Mathematics-10
Exercise - 1.4

Radical Equation **(K.B)**

(LHR 2016, GRW 2014, 17, BWP 2015, 17, MTN 2015, SWL 2016, SGD 2017, D.G.K 2016, 17)

An equation in which a variable or an algebraic expression occurs under radical sign is called radical equation.

e.g. $\sqrt{ax+b}=cx+d,\ 2\sqrt{x}-3=0$ etc.

Extraneous Root **(K.B)**

Value of variable obtained by solving the equation not satisfying it is called extraneous root.

Note **(U.B + K.B)**

- Roots of radical equation must be verified.
- Extraneous is introduced by either squaring the given equation or clearing it of fractions.

Type 1 **(K.B)**

Equation of the type:

$\sqrt{ax+b}=cx+d$

Example 1: (Page # 12)

Solve the equation $\sqrt{3x+7}=2x+3$

Solution:

$$\left(\sqrt{3x+7}\right)^2=(2x+3)^2$$

$$3x+7=4x^2+12x+9$$

$$4x^2+9x+2=0$$

$$x=\frac{-9\pm\sqrt{(9)^2-4(4)(2)}}{2(4)}$$

$$=\frac{-9\pm\sqrt{81-32}}{8}$$

$$=\frac{-9\pm\sqrt{49}}{8}$$

$$=\frac{-9\pm7}{8}$$

Either

$$x=\frac{-9+7}{8} \quad \text{or} \quad x=\frac{-9-7}{8}$$

$$=\frac{-2}{8} \qquad\qquad =\frac{-16}{8}$$

$$=\frac{-1}{4} \qquad\qquad =-2$$

Checking:

Putting $x=-\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7}=2\left(-\frac{1}{4}\right)+3$$

$$\sqrt{\frac{-3+28}{4}}=-\frac{1}{2}+3$$

$$\sqrt{\frac{25}{4}}=\frac{5}{2}$$

$$\frac{5}{2}=\frac{5}{2} \quad \text{(True)}$$

Now putting $x=-2$ in equation (i)

$$\sqrt{3(-2)+7}=2(-2)+3$$

$$\sqrt{1}=-1 \quad \text{(False)}$$

Thus, the solution set is $\left\{-\frac{1}{4}\right\}$**.** -2 **is an extraneous root.**

Type 2

Equation of the type:

$\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Example 2: (Page # 13)

Solve the equation

$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$

(A.B + K.B + U.B)

Solution:

$$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$$

Squaring both sides

$$x+3+x+6+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$2\sqrt{x^2+9x+18}=-x+2$$

Squaring both sides of the equation (ii), we get

$$4\left(x^2+9x+18\right)=x^2-4x+4$$

$$3x^2+40x+68=0$$

Applying quadratic formula,

$$x=\frac{-40\pm\sqrt{(40)^2-4(3)(68)}}{2(3)}$$

$$=\frac{-40\pm\sqrt{1600-816}}{6}$$

$$=\frac{-40\pm\sqrt{784}}{6}$$

$$=\frac{-40\pm 28}{6}$$

Either

$x=\frac{-40+28}{6}$ or $x=\frac{-40-28}{6}$

$=\frac{-12}{6}$ or $=\frac{-68}{6}$

$=-2$ $\quad$ $=\frac{-34}{3}$

Checking:

Putting $x=-2$ in the equation (i)

$$\sqrt{-2+3}+\sqrt{-2+6}=\sqrt{-2+11}$$

$$\sqrt{1}+\sqrt{4}=\sqrt{9}$$

$$1+2=3$$

$3=3$ (True)

Now, putting $x=\frac{-34}{3}$ in the equation (i)

$$\sqrt{\frac{-34}{3}+3}+\sqrt{\frac{-34}{3}+6}=\sqrt{\frac{-34}{3}+11}$$

$$\sqrt{\frac{-34+9}{3}}+\sqrt{\frac{-34+18}{3}}=\sqrt{\frac{-34+33}{3}}$$

$$\sqrt{\frac{-25}{3}}+\sqrt{\frac{-16}{3}}=\sqrt{\frac{-1}{3}}$$

$$\frac{5i}{\sqrt{3}}+\frac{4i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$\frac{9i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ (False)

Thus, solution set is $\{-2\}$, $\frac{-34}{3}$ is an extraneous root.

Type 3

Equation of the type:

$$\sqrt{x^2+px+m}+\sqrt{x^2+px+n}=q$$

Example 3: (Page # 14)

Solve the equation

$$\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3$$

(A.B + K.B + U.B)

Solution:

$$\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3 \rightarrow \text{(i)}$$

Let $x^2-3x=y \rightarrow$ (ii)

Equation (i) $\Rightarrow$

$$\sqrt{y+36}-\sqrt{y+9}=3$$

$\Rightarrow x=3$

Squaring both sides

$$\left(\sqrt{y+36}-3\right)^2=\left(\sqrt{y+9}\right)^2$$

$$\left(\sqrt{y+36}\right)^2+(3)^2-2(3)\sqrt{y+36}=y+9$$

$$y+36+9-6\sqrt{y+36}=y+9$$

$$36=6\sqrt{y+36}$$

$$6=\sqrt{y+36}$$

Again squaring both sides

$36=y+36$

$\Rightarrow y=0$

Putting the value of y in equation (ii)

$$x^2-3x=0$$

$$x(x-3)=0$$

Either

$x=0$ or $x-3=0$

$\Rightarrow\ x=3$

On checking it is found that 0 and 3 are roots of the given equation.

Thus, solution set $=\{0,3\}$

Exercise 1.4

Solve the following equations.

Q.1 $2x+5=\sqrt{7x+16}$ **(A.B)**

Solution:

$2x+5=\sqrt{7x+16} \longrightarrow$ (i)

Taking square on both sides

$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$

$4x^2+20x+25=7x+16$

$4x^2+20x-7x+25-16=0$

$4x^2+13x+9=0$

$4x^2+9x+4x+9=0$

$x(4x+9)+1(4x+9)=0$

$(4x+9)(x+1)=0$

Either

$4x+9=0$ or $x+1=0$

$4x=-9 \quad x=-1$

$x=\frac{-9}{4}$

Check

Put $x=\frac{-9}{4}$ in equation (i)

$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$

$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$

$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$

$\frac{2}{4}=\sqrt{\frac{1}{4}}$

$\frac{1}{2}=\frac{1}{2}$ Satisfied

Put $x=-1$ in equation (i)

$2(-1)+5=\sqrt{7(-1)+16}$

$-2+5=\sqrt{-7+16}$

$3=\sqrt{9}$

$3 = 3$ Satisfied

$\therefore$ **Solution Set** = $\left\{\frac{-9}{4},-1\right\}$

Q.2 $\sqrt{x+3}=3x-1$ **(FSD 2014)** **(A.B)**

Solution:

$\sqrt{x+3}=3x-1 \longrightarrow$(i)

Taking square on both sides

$\left(\sqrt{x+3}\right)^2=(3x-1)^2$

$x+3=9x^2-6x+1$

$-9x^2+x+6x+3-1=0$

$-9x+7x+2=0$

$-(9x-7x-2)=0$

$9x-7x-2=0$

$9x^2-9x+2x-2=0$

$9x(x-1)+2(x-1)=0$

$(x-1)(9x+2)=0$

Either

$x-1=0$ or $9x+2=0$

$x=1 \quad 9x=-2$

$x=\frac{-2}{9}$

Check

Put $x = 1$ in equation (i)

$\sqrt{(1)+3}=3(1)-1$

$\sqrt{4}=3-1$

2 = 2Satisfied

Put $x=\frac{-2}{9}$ in equation (i)

$\sqrt{\left(\frac{-2}{9}\right)+3}=3\left(\frac{-2}{9}\right)-1$

$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$

$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$

$\frac{5}{3}=\frac{-15}{9}$

$\frac{5}{3}=\frac{-5}{3}$ Not Satisfied

$\therefore$ **Solution Set = {1},**

$\frac{-2}{9}$ **is extraneous root.**

Q.3 $4x+3=\sqrt{13x+14}$ **(A.B)**

Solution:

$4x+3=\sqrt{13x+14} \rightarrow (\text{i})$

Squaring both sides, we get

$(4x+3)^2=\left(\sqrt{13x+14}\right)^2$

$(4x)^2+2(4x)(3)+(3)^2=13x+14$

$16x^2+24x+9-13x-14=0$

$16x^2+11x-5=0$

$16x^2+16x-5x-5=0$

$16x(x+1)-5(x+1)=0$

$(x+1)(16x-5)=0$

Either

$x+1=0$ or $16x-5=0$

$x=-1$ $16x=5$

$x=\frac{5}{16}$

Check

Put $x=-1$ in equation (i)

$4x=\sqrt{13x+14}-3$

$4(-1)=\sqrt{13(-1)+14}-3$

$-4=\sqrt{-13+14}-3$

$-4=\sqrt{1}-3$

$-4=1-3$

$-4=-2$ (Not true)

Put $x=\frac{5}{16}$ in equation (i)

$4x=\sqrt{13x+14}-3$

$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14}-3$

$\frac{5}{4}=\sqrt{\frac{65}{16}+14}-3$

$\frac{5}{4}=\sqrt{\frac{65+224}{16}}-3$

$\frac{5}{4}=\sqrt{\frac{289}{16}}-3$

$\frac{5}{4}=\frac{17}{4}-3$

$\frac{5}{4}=\frac{17-12}{4}$

$\frac{5}{4}=\frac{5}{4}$ True

$\therefore$ **Solution Set** $=\left\{\frac{5}{16}\right\}$,

Extraneous root = -1

Q.4 $\sqrt{3x+100}-x=4$ **(A.B)**

Solution:

$\sqrt{3x+100}-x=4 \rightarrow (\text{i})$

$\sqrt{3x+100}=4+x$

Taking square on both sides

$\left(\sqrt{3x+100}\right)^2=(4+x)^2$

$3x+100=16+8x+x^2$

$-x^2+3x-8x+100-16=0$

$-x^2-5x+84=0$

$-(x^2+5x-84)=0$

Or $x^2+5x-84=0$

$x^2+12x-7x-84=0$

$x(x+12)-7(x+12)=0$

$(x-7)(x+12)=0$

Either

$x-7=0$ or $x+12=0$

$x=7$ $x=-12$

Check

Put x = 7 in equation (i)

$\sqrt{3(7)+100}-7=4$

$\sqrt{21+100}-7=4$

$\sqrt{121}-7=4$

$11-7=4$

4 = 4 Satisfied

Put x = -12 in equation (i)

$\sqrt{3(-12)+100}-(-12)=4$

$\sqrt{-36+100}+12=4$

$\sqrt{64}+12=4$

8 + 12 = 4

20 = 4Not satisfied

$\therefore$ **Solution Set** $=\{7\}, -12$ **is extraneous root.**

Q.5 $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ **(A.B + U.B)**

Solution:

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \rightarrow$ (i)

Taking square on both sides

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$x+5+x+21+2\sqrt{(x+5)(x+21)}=x+60$$

$$2x+26+2\sqrt{x^2+26x+105}=x+60$$

$$2x-x+26-60=-2\sqrt{x^2+26x+105}$$

$$x-34=-2\sqrt{x^2+26x+105}$$

Again squaring both sides

$$(x-34)^2=\left(-2\sqrt{x^2+26x+105}\right)^2$$

$$x^2-68x+1156=4(x^2+26x+105)$$

$$x^2-68x+1156=4x^2+104x+420$$

$$x^2-4x^2-68x-104x+1156-420=0$$

$$-3x^2-172x+736=0$$

$$-(3x^2+172x-736)=0$$

Or $3x^2+172x-736=0$

As $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here $a=3, b=172, c=-736$

Putting the values

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$=\frac{-172\pm\sqrt{38416}}{6}$$

$$=\frac{-172\pm196}{6}$$

Either

$x=\dfrac{-176-196}{6}$ or $x=\dfrac{-176+196}{6}$

$=\dfrac{-368}{6}$ $\quad x=\dfrac{24}{6}$

$x=\dfrac{-184}{3}$ $\quad x=4$

Check

Put $x=\dfrac{-184}{3}$ in equation (i)

$$\sqrt{\frac{-184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$

$$\frac{13}{\sqrt{3}}\text{i}+\frac{11}{\sqrt{3}}\text{i}=\frac{2}{3}\text{i}$$

$$\frac{24}{\sqrt{3}}i=\frac{2}{3}i$$

Not satisfied

Put $x=4$ in equation (i)

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

3 + 5 = 8

8 = 8 Satisfied

$\therefore$ **Solution Set** $=\{4\}, \dfrac{-184}{3}$

is extraneous root

Q.6 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ **(A.B + U.B)**

Solution:

$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6} \rightarrow$ (i)

Taking square on both sides

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$$
$$x+1+x-2+2\sqrt{x^2-x-2}=x+6$$
$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2x-x-1-6=-2\sqrt{x^2-x-2}$$
$$x-7=-2\sqrt{x^2-x-2}$$

Again taking square on both sides

$$(x-7)^2=\left(-2\sqrt{x^2-x-2}\right)^2$$
$$x^2-14x+49=4(x^2-x-2)$$
$$x^2-14x+49=4x^2-4x-8$$
$$x^2-4x^2-14x+4x+49+8=0$$
$$-3x^2-10x+57=0$$
$$-(3x^2+10x-57)=0$$

Or $3x^2+10x-57=0$

$$3x^2+19x-9x-57=0$$
$$x(3x+19)-3(3x+19)=0$$
$$(x-3)(3x+19)=0$$

Either

$x-3=0$ or $3x+19=0$

$x=3$ $3x=-19$

$$x=\frac{-19}{3}$$

Check

Put x = 3 in equation (i)

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

2 + 1 = 3

3 = 3 Satisfied

Put x $\frac{-19}{3}$ in equation (i)

$$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$$
$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$
$$\sqrt{\frac{-16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{\frac{-1}{3}}$$
$$\frac{4}{\sqrt{3}}i+\frac{5}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$$
$$\frac{4i+5i}{\sqrt{3}}=\frac{1}{\sqrt{3}}i$$

$\frac{9}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$, Not satisfied

$\therefore$ **Solution Set** $=\{3\}$, $\frac{-19}{3}$ **is extraneous root**

Q.7 $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ **(A.B + U.B)**

Solution:

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}\rightarrow(i)$$

Squaring both sides, we get

$$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$
$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$
$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$
$$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$$
$$-2\sqrt{66-17x+x^2}=27-x-17+2x$$
$$-2\sqrt{x^2-17x+66}=x+10$$

Again squaring both sides

$$\left(-2\sqrt{x^2-17x+66}\right)^2=(x+10)^2$$
$$4\left(x^2-17x+66\right)=x^2+20x+100$$
$$4x^2-68x+264-x^2-20x-100=0$$
$$3x^2-88x+164=0$$

$a=3,\quad b=-88,\quad c=164$

Using Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2\times 3}$$
$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$
$$x=\frac{88\pm\sqrt{5776}}{6}$$
$$x=\frac{88\pm 76}{6}$$
$$x=\frac{88+76}{6}\quad,\quad x=\frac{88-76}{6}$$

$x=\frac{164}{6}$, $x=\frac{12}{6}$

$x=\frac{82}{3}$, $x=2$

Check

When $x=2$

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$

$\sqrt{9}-\sqrt{4}=\sqrt{25}$

$3-2=5$

$1=5$ (Not true)

Put $x=\frac{82}{3}$ in equation (i)

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$

$\sqrt{\frac{33-82}{3}}-\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$

$\sqrt{\frac{-49}{3}}-\sqrt{\frac{-64}{3}}=\sqrt{\frac{-1}{3}}$

$\frac{7}{\sqrt{3}}i-\frac{8}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$

$\frac{7i-8i}{\sqrt{3}}=\frac{1}{\sqrt{3}}i$

$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ (Not true)

$\therefore$ **Solution Set** $=\{\ \}$

Extraneous roots $=\frac{82}{3},2$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ **(A.B)**

Solution:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}\rightarrow(i)$

Taking sq. on both sides

$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$

$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$

$4a+x+a-x-2\sqrt{(4a+x)(a-x)}=a$

$5a-2\sqrt{4a^2-3ax-x^2}=a$

$5a-a=2\sqrt{4a^2-3ax-x^2}$

$4a=2\sqrt{4a^2-3ax-x^2}$

Again taking square on both sides

$(4a)^2=(2\sqrt{4a^2-3ax-x^2})^2$

$16a^2=4(4a^2-3ax-x^2)$

$16a^2=16a^2-12ax-4x^2$

$16a^2-16a^2+12ax+4x^2=0$

$12ax+4x^2=0$

$4x(3a+x)=0$

Either

$4x=0$ or $3a+x=0$

$x=\frac{0}{4}$ $x=-3a$

$x=0$

Check

Put $x=0$ in equation (i)

$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$

$\sqrt{4a}-\sqrt{a}=\sqrt{a}$

$2\sqrt{a}-\sqrt{a}=\sqrt{a}$

$\sqrt{a}=\sqrt{a}$ $\therefore$ Satisfied

Put $x=-3a$ in equation (i)

$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$

$\sqrt{a}-\sqrt{4a}=\sqrt{a}$

$\sqrt{a}-2\sqrt{a}=\sqrt{a}$

$-\sqrt{a}=\sqrt{a}$ Not satisfied

$\therefore$ **Solution Set = {0},** $-3a$ **is extraneous root**

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ **(A.B)**

Solution:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1\rightarrow$(i)

Let $x^2+x=y\rightarrow$(ii)

Equation (i) $\Rightarrow \sqrt{y+1}-\sqrt{y-1}=1$

Taking square on both sides

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$$

$$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$$

$$y+1+y-1-2\sqrt{(y+1)(y-1)}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$2y-1=2\sqrt{y^2-1}$$

Again taking square on both sides

$$(2y-1)^2=(2\sqrt{y^2-1})^2$$

$$4y^2-4y+1=4(y^2-1)$$

$$4y^2-4y+1=4y^2-4$$

$$4y^2-4y^2-4y+1+4=0$$

$$-4y+5=0$$

$$-4y=-5$$

$$y=\frac{5}{4}$$

Put $y=\frac{5}{4}$ in equation (ii)

$$x^2+x=\frac{5}{4}$$

$$4\left(x^2+x\right)=5$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

Here a = 4, b = 4, c = -5

As $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Putting the values

$$x=\frac{-(4)\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}$$

$$x=4\frac{(-1\pm\sqrt{6})}{8}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

On checking, we know that these values satisfy the equation.

$\therefore$ **Solution Set** $=\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ **(A.B)**

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3\rightarrow(i)$$

Let $x^2+3x=y\rightarrow(ii)$

Equation (i) $\Rightarrow \sqrt{y+8}+\sqrt{y+2}=3$

Taking square on both sides

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$$

$$(\sqrt{y+8})^2+(\sqrt{y+2})^2+2(\sqrt{y+8})(\sqrt{y+2})=9$$

$$y+8+y+2+2\sqrt{(y+8)(y+2)}=9$$

$$2y+10+2\sqrt{y^2+10y+16}=9$$

$$2y+10-9=-2\sqrt{y^2+10y+16}$$

$$2y+1=-2\sqrt{y^2+10y+16}$$

Again taking square on both sides

$$(2y+1)^2=(-2\sqrt{y^2+10y+16})^2$$

$$4y^2+4y+1=4(y^2+10y+16)$$

$$4y^2+4y+1=4y^2+40y+64$$

$$4y^2-4y^2+4y-40y+1-64=0$$

$$-36y-63=0$$

$$-36y=63$$

$$y=\frac{63}{-36}$$

$$y=\frac{-7}{4}$$

Put $y=\frac{-7}{4}$ in equation (i)

$$x^2+3x=\frac{-7}{4}$$

$4(x^2+3x)=-7$

$4x^2+12x=-7$

$4x^2+12x+7=0$

Here a = 4, b = 12, c = 7

As $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Putting the values

$x=\dfrac{-12\pm\sqrt{(12)^2-4(4)(7)}}{2(4)}$

$x=\dfrac{-12\pm\sqrt{144-112}}{8}$

$x=\dfrac{-12\pm\sqrt{32}}{8}$

$x=\dfrac{-12\pm4\sqrt{2}}{8}$

$x=4\dfrac{(-3\pm\sqrt{2})}{8}$

$x=\dfrac{-3\pm\sqrt{2}}{2}$

On checking, we know that these values satisfy the equation.

$\therefore$ **Solution Set** = $\left\{\dfrac{-3\pm\sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$ **(A.B)**

Solution:

$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5\rightarrow(\text{i})$

Let $x^2+3x=y\rightarrow(\text{ii})$

Put in equation $(\text{i})\Rightarrow$

$\sqrt{y+9}+\sqrt{y+4}=5$

Squaring on both sides, we get

$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$

$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\sqrt{y+9}\sqrt{y+4}=25$

$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$

$2y+13+2\sqrt{y^2+13y+36}=25$

$2\sqrt{y^2+13y+36}=25-13-2y$

$2\sqrt{y^2+13y+36}=12-2y$

$2\sqrt{y^2+13y+36}=2(6-y)$

$\sqrt{y^2+13y+36}=(6-y)$

Again squaring both sides.

$\left(\sqrt{y^2+13y+36}\right)^2=(6-y)^2$

$y^2+13y+36=36+y^2-12y$

$13y+12y=36-36$

$25y=0$

$y=0$

Put in equation (ii)

$x^2+3x=y$

$x^2+3x=0$

$x(x+3)=0$

Either

$x=0$ *or* $x+3=0$

$x=0$ *or* $x=-3$

Check

When $x=0$

$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

$\sqrt{0^2+3(0)+9}+\sqrt{0^2+3(0)+4}=5$

$\sqrt{0+9}+\sqrt{0+4}=5$

$\sqrt{9}+\sqrt{4}=5$

$3+2=5$

$5=5$ (True)

When $x=-3$

$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

$\sqrt{(-3)^2+3(-3)+9}+\sqrt{(-3)^2+3(-3)+4}=5$

$\sqrt{9-9+9}+\sqrt{9-9+4}=5$

$\sqrt{9}+\sqrt{4}=5$

$3+2=5$

$5=5$ (True)

$\therefore$ **Solution Set** $=\{0,-3\}$