Mathematics-10

Exercise - 1.3

Reciprocal Equation (U.B)

(LHR 2015, 16, 17, FSD 2017, MTN 2015, 17, RWP 2017, BWP 2017, D.G.K 2016, 17)

An equation, which remains unchanged when 'x' is replaced by $\frac{1}{x}$ is called reciprocal equation.

e.g. $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

$ax^4+bx^3+cx^2+bx+a=0$

Exponential Equation (U.B)

(LHR 2015, GRW 2017 SWL 2017, SGD 2017, D.G.K 2015, MTN 2016)

An equation, in which a variable or an algebraic expression occurs in exponent is called exponential equation.

e.g. $2^x+64.2^{-x}-20=0$

Or

$3^{2x+2}=12.3^x-3$ etc.

Equations Reducible to Quadratic Form (K.B + A.B + U.B)

Type (i)

The Equations of the type

$ax^4+bx^2+c=0$

Example 1: (Page # 8)

Solve the equation $x^4-13x^2+36=0$

Solution:

$x^4-13x^2+36=0$

$(x^2)^2-13x^2+36=0\rightarrow$ (i)

Let $x^2=y\rightarrow$ (ii)

Equation (i) $\Rightarrow$

$y^2-13y+36=0$

$y^2-9y-4y+36=0$

$y(y-9)-4(y-9)=0$

$(y-9)(y-4)=0$

Either

$y-9=0$ or $y-4=0$

$\Rightarrow y=9$ or $y=4$

Putting the value of y in equation (ii)

$\Rightarrow x^2=9$ or $x^2=4$

Taking square root on both sides

$\Rightarrow x=\pm 3$ or $x=\pm 2$

$\therefore$ **Solution Set** $=\{\pm 3,\pm 2\}$

Type (ii)

The Equations of the type

$ap(x)+\frac{b}{p(x)}=c$

Example 2: (Page # 8) **(K.B + A.B)**

Solve the equation $2(2x-1)+\frac{3}{2x-1}=5$

Solution:

$2(2x-1)+\frac{3}{2x-1}=5\rightarrow$ (i)

Let $2x-1=y\rightarrow$ (ii)

Equation (i) $\Rightarrow$

$2y+\frac{3}{y}=5$

$2y^2+3=5y$

(by Mul both sides by y)

$2y^2-5y+3=0$

$2y^2-2y-3y+3=0$

$2y(y-1)-3(y-1)=0$

$(y-1)(2y-3)=0$

Either

$y-1=0$ or $2y-3=0$

$\Rightarrow y=1$ or $2y=3$

$\Rightarrow y=\frac{3}{2}$

Putting the values of y in equation (ii)

$\Rightarrow 2x-1=1$ or $2x-1=\frac{3}{2}$

$2x = 1+1$ or $2x = \frac{3}{2}+1$

$2x = 2$ or $2x = \frac{5}{2}$

$x = 1$ or $x = \frac{5}{4}$

$\therefore$ **Solution Set** $= \left\{1, \frac{5}{4}\right\}$

Type (iii) **(A.B + K.B)**

Reciprocal Equation of the type:

$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$ or

$ax^4+bx^3+cx^2+bx+a=0$

Example 3: (Page # 9) **(A.B)**

Solve the equation

$2x^4-5x^3-14x^2-5x+2=0$

Solution:

$2x^4-5x^3-14x^2-5x+2=0$

$2x^4+2-5x^3-5x-14x^2=0$

Dividing each term by x^2

$2x^2+\frac{2}{x^2}-5x-\frac{5}{x}-14=0$

$2\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)-14=0 \rightarrow \text{(i)}$

Let $x+\frac{1}{x}=y \rightarrow \text{(ii)}$

Squaring both sides

$y=-2$

$x^2+\frac{1}{x^2}+2=y^2$

$x^2+\frac{1}{x^2}=y^2-2$

Equation (i) $\Rightarrow$

$2(y^2-2)-5y-14=0$

$2y^2-4-5y-14=0$

$2y^2-5y-18=0$

$2y^2-9y+4y-18=0$

$y(2y-9)+2(2y-9)=0$

$(2y-9)(y+2)=0$

Either

$2y-9=0$ or $y+2=0$

$\Rightarrow 2y=9$ or $y=-2$

$\Rightarrow y=\frac{9}{2}$

Putting the values of y in equation (ii)

When $y=\frac{9}{2}$

$x+\frac{1}{x}=\frac{9}{2}$

$2x^2+2=9x$

$2x^2-9x+2=0$

By using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(2)}}{2(2)}$$

$$x=\frac{9\pm\sqrt{81-16}}{4}$$

$$x=\frac{9\pm\sqrt{65}}{4}$$

When $y=-2$

$x+\frac{1}{x}=-2$

$x^2+1=-2x$

$x^2+2x+1=0$

$(x+1)^2=0$

$\Rightarrow x+1=0$

Or $x=-1$

Thus, Solution Set $=\left\{-1, \frac{9\pm\sqrt{65}}{4}\right\}$

Type (iv) **(A.B + K.B + U.B)**

Exponential Equation:

$5y+\frac{5}{y}=26$ or $5^{1+x}+5^{1-x}=26$

Example 4: (Page # 10)

Solve the equation $5^{1+x}+5^{1-x}=26$

Solution:

$5^{1+x}+5^{1-x}=26$

$5^1.5^x+5^1.5^{-x}=26$

$$5.5^x+\frac{5}{5^x}=26\rightarrow(\text{i})$$

Let $5^x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow$

$$5y+\frac{5}{y}=26$$
$$5y^2-26y+5=0$$
$$5y^2-26y+5=0$$
$$5y^2-25y-y+5=0$$
$$(y-5)(5y-1)=0$$
$$(y-5)(5y-1)=0$$

Either

$y-5=0$ or $5y-1=0$

$y=5$ or $y=\frac{1}{5}$

Putting the values of y in equation (ii)

$\Rightarrow 5^x=5$ or $5^x=\frac{1}{5}$

$\Rightarrow 5^x=5^1$ or $5^x=5^{-1}$

$\because$ Bases are same

$x=1$ or $x=-1$

Thus, Solution Set $=\{\pm1\}$

Type (v) (A.B + U.B + K.B)

The Equation of the type of $(x+a)(x+b)(x+c)(x+d)=k$ where $a+b=c+d$

Example 5: (Page # 11)

Solve the equation $(x-1)(x+2)(x+8)(x+5)=19$

Solution:

$$(x-1)(x+2)(x+8)(x+5)=19$$
$$(x-1)(x+8)(x+2)(x+5)=19\quad(\because -1+8=2+5)$$
$$(x^2+7x-8)(x^2+7x+10)-19=0\rightarrow(\text{i})$$

Let $x^2+7x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow$

$$(y-8)(y+10)-19=0$$
$$y^2+10y-8y-80-19=0$$
$$y^2+2y-99=0$$
$$y^2+11y-9y-99=0$$
$$y(y+11)-9(y+11)=0$$
$$(y+11)(y-9)=0$$

Putting the values of y

$$(x^2+7x+11)(x^2+7x-9)=0$$

Either

$x^2+7x+11=0$ or $x^2+7x-9=0$

When $x^2+7x+11=0$

Solving by quadratic formula, we have

$$x=\frac{\sqrt{-7\pm-(7)^2-(1)(11)}}{2(1)}$$
$$=\frac{-7\pm\sqrt{49-44}}{2}$$
$$=\frac{-7\pm\sqrt{5}}{2}$$

When $x^2+7x-9=0$

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-9)}}{2(1)}$$
$$=\frac{-7\pm\sqrt{49+36}}{2}$$
$$=\frac{-7\pm\sqrt{85}}{2}$$

$\therefore$ **Solution Set** $=\left\{\frac{-7\pm\sqrt{5}}{2},\frac{-7\pm\sqrt{85}}{2}\right\}$

Exercise 1.3

Solve the following equations.

Q.1 $2x^4-11x^2+5=0$ **(A.B)**

Solution:

$$2x^4-11x^2+5=0$$
$$2(x^2)^2-11x^2+5=0\rightarrow(\text{i})$$

Let $x^2=y$

Putting $x^2=y$ in equation (i)

$$2y^2-11y+5=0$$
$$2y^2-10y-y+5=0$$
$$2y(y-5)-1(y-5)=0$$

$(y-5)(2y-1)=0$

Either

$y-5=0$ or $2y-1=0$

$y=5$ $\quad 2y=1$

$y=\frac{1}{2}$

Putting $y=x^2$

$x^2=5$ or $x^2=\frac{1}{2}$

Taking square root on both sides

$x=\pm\sqrt{5}$ $\quad x=\pm\frac{1}{\sqrt{2}}$

$\therefore$ **Solution Set** = $\left\{\pm\sqrt{5},\pm\frac{1}{\sqrt{2}}\right\}$

Q.2 $2x^4=9x^2-4$ **(A.B)**

Solution:

$2x^4=9x^2-4$

$2(x^2)^2-9x^2+4=0\rightarrow(i)$

Let $x^2=y$

Putting $x^2=y$ in equation (i)

$2y^2-9y+4=0$

$2y^2-8y-y+4=0$

$2y(y-4)-1(y-4)=0$

$(y-4)(2y-1)=0$

Either

$y-4=0$ or $2y-1=0$

$y=4$ $\quad 2y=1$

$y=\frac{1}{2}$

Putting $y=x^2$

$x^2=4$ or $x^2=\frac{1}{2}$

Taking square root on both sides

$x=\pm 2$ $\quad x=\pm\frac{1}{\sqrt{2}}$

$\therefore$ **Solution set** = $\left\{\pm 2,\pm\frac{1}{\sqrt{2}}\right\}$

Q.3 $5x^{\frac{1}{2}}=7x^{\frac{1}{4}}-2$ **(FSD 2016)** **(A.B)**

Solution:

$5x^{\frac{1}{2}}=7x^{\frac{1}{4}}-2$

$5(x^{\frac{1}{4}})^2-7x^{\frac{1}{4}}+2=0\rightarrow(\text{i})$

Let $x^{\frac{1}{4}}=y$

Putting $x^{\frac{1}{4}}=y$ in equation (i)

$5y^2-7y+2=0$

$5y^2-5y-2y+2=0$

$5y(y-1)-2(y-1)=0$

$(y-1)(5y-2)=0$

Either

$y-1=0$ or $5y-2=0$

$y=1$ or $5y=2$

$y=\frac{2}{5}$

Putting $y=x^{1/4}$

$x^{\frac{1}{4}}=1$ or $x^{\frac{1}{4}}=\frac{2}{5}$

$x=(1)^4$ $\quad \left(x^{\frac{1}{4}}\right)^4=\left(\frac{2}{5}\right)^4$

$x=1$ $\quad x=\frac{16}{625}$

$\therefore$ **Solution Set** $=\left\{1,\frac{16}{625}\right\}$

Q.4 $x^{2/3}+54=15x^{1/3}$ **(A.B)**

Solution:

$x^{2/3}+54=15x^{1/3}$

$\left(x^{\frac{1}{3}}\right)^2-15x^{\frac{1}{3}}+54=0\rightarrow(\text{i})$

Let $x^{\frac{1}{3}}=y$

Putting $x^{\frac{1}{3}}=y$ in equation (i)

$y^2-15y+54=0$

$y^2-9y-6y+54=0$

$y(y-9)-6(y-9)=0$

$(y-9)(y-6)=0$

Either

$y-9=0$ or $y-6=0$

$y=9$ or $y=6$

Putting $y=x^{\frac{1}{3}}$

$x^{\frac{1}{3}}=9$ or $x^{\frac{1}{3}}=6$

Taking cube on both sides

$\left(x^{\frac{1}{3}}\right)^3=(9)^3$ or $\left(x^{\frac{1}{3}}\right)^3=(6)^3$

$x=729$ or $x=216$

$\therefore$ **Solution Set** $=\{729,216\}$

Q.5 $3x^{-2}+5=8x^{-1}$ **(SWL 2014)** **(A.B)**

Solution:

$3x^{-2}+5=8x^{-1}$

$3(x^{-1})^2-8x^{-1}+5=0\rightarrow(\text{i})$

Let $x^{-1}=y$

Putting $x^{-1}=y$ in equation (i)

$3y^2-8y+5=0$

$3y^2-3y-5y+5=0$

$3y(y-1)-5(y-1)=0$

$(y-1)(3y-5)=0$

Either

$y-1=0$ or $3y-5=0$

$y=1$ or $3y=5$

$y=\frac{5}{3}$

Putting $y=x^{-1}$

$x^{-1}=1$, $x^{-1}=\frac{5}{3}$

$\frac{1}{x}=1$, $\frac{1}{x}=\frac{5}{3}$

Taking reciprocal on both sides

$\Rightarrow x=1$ $x=\frac{3}{5}$

$\therefore$ **Solution Set** $=\left\{1,\frac{3}{5}\right\}$

Q.6 $(2x^2+1)+\frac{3}{2x^2+1}=4$ **(A.B)**

Solution:

$(2x^2+1)+\frac{3}{2x^2+1}=4\rightarrow(\text{i})$

Let $a=2x^2+1$

Putting the value of 'a' in equation (i)

$a+\frac{3}{a}-4=0$ (Mul both sides by a)

$a^2+3-4a=0$

$a^2-4a+3=0$

$a^2-3a-a+3=0$

$a(a-3)-1(a-3)=0$

$(a-3)(a-1)=0$

Either

$a-3=0$ or $a-1=0$

$a=3$ $a=1$

Putting $a=2x^2+1$

$2x^2+1=3$ or $2x^2+1=1$

$2x^2=3-1$ $2x^2=1-1$

$2x^2=2$ $2x^2=0$

$x^2=1$ $x^2=0$

Taking square root on both sides

$x=\pm1$ $x=0$

$\therefore$ **Solution Set** $=\{0,\pm1\}$

Q.7 $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ **(A.B)**

Solution:

$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4\rightarrow(\text{i})$

If $y=\frac{x}{x-3}\rightarrow(\text{ii})$

Equation (i) $\Rightarrow y+\frac{4}{y}=4$

$y^2+4=4y$

$y^2-4y+4=0$

$(y^2)-2(y)(2)+(2)^2=0$

$(y-2)^2=0$

Taking square root on both sides

$y-2=0$

$y=2$

Putting value of y in equation (ii)

$\frac{x}{x-3}=2$

$x=2(x-3)$

$x=2x-6$

$6=2x-x$

$6=x$

Or $x=6$

$\therefore$ **Solution Set** $=\{6\}$

Q.8 $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$ **(A.B)**

Solution:

$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}\rightarrow$ (i)

Let $a=\frac{4x+1}{4x-1}$

Then,

$\frac{1}{a}=\frac{4x-1}{4x+1}$

Putting the values in equation (i)

$a+\frac{1}{a}=\frac{13}{6}$

$\frac{a^2+1}{a}=\frac{13}{6}$

$6a^2+6=13a$

$6a^2-13a+6=0$

$6a^2-9a-4a+6=0$

$3a(2a-3)-2(2a-3)=0$

$(2a-3)(3a-2)=0$

Either

$2a-3=0$ or $3a-2=0$

$2a=3$ $3a=2$

$a=\frac{3}{2}$ $a=\frac{2}{3}$

Putting $a=\frac{4x+1}{4x-1}$

$\frac{4x+1}{4x-1}=\frac{3}{2}$ or $\frac{4x+1}{4x-1}=\frac{2}{3}$

$2(4x+1)=3(4x-1)$ or $3(4x+1)=2(4x-1)$

$8x+2=12x-3$ $12x+3=8x-2$

$12x-8x=2+3$ $12x-8x=-2-3$

$4x=5$ $4x=-5$

$x=\frac{5}{4}$ $x=\frac{-5}{4}$

$\therefore$ **Solution Set** $=\left\{\pm\frac{5}{4}\right\}$

Q.9 $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ **(A.B)**

Solution:

$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}\rightarrow$ (i)

Let $\frac{x-a}{x+a}=b$

$\frac{1}{b}=\frac{x+a}{x-a}$

Putting the values in equation (i)

$b-\frac{1}{b}=\frac{7}{12}$

$b-\frac{1}{b}-\frac{7}{12}=0$

Multiplying by $12b$

$12b^2-12-7b=0$

$12b^2-7b-12=0$

$12b^2-16b+9b-12=0$

$4b(3b-4)+3(3b-4)=0$

$(3b-4)(4b+3)=0$

Either

$3b-4=0$ or $4b+3=0$

$3b=4$ $4b=-3$

$b=\frac{4}{3}$ $b=-\frac{3}{4}$

Put $b=\frac{x-a}{x+a}$

$\frac{x-a}{x+a}=-\frac{3}{4}$ $\frac{x-a}{x+a}=+\frac{4}{3}$

$4(x-a)=-3(x+a)$ $3(x-a)=4(x+a)$

$4x-4a=-3x-3a$ $3x-3a=4x+4a$

$4x+3x=-3a+4a$ $3x-4x=4a+3a$

$7x=a$ $-x=7a$

$x=\frac{a}{7}$ $x=-7a$

$\therefore$ **Solution Set** = $\left\{-7a\ ,\ \frac{a}{7}\right\}$

Q.10 $x^4-2x^3-2x^2+2x+1=0$

(A.B + K.B)

Solution:

$x^4-2x^3-2x^2+2x+1=0$

Divide by x^2

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0\to(\text{i})$$

Let $x-\frac{1}{x}=\text{a}\to(\text{ii})$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(a)^2$$

$$x^2+\frac{1}{x^2}-2=a^2$$

$$x^2+\frac{1}{x^2}=a^2+2$$

By putting values in equation (i)

$$\left(a^2+2\right)-2a-2=0$$

$a^2+2-2a-2=0$

$a^2-2a=0$

$a(a-2)=0$

Either

$a=0$ or $a-2=0$

$a=2$

putting the value of a in equation (ii)

when a = 0 when a = 2

$x-\frac{1}{x}=0\to(\text{iii})$ $x-\frac{1}{x}=2\to(\text{iv})$

Equation (iii) $\Rightarrow x=\frac{1}{x}$

$x^2=1$

Taking square root on both sides

$x=\pm1$

Equation (iv) $\Rightarrow$

$x^2-1=2x$

$x^2-2x-1=0$

Here

$a=1, b=-2, c=-1$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$=\frac{2\pm\sqrt{4+4}}{2}$$

$$=\frac{2\pm\sqrt{8}}{2}$$

$$=\frac{2\pm2\sqrt{2}}{2}$$

$$=\frac{2\left(1\pm\sqrt{2}\right)}{2}$$

$$=1\pm\sqrt{2}$$

$\therefore$ **Solution Set** = $\left\{\pm1, 1\pm\sqrt{2}\right\}$

Q.11 $2x^4+x^3-6x^2+x+2=0$

(A.B + K.B)

Solution:

$2x^4+x^3-6x^2+x+2=0 \rightarrow (\text{i})$

Dividing by x^2, we get

$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$

$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$

Let $x+\frac{1}{x}=y \rightarrow (\text{ii})$

Squaring both sides

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2$

$x^2+\frac{1}{x^2}=y^2-2$

Putting the values in equation (i)

$2(y^2-2)+y-6=0$

$2y^2-4+y-6=0$

$2y^2+y-10=0$

$2y^2+5y-4y-10=0$

$y(2y+5)-2(2y+5)=0$

$(2y+5)(y-2)=0$

Either

$2y+5=0$ or $y-2=0$

$2y=-5,$ $\quad y=2$

$y=\frac{-5}{2}$

Putting the values of y in equation (ii)

When $y=\frac{-5}{2}$

$x+\frac{1}{x}=y$

$x+\frac{1}{x}=\frac{-5}{2}$

$\frac{x^2+1}{x}=\frac{-5}{2}$

$2x^2+2=-5x$

$2x^2+5x+2=0$

$2x^2+4x+x+2=0$

$2x(x+2)+1(x+2)=0$

$(x+2)(2x+1)=0$

Either

$x+2=0$ or $2x+1=0$

$x=-2$ $\quad 2x=-1$

$x=\frac{-1}{2}$

When $y=2$

$x+\frac{1}{x}=y$

$x+\frac{1}{x}=2$

$\frac{x^2+1}{x}=2$

$x^2+1=2x$

$x^2-2x+1=0$

$(x-1)^2=0$

Taking square root on both sides

$x-1=0$

$x=1$

$\therefore$ **Solution Set** $=\left\{1,-2,-\frac{1}{2}\right\}$

Q.12 $4.2^{2x+1}-9.2^x+1=0$ **(A.B + K.B)**

Solution:

$4.2^{2x+1}-9.2^x+1=0$

$4.2^{2x}.2-9.2^x+1=0$

$8.(2^x)^2-9.2^x+1=0 \rightarrow (\text{i})$

Let $2^x=y \rightarrow (ii)$

Equation $(\text{i}) \Rightarrow$

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(y-1)(8y-1)=0$

Either

$y-1=0$ or $8y-1=0$

$y=1$ $\quad 8y=1$

$y=\frac{1}{8}$

Putting the value of y in equation (ii)

when $y=1$, when $y=\frac{1}{8}$

$2^x = y$, $2^x = y$

$2^x = 1$, $2^x = \frac{1}{8}$

$2^x = 2^o$, $2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

$\therefore$ Bases are same

$\Rightarrow x=0$ $x=-3$

$\therefore$ **Solution Set** $=\{0,-3\}$

Q.13 $3^{2x+2} = 12.3^x - 3$ **(A.B + K.B)**

Solution:

$3^{2x+2} = 12.3^x - 3$

$3^{2x+2} - 12.3^x + 3 = 0$

$3^{2x}.\ 3^2 - 12.3^x + 3 = 0$

$9(3^x)^2 - 12.3^x + 3 = 0 \rightarrow (\text{i})$

Let $3^x = y \rightarrow (\text{ii})$

Putting $3^x = y$ in equation (i)

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(y-1)(9y-3) = 0$

Either

$y - 1 = 0$ or $9y - 3 = 0$

$y = 1$ $9y = 3$

$y = \frac{3}{9}$

$y = \frac{1}{3}$

Putting the value of y in equation (ii)

when $y=1$ when $y=\frac{1}{3}$

$3^x = y$ $3^x = y$

$3^x = 1$ $3^x = \frac{1}{3}$

$3^x = 3^0$ $3^x = \frac{1}{3^1}$

$3^x = 3^{-1}$

$\therefore$ Bases are same

$x = 0$ $x = -1$

$\therefore$ **Solution Set** $=\{0,-1\}$

Q.14 $2^x + 64\times 2^{-x} - 20 = 0$ **(A.B + K.B)**

Solution:

$2^x + 64\times 2^{-x} - 20 = 0$

$2^x + 64\times 2^{-x} - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0 \rightarrow (\text{i})$

Let $2^x = y \rightarrow (\text{ii})$

Put $2^x = y$ in equation (i)

$y + \frac{64}{y} - 20 = 0$

Multiplying throughout by y

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 4y - 16y + 64 = 0$

$y(y-4) - 16(y-4) = 0$

$(y-4)(y-16) = 0$

Either

$y - 4 = 0$ or $y - 16 = 0$

$y = 4$ $y = 16$

Putting the value of y in equation (ii)

when $y = 4$ when $y = 16$

$2^x = y$ $2^x = y$

$2^x = 4$ $2^x = 16$

$2^x = 2^2$ $2^x = 2^4$

$x = 2$ $x = 4$

$\therefore$ **Solution Set** $=\{2,4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7) = 192$

(A.B + K.B + U.B)

Solution:

$(x+1)(x+3)(x-5)(x-7) = 192$

$\because (1-5=-4 \quad 3-7=-4)$

$(x+1)(x-5)(x+3)(x-7) = 192$

$(x^2-4x-5)(x^2-4x-21) = 192 \rightarrow (\text{i})$

Let $x^2 - 4x = y \rightarrow (\text{ii})$

Equation (i) $\Rightarrow$

$(y-5)(y-21)=192$

$y^2-26y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2-29y+3y-87=0$

$y(y-29)+3(y-29)=0$

$(y-29)(y+3)=0$

Either

$y-29=0$ or $y+3=0$

$y=29$ or $y=-3$

Putting the values of y in equation (ii)

when $y=29$

$x^2-4x=y$

$x^2-4x=29$

$x^2-4x-29=0$

$a=1, b=-4, c=-29$

Using quadratic formula

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{29}$

$x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4\times1\times(-29)}}{2\times1}$

$x=\dfrac{4\pm\sqrt{16+116}}{2}$

$x=\dfrac{4\pm\sqrt{132}}{2}$

$x=\dfrac{4\pm\sqrt{4\times33}}{2}$

$x=\dfrac{4\pm2\sqrt{33}}{2}=2\left(\dfrac{2\pm\sqrt{33}}{2}\right)$

$x=2\pm\sqrt{33}$

when $y=-3$

$x^2-4x=y$

$x^2-4x=-3$

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

Either

$x-3=0$ or $x-1=0$

$x=3$, $x=1$

$\therefore$ **Solution Set** = $\{1,3,2\pm\sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5)+360=0$

(U.B + K.B)

Solution:

$(x-1)(x-2)(x-8)(x+5)+360=0$

$(x^2-2x-x+2)(x^2-8x+5x-40)+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0\rightarrow$(i)

Let $x^2-3x=a\rightarrow$(ii)

Equation (i) $\Rightarrow$

$(a+2)(a-40)+360=0$

$a^2-40a+2a-80+360=0$

$a^2-38a+280=0$

$a^2-28a-10a+280=0$

$a(a-28)-10(a-28)=0$

$(a-28)(a-10)=0$

Either

$a-28=0$ or $a-10=0$

$a=28$ $a=10$

Putting the values of a in equation (ii)

When $a=28$

$x^2-3x=28$

$x^2-3x-28=0$

$x^2-7x+4x-28=0$

$x(x-7)+4(x-7)=0$

$(x-7)(x+4)=0$

Either

$x-7=0$ or $x+4=0$

$x=7$ $x=-4$

when $a=10$

$x^2-3x=10$

$x^2-3x-10=0$

$x^2-5x+2x-10=0$

$x(x-5)+2(x-5)=0$

$(x-5)(x+2)=0$

Either

$x-5=0$ or $x+2=0$

$x=5$ $x=-2$

$\therefore$ **Solution Set** $=\{7,-4,5,-2\}$