0$, we have

$a=5,\ b=-9,\ c=-2$

Quadratic formula is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Putting the values in quadratic formula

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(5)(-2)}}{2(5)}$$

$$x=\frac{9\pm\sqrt{81+40}}{10}$$

$$=\frac{9\pm\sqrt{121}}{10}$$

$$=\frac{9\pm 11}{10}$$

Either

$x=\frac{9+11}{10}$ or $x=\frac{9-11}{10}$

$x=\frac{20}{10}=2$ or $x=\frac{-2}{10}=-\frac{1}{5}$

$\therefore 2,-\frac{1}{5}$ are the roots of the given equation.

Thus, the solution set is $\left\{-\frac{1}{5},2\right\}$

Exercise 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2-x^2=7x$ **(A.B)**

Solution:

$2-x^2=7x$

$0=x^2+7x-2$

$\Rightarrow x^2+7x-2=0$

By comparing given equation with ax^2+bx+c, we get

$a=1, b=7, c=-2$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{(7)^2-4(1)(-2)}}{2\times1}$$

$$=\frac{-7\pm\sqrt{49+8}}{2}=\frac{-7\pm\sqrt{57}}{2}$$

$\therefore$ **Solution Set** $=\left\{\frac{-7\pm\sqrt{57}}{2}\right\}$

(ii) $5x^2+8x+1=0$ **(A.B)**

Solution:

$5x^2+8x+1=0$

Here $a=5, b=8, c=1$

Putting the values in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-8\pm\sqrt{(8)^2-4\times5\times1}}{2\times5}$$

$$=\frac{-8\pm\sqrt{64-20}}{10}=\frac{-8\pm\sqrt{44}}{10}$$

$$=\frac{-8\pm\sqrt{4\times11}}{10}=\frac{-8\pm2\sqrt{11}}{10}$$

$$=\frac{\not{2}\left(-4\pm\sqrt{11}\right)}{\not{10}_5}$$

$$=\frac{-4\pm\sqrt{11}}{5}$$

$\therefore$ **Solution Set** $=\left\{\frac{-4\pm\sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2+x=4\sqrt{3}$ **(A.B + U.B + K.B)**

Solution:

$\sqrt{3}x^2+x=4\sqrt{3}$

$\sqrt{3}x^2+x-4\sqrt{3}=0$

Here $a=\sqrt{3}, \quad b=1, \quad c=-4\sqrt{3}$

Putting in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{(1)^2-4\sqrt{3}\left(-4\sqrt{3}\right)}}{2\times\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{1+16\times\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{1+48}}{2\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{49}}{2\sqrt{3}}$$

$$x=\frac{-1\pm7}{2\sqrt{3}}$$

Either

$x=\frac{-1+7}{2\sqrt{3}}$ or $x=\frac{-1-7}{2\sqrt{3}}$

$x=\frac{\cancel{6}^{3}}{\cancel{2}\sqrt{3}}$ $x=\frac{\cancel{-8}^{4}}{\cancel{2}\sqrt{3}}$

$x=\frac{3}{\sqrt{3}}$ $x=-\frac{4}{\sqrt{3}}$

$x=\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$ **or** $x=-\frac{4}{\sqrt{3}}$

$x=\sqrt{3}$

$\therefore$ **Solution of Set** $=\left\{\sqrt{3},-\frac{4}{\sqrt{3}}\right\}$

(iv) $4x^2-14=3x$ **(A.B)**

Solution:

$4x^2-14=3x$

$4x^2-3x-14=0$

Here $a=4,\quad b=-3,\quad c=-14$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\times4\times(-14)}}{2\times4}$$

$$x=\frac{3\pm\sqrt{9+224}}{8}$$

$$x=\frac{3\pm\sqrt{233}}{8}$$

$\therefore$ **Solution Set** $=\left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $6x^2-3-7x=0$ **(A.B)**

Solution:

$6x^2-3-7x=0$

$6x^2-7x-3=0$

Here $a=6,\quad b=-7,\quad c=-3$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2\times6}$$

$$x=\frac{7\pm\sqrt{49+72}}{12}$$

$$x=\frac{7\pm\sqrt{121}}{12}$$

$$x=\frac{7\pm11}{12}$$

Either

$x=\frac{7+11}{12}$ or $x=\frac{7-11}{12}$

$x=\frac{18}{12}$, $x=\frac{-4}{12}$

$x=\frac{3}{2}$, $x=\frac{-1}{3}$

$\therefore$ **Solution Set** $=\left\{\frac{3}{2},\frac{-1}{3}\right\}$

(vi) $3x^2+8x+2=0$ **(A.B)**

Solution:

$3x^2+8x+2=0$

Here

$a=3,\quad b=8,\quad c=2$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4\times3\times2}}{2\times3}$$

$$x=\frac{-8\pm\sqrt{64-24}}{6}$$

$$x=\frac{-8\pm\sqrt{40}}{6}$$

$$x=\frac{-8\pm2\sqrt{10}}{6}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{10}\right)}{\cancel{6}_3}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

$\therefore$ **Solution Set** $= \left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6} - \frac{4}{x-5} = 1$ **(A.B + K.B)**

Solution:

$$\frac{3}{x-6} - \frac{4}{x-5} = 1$$

Multiplying both sides by $(x-6)(x-5)$

$$3(x-5) - 4(x-6) = 1(x-6)(x-5)$$

$$3x - 15 - 4x + 24 = x^2 - 6x - 5x + 30$$

$$-x + 9 = x^2 - 11x + 30$$

$$0 = x^2 - 11x + 30 + x - 9$$

$$0 = x^2 - 10x + 21$$

$$x^2 - 10x + 21 = 0$$

Here

$a = 1, \quad b = -10, \quad c = 21$

Putting values of a, b, c in Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4\times1\times21}}{2\times1}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{10 \pm 4}{2}$$

Either

$x = \frac{10+4}{2}$ or $x = \frac{10-4}{2}$

$x = \frac{14}{2}$, $x = \frac{6}{2}$

$x = 7$, $x = 3$

$\therefore$ **Solution Set** $= \{3, 7\}$

(viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$ **(A.B + K.B)**

Solution:

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$$

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$$

Multiplying throughout by $3(2x)(x-1)$

$$3(2x)(x+2) - 3(x-1)(4-x) = 7(2x)(x-1)$$

$$6x(x+2) + 3(x-1)(x-4) = 14x(x-1)$$

$$6x^2 + 12x + 3(x^2 - 5x + 4) = 14x^2 - 14x$$

$$6x^2 + 12x + 3x^2 - 15x + 12 = 14x^2 - 14x$$

$$9x^2 - 3x + 12 = 14x^2 - 14x$$

$$0 = 14x^2 - 14x - 9x^2 + 3x - 12$$

$$0 = 5x^2 - 11x - 12$$

$$5x^2 - 11x - 12 = 0$$

Here

$a = 5, \quad b = -11, \quad c = -12$

Putting values of a, b, c in Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4\times5\times(-12)}}{2\times5}$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10}$$

$$x = \frac{11 \pm 19}{10}$$

Either

$x = \frac{11+19}{10}$ or $x = \frac{11-19}{10}$

$x = \frac{30}{10}$, $x = \frac{-8}{10}$

$x = 3$, $x = -\frac{4}{5}$

$\therefore$ **Solution Set** $= \left\{-\frac{4}{5}, 3\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$ **(A.B + K.B)**

(FSD 2016)

Solution:

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$

Multiplying throughout by $(x-a)(x-b)$

$$a(x-a)+b(x-b)=2(x-a)(x-b)$$
$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$
$$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$$
$$0=2x^2-2ax-2bx+2ab-ax-bx+a^2+b^2$$
$$0=2x^2-3ax-3bx+a^2+b^2+2ab$$
$$2x^2-3(a+b)x+(a+b)^2=0$$

Here

$A=2,\ \ B=-3(a+b), C=(a+b)^2$

Putting values of A, B, C in Quadratic Formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4\times 2(a+b)^2}}{2\times 2}$$
$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm(a+b)}{4}$$

Either

$$x=\frac{3(a+b)+(a+b)}{4} \quad \text{or} \quad x=\frac{3(a+b)-(a+b)}{4}$$
$$x=\frac{\cancel{4}(a+b)}{\cancel{4}} \quad , \quad x=\frac{\cancel{2}(a+b)}{{}_2\cancel{4}}$$
$$x=a+b \quad , \quad x=\frac{a+b}{2}$$

$\therefore$ **Solution Set** $=\left\{a+b,\frac{a+b}{2}\right\}$

(x) $-(l+m)-lx^2+(2l+m)x=0$ **(A.B + K.B + U.B)**

Solution:

$$-(l+m)-lx^2+(2l+m)x=0$$
$$-lx^2+(2l+m)x-(l+m)=0$$
$$lx^2-(2l+m)x+(l+m)=0$$

Here $a=l,\ \ b=-(2l+m),\ \ \ c=l+m$

Putting the values of a, b, c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-[-(2l+m)]\pm\sqrt{[-(2l+m)]^2-4\times l\times(l+m)}}{2\times l}$$
$$x=\frac{(2l+m)\pm\sqrt{(2l+m)^2-4l(l+m)}}{2l}$$
$$x=\frac{(2l+m)\pm\sqrt{\cancel{4l^2}+m^2+\cancel{4lm}-\cancel{4l^2}-\cancel{4lm}}}{2l}$$
$$x=\frac{(2l+m)\pm\sqrt{m^2}}{2l}$$
$$x=\frac{2l+m\pm m}{2l}$$
$$x=\frac{2l+m+m}{2l} \quad , \quad x=\frac{2l+\cancel{m}-\cancel{m}}{2l}$$
$$x=\frac{2l+2m}{2l} \quad , \quad x=\frac{2l}{2l}$$
$$x=\frac{\cancel{2}(l+m)}{\cancel{2}l} \quad , \quad x=1$$
$$x=\frac{l+m}{l}$$

$\therefore$ **Solution Set** $=\left\{1,\frac{l+m}{l}\right\}$