10th کلاس Maths کی سپلی (2024) کلیر کرنے کیلیے یہ نوٹس تیار کرلیں
انشاءاللہ پاس ہو جاییں گے۔

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Unit-1

QUADRATIC EQUATIONS

Quadratic Equation (U.B)

(LHR 2014, 16, GRW 2014, FSD 2016, 17, MTN 2015, BWP 2015, D.G.K 2016)

"A polynomial equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation, or an equation of degree two is called quadratic equation".

i.e., $ax^2 + bx + c = 0$ where

Pure Quadratic Equation (U.B)

A quadratic equation in which coefficient of 'x' is zero is called pure quadratic equation.

Methods to Solve a Quadratic Equation (K.B)

(LHR 2017, GRW 2016, 17, SWL 2016, SGD 2013, 14, 15, MTN 2015, 17, RWP, 2016, D.G.K 2014, 17)

There are three methods to solve a quadratic equation.

(i) Factorization method

(ii) Completing square method

(iii) Using quadratic formula

Reciprocal Equation (U.B)

(LHR 2015, 16, 17, FSD 2017, MTN 2015, 17, RWP 2017, BWP 2017, D.G.K 2016, 17)

An equation, which remains unchanged when 'x' is replaced by $\frac{1}{x}$ is called reciprocal equation.

$$ax^4 + bx^3 + cx^2 + bx + a = 0$$

Exponential Equation (U.B)

(LHR 2015, GRW 2017 SWL 2017, SGD 2017, D.G.K 2015, MTN 2016)

An equation, in which a variable or an algebraic expression occurs in exponent is called exponential equation.

e.g. $2^x + 64.2^{-x} - 20 = 0$

Radical Equation (K.B)

(LHR 2016, GRW 2014, 17, BWP 2015, 17, MTN 2015, SWL 2016, SGD 2017, D.G.K 2016, 17)

An equation in which a variable or an algebraic expression occurs under radical sign is called radical equation.

e.g. $\sqrt{ax+b} = cx + d$, $2\sqrt{x} - 3 = 0$ etc.

(ii) Solve by factorization $5x^2 = 15x$

(LHR 2015, 16, GRW 2014, 16, 17, SWL 2016, 17, BWP 2014, 16, D.G.K 2017)

Solution:

$5x^2 = 15x$

$5x^2 - 15x = 0$

$5x(x-3) = 0$

Either

$5x = 0$ or $x - 3 = 0$

$x = 0$ $\quad$ $x = 3$

$\therefore$ **Solution Set** $= \{0, 3\}$

Example 2: Solve $5x^2 = 30x$ by factorization.

Solution: $5x^2 = 30x$

$5x^2 - 30x = 0$ which is factorized as

$5x(x-6) = 0$

Either $5x = 0$ or $x - 6 = 0$ $\Rightarrow x = 0$ or $x = 6$

$\therefore$ $x = 0, 6$ are the roots of the given equation.

Thus, the solution set is $\{0, 6\}$.

Q.1 Write the following quadratic equations in the standard form and point out pure quadratic equations.

(SGD 2015, 17, RWP 2016) (A.B)

(i) $(x+7)(x-3) = -7$

Solution:

$(x+7)(x-3) = -7$

$x^2 + 7x - 3x - 21 = -7$

$x^2 + 4x - 21 + 7 = 0$

$x^2 + 4x - 14 = 0$

Which is the required standard form of quadratic equation.

Unit-2

THEORY OF QUADRATIC EQUATIONS

Exercise 2.1

Q.1 Find the discriminant of the following given quadratic equations:

Solution:

(i) $2x^2 + 3x - 1 = 0$ **(A.B)**

(GRW 2017, FSD 2016, MTN 2014, D.G.K 2016)

By comparing given equation with $ax^2 + bx + c = 0$**, we get**

$a = 2,\ b = 3,\ c = -1$

Disc $= b^2 - 4ac$

$= (3)^2 - 4(2)(-1)$

$= 9 + 8$

$= 17$

(ii) $6x^2 - 8x + 3 = 0$ **(A.B)**

(LHR 2016, SWL 2016, D.G.K 2015, 17)

By comparing given equation with $ax^2 + bx + c = 0$**, we get**

$a = 6,\ b = -8,\ c = 3$

Disc $= b^2 - 4ac$

$= (-8)^2 - 4(6)(3)$

$= 64 - 72$

$= -8$

(iii) $9x^2 - 30x + 25 = 0$ **(A.B)**

(LHR 2017, MTN 2015)

By comparing given equation with $ax^2 + bx + c = 0$**, we get**

$a = 9,\ b = -30,\ c = 25$

Disc $= (-30)^2 - 4(9)(25)$

$= 900 - 900$

$= 0$

(iv) $4x^2 - 7x - 2 = 0$ **(A.B)**

(GRW 2014, SGD 2017, MTN 2016)

By comparing given equation with $ax^2 + bx + c = 0$**, we get**

$a = 4,\ b = -7,\ c = -2$

(i) Discuss the nature of the roots of the following equations. (A.B)

Solution:

(a) $x^2 + 3x + 5 = 0$

Here $a = 1, b = 3, c = 5$

Disc $= b^2 - 4ac$

$= (3)^2 - 4(1)(5)$

$= 9 - 20$

$= -11$

< 0

$\therefore$ Roots are complex conjugate or imaginary.

(b) $2x^2 - 7x + 3 = 0$ **(A.B)**

(GRW 2016, SGD 2014, RWP 2017, D.G.K 2016)

Here $a = 2, b = -7, c = 3$

Disc $= b^2 - 4ac$

$= (-7)^2 - 4(2)(3)$

$= 49 - 24$

$= 25$

Since disc >0 and perfect square roots are rational and unequal.

(c) $x^2 + 6x - 1 = 0$ **(A.B)**

Here $a = 1, b = 6, c = -1$

Disc $= b^2 - 4ac$

$= (6)^2 - 4(1)(-1)$

$= 36 + 4$

$= 40$

Since Disc. >0 and not a perfect square roots are irrational and unequal.

(d) $16x^2 - 8x + 1 = 0$ **(FSD 2017) (A.B)**

Here $a = 16, b = -8, c = 1$

Disc. $= b^2 - 4ac$

$= (-8)^2 - 4(16)(1)$

$= 64 - 64$

$= 0$

Since, Disc. = 0, roots are rational and equal.

Disc $= b^2 - 4ac$

$= (-7)^2 - 4(4)(-2)$

$= 49 + 32$

$= 81$

$h = -1$

Exercise 2.3

Q.1 Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2 - 5x + 3 = 0$ **(MTN 2017)** **(A.B)**

Here $a = 1, b = -5, c = 3$

Sum of roots $= -\frac{b}{a}$

$= -\frac{(-5)}{1}$

$= 5$

Product of roots $= \frac{c}{a}$

$= \frac{3}{1}$

$= 3$

(ii) $3x^2 + 7x - 11 = 0$ **(A.B)**

(LHR 2017, SWL 2017, SGD 2016, D.G.K 2014)

Here

$a = 3,\ b = 7,\ c = -11$

Sum of roots $= -\frac{b}{a}$

$= -\frac{7}{3}$

Product of roots $= \frac{c}{a}$

$= \frac{-11}{3}$

$= -\frac{11}{3}$

(iii) $px^2 - qx + r = 0$ **(A.B)**

(LHR 2014, GRW 2014, SWL 2016, MTN 2017, SGD 2016)

Here $a = p, b = -q, c = r$

Sum of roots $= -\frac{b}{a}$

$= -\frac{(-q)}{p}$

$= \frac{q}{p}$

Product of roots $= \frac{c}{a}$

$= \frac{r}{p}$

(iv) $(a+b)x^2 - ax + b = 0$ **(A.B**

(BWP 2014, 1

Here

$A = a+b,\ B = -a,\ C = b$

Sum of roots $= -\frac{B}{A}$

$= -\frac{-a}{a+b}$

$= \frac{a}{a+b}$

Product of roots $= \frac{C}{A}$

$= \frac{b}{a+b}$

(v) $(l+m)x^2 + (m+n)x + n - 1 = 0$

Here

$a = l+m,\quad b = m+n,\quad c = n-l$

Sum of roots $= S = -\frac{b}{a}$

$= -\frac{m+n}{l+m}$

Product of roots $= P = \frac{c}{a}$

$= \frac{n-l}{l+m}$

(vi) $7x^2 - 5mx + 9n = 0$ **(A.B**

Here

$a = 7, b = -5\text{m}, c = 9n$

Sum of roots $= -\frac{b}{a}$

$= -\frac{-5\text{m}}{7}$

$= \frac{5\text{m}}{7}$

Product of roots $= \frac{c}{a}$

$= \frac{9n}{7}$

Q.2 Find the value of k, if

Q.1 Write the quadratic equations having following roots.

(a) 1, 5 **(K.B + A.B)**
(b) 4, 9 **(K.B + A.B)**
(c) –2, 3 **(K.B + A.B)**
(d) 0, –3 **(K.B + A.B)**
(e) 2, –6 **(K.B + A.B)**
(f) –1, –7 **(K.B + A.B)**
(g) $1+i, 1-i$ **(K.B + A.B)**
(h) $3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**

Solution:

(a) Roots of required equation are 1, 5
Then sum of roots = S = 1 + 5 = 6
And product of roots = P = 1 × 5 = 5
∴ Required quadratic equation is:
$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 5 = 0$$

(b) **(FSD 2016, 17, RWP 2017, RWP 2017)**
Roots of required equation are 4, 9
Then sum of roots = S = 4 + 9 = 13
And product of roots $= P = 4 \times 9 = 36$
∴ Required quadratic equation is:
$$x^2 - Sx + P = 0$$
$$x^2 - 13x + 36 = 0$$

(c) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are –2, 3
Then sum of roots = S = –2 + 3 = 1
And product of roots = P = –2(3) = –6
∴ Required quadratic equation is:
$$x^2 - Sx + P = 0$$
$$x^2 - 1x + (-6) = 0$$
(K.B + A.B)
$$x^2 - x - 6 = 0$$

(d) **(SGD 2014, BWP 2017)**
Roots of required equation are 0, –3
Then sum of roots = S = 0 + (–3) = –3
And product of roots = P = 0(–3) = 0
∴ Required quadratic equation is:
$$x^2 - Sx + P = 0$$
$$x^2 - (-3)x + 0 = 0$$ **(K.B + A.B)**
$$x^2 + 3x = 0$$

(e) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are 2, –6
Sum of roots = S = 2 + (–6) = –4
Product of roots = P = 2(–6) = –12
∴ Required quadratic equation is:
$$x^2 - Sx + P = 2$$
$$x^2 - (-4)x + (-12) = 0$$
$$x^2 + 4x - 12 = 0$$ **(K.B + A.B)**

(f) **(LHR 2015, 17, RWP 2016)**
Roots of required equation are –1, –7
Sum of roots = S = –1 + (–7) = –8
Product of roots = P = –1(–7) = 7
∴ Required quadratic equation is:
$$x^2 - Sx + P = 0$$
$$x^2 - (-8)x + 7 = 0$$ **(K.B + A.B)**
$$x^2 + 8x + 7 = 0$$

(g) **(D.G.K 2014, SGD 2017)**
$1+i, 1-i$ **(K.B + A.B)**
Roots of the required equation are $1+i,\ 1-i$

Sum of roots $= S = (1+i) + (1-i)$
$= 1 + i + 1 - i$
$= 2$

Product of roots $= P = (1+i)(1-i)$
$= (1)^2 - (i)^2$
$= 1 - i^2$
$= 1 - (-1)$
$= 1 + 1$
$= 2$

Required quadratic equation is
$$x^2 - Sx + P = 0$$
$$x^2 - 2x + 2 = 0$$

(h) Roots of required equation are $3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**

Sum of roots $= (3+\sqrt{2}) + (3-\sqrt{2})$
$= 3 + \sqrt{2} + 3 - \sqrt{2}$
$= 6$

Product of roots $= (3+\sqrt{2})(3-\sqrt{2})$
$= (3)^2 - (\sqrt{2})^2$
$= 9 - 2$
$= 7$

Unit-3

VARIATIONS

Ratio: A relation between two quantities of the same kind is called *ratio*.

Proportion: A *proportion* is a statement, which is expressed as equivalence of two ratios.

Direct variation: If two quantities are related in such a way that when one changes in any ratio so does the other is called *direct variation*.

Inverse variation If two quantities are related in such a way that when one quantity increases, the other decreases is called *inverse variation*.

Joint variation: A combination of direct and inverse variations of one or more than one variables forms *joint variation*.

Exercise 3.1

Q.3 If $3(4x-5y)=2x-7y$, find the ratios $x:y$. **(A.B)**

Given

$3(4x-5y)=2x-7y$

Required **(LHR 2015)**

$x:y=?$ **(MTN 2016)**

Solution:

Here

$3(4x-5y)=2x-7y$

$12x-15y=2x-7y$

$12x-2x=15y-7y$

$10x=8y$

$\frac{x}{y}=\frac{8}{10}$

$\frac{x}{y}=\frac{4}{5}$

$\Rightarrow x:y=4:5$

Result:

$x:y=4:5$

Q.4 Find the value of p, if the ratios $2p+5:3p+4$ and $3:4$ are equal. Find value of 'p' **(A.B)**

(GRW 2015, SWL 2016, RWP 2015, 17)

Solution:

According to given condition.

$2p+5:3p+4=3:4$

$\frac{2p+5}{3p+4}=\frac{3}{4}$

By cross multiplication

$4(2p+5)=3(3p+4)$

$8p+20=9p+12$

$8p-9p=12-20$

$-P=-8$

$\Rightarrow P=8$

Result:

$p=8$

Q.5 If the ratios $3x+1:6+4x$ and $2:5$ are equal. Find the value of x.

Solution: **(D.G.K 2015)** **(A.B + K.B)**

Here

$3x+1:6+4x=2:5$

$\Rightarrow \frac{3x+1}{6+4x}=\frac{2}{5}$

By cross multiplication, we get

$5(3x+1)=2(6+4x)$

$15x+5=12+8x$

$15x-8x=12-5$

$7x=7$

$x=\frac{7}{7}$

$x=1$

Result

$x=1$

Exercise 3.3

Q.1 Find a third proportional to

(i) 6, 12 **(A.B)**

(SGD 2017, RWP 2017, D.G.K 2016, 17)

Let, the third proportional $= a$

According to the given condition;

$6:12::12:a$

$\because$ Product of means = product of extremes

$12(12) = 6a$

$144 = 6a$

$\frac{144}{6} = a$

$\Rightarrow a = 24$

$\therefore$ **Third proportional is** 24

(ii) $a^3, 3a^2$ **(A.B)**

(MTN 2014, 16, RWP 2017, D.G.K 2014)

Let, third proportional $= x$

According to the given condition;

$a^3:3a^2::3a^2:x$

$\because$ Product of extremes = Product of means

$(a^3)x=(3a^2)(3a^2)$

$a^3x=9a^4$

$x=\frac{9a^4}{a^3}$

$x=9a$

$\therefore$ **Third proportional is** $9a$

(iii) $a^2-b^2, a-b$ **(A.B)** **(LHR 2015, GRW 2014, 16, SWL 2016, BWP 2015)**

Let, third proportional $= x$

According to the given condition;

$a^2-b^2:a-b::a-b:x$

$\because$ Product of extremes = product of means

$(a^2-b^2)(x)=(a-b)^2$

$(a-b)(a+b)(x)=(a-b)^2$

$(a+b)x=a-b$

$x=\frac{a-b}{a+b}$

$\therefore$ **Third proportional is** $\frac{a-b}{a+b}$

Q.2 Find a fourth proportional to

(i) 5, 8, 15 **(A.B)**

(LHR 2014, GRW 2017, BWP 2016)

Let, the fourth proportional = x

According to the given condition:

$5:8::15:x$

$\because$ Product of mean = product of extreme

$(5)x = 8(15)$

$5x = 120$

$x = \frac{120}{5}$

$x = 24$

$\therefore$ **Fourth proportional is** 24

Q.3 Find a mean proportional between

(i) 20, 45 **(A.B)**

(LHR 2016, GRW 2014, D.G.K 2016)

Let, the mean proportional = x

According to the given condition;

$20:x::x:45$

$\because$ Product of means = Product of extremes

$(x)(x) = (20)(45)$

$x^2 = 900$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{900}$

$x = \pm 30$

$\therefore$ **The mean proportional is** ± 30

Unit-4

PARTIAL FRACTIONS

Identity (K.B)

(GRWP 2014, 15, 17, RWP 2016, SGD 2016, D.G.K 2015, 17)

An identity is an equation, which is satisfied by all the values of the variables involved

For example: $(x+3)^2 = x^2 + 6x + 9$,

Rational Fraction (K.B)

(LHR 2014, 16, GRW 2016, FSD 2015, SGD 2015, 16, MTN 2015, D.G.K 2016)

An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients is called a rational fraction. The polynomial $D(x) \neq 0$

For example $\frac{x^2+4}{x-2}$ where $x \neq 2$

Proper Fraction (K.B)

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq 0$ is called proper fraction, if degree of the polynomial N(x) is less than degree of the polynomial D(x)

For example: $\frac{2}{x+1}, \frac{5x-3}{x^2+4}$ etc.

Improper Fraction (U.B + K.B)

(LHR 2014, 15, GRW 2014, 17, FSD 2015, SGD 2017, RWP 2017, MTN 2015)

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq 0$ is called an improper fraction, if degree of the polynomial N(x) is greater than or equal to degree of the polynomial D(x).

For example: $\frac{5x}{x+2}, \frac{6x^4}{x^3+1}$ etc.

Partial Fraction (K.B)

(LHR 2014, 16, 17, GRW 2015, FSD 2015, 17, RWP 2015, 16, BWP 2015,)

Decomposition of resultant fraction into its components or into different fractions is called partial fraction.

Unit-5

SETS AND FUNCTIONS

Exercise 5.1

Q.1 **(A.B)**

Given

$X = \{1, 4, 7, 9\}$

$Y = \{2, 4, 5, 9\}$

To Find

(i) X∪Y

(ii) X∩Y

(iii) Y∪X

(iv) Y∩X

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$

$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$

$= \{4, 9\}$

Q.3

Given

$X=\phi,\quad Y=Z^{+},\quad T=O^{+}$

To Find

(i) X∪Y
(ii) X∪T
(iii) Y∪T
(iv) X∩Y
(v) X∩T
(vi) Y∩T

Solution:

(i) $X\cup Y=\phi\cup Z^{+}$
$=Z^{+}$

(ii) $X\cup T=\phi\cup O^{+}$
$=O^{+}$

(iii) $Y\cup T=Z^{+}\cup O^{+}$
$=Z^{+}$

(iv) $X\cap Y=\phi\cap Z^{+}$
$=\phi$

(v) $X\cap T=\phi\cap O^{+}$
$=\phi$

(vi) $Y\cap T=Z^{+}\cap O^{+}$
$=O^{+}$

Q.5 Given **(A.B)**

$X=\{2,4,6,.....,20\}$ **(LHR 2014)**

$Y=\{4,8,12,...,24\}$ **(FSD 2015)**

To Find

(i) X – Y
(ii) Y – X

Solution:

(i) $X-Y=\{2,4,6,.....,20\}-\{4,8,12,....,24\}$
$=\{2,6,10,14,18\}$

(ii) $Y-X=\{4,8,12,.....,24\}-\{2,4,6,.....,20\}$
$=\{24\}$

Q.6 Given **(BWP 2014)** **(A.B)**

$A=N,B=W$ **(LHR 2015)**

To Find **(D.G.K 2014)**

(i) A – B
(ii) B – A

Solution:

(i) $A-B=N-W=\{1,2,3\ldots\}-\{0,1,2,3,\ldots\}$
$=\phi$

(ii) $B-A=W-N=\{0,1,2,3,....\}-\{1,2,3,....\}$
$=\{0\}$

Exercise 5.4

Q.1 **Given** $A = \{a,b\}$ **(GRW 2014)** **(A.B)**

$B = \{c,d\}$ **(RWP 2015)**

To Find

(i) $A \times B$

(ii) $B \times A$

Solution:

(i) $A \times B = \{a,b\} \times \{c,d\}$

$= \{(a,c),(a,d),(b,c),(b,d)\}$

(ii) $B \times A = \{c,d\} \times \{a,b\}$

$= \{(c,a),(c,b),(d,a),(d,b)\}$

Q.2 **Given** $A = \{0,2,4\}$ **(A.B)**

$B = \{-1,3\}$

(FSD 2015, SWL 2017, BWP 2015)

To Find

$A \times B \quad B \times A \quad A \times A \quad B \times B$

Solution:

(i) $A \times B = \{0,2,4\} \times \{-1,3\}$

$= \{(0,-1),(0,3),(2,-1),(2,3),(4,-1),(4,3)\}$

(ii) $B \times A = \{-1,3\} \times \{0,2,4\}$

$= \{(-1,0),(-1,2),(-1,4),(3,0),(3,2),(3,4)\}$

(iii) $A \times A = \{0,2,4\} \times \{0,2,4\}$

$= \{(0,0),(0,2),(0,4),(2,0),(2,2),(2,4),(4,0),(4,2),(4,4)\}$

(iv) $B \times B = \{-1,3\} \times \{-1,3\}$

$= \{(-1,-1),(-1,3),(3,-1),(3,3)\}$

Q.3 **(A.B)**

(i) **Given** $(a-4, b-2)=(2,1)$

(GRW 2016, 17, FSD 2017, SWL 2015, SGD 2017, MTN 2016)

Required

Values of a and b

Solution:

Given that

$$(a-4, b-2)=(2,1)$$

By comparing, we get

$a-4=2$ and $b-2=1$

$a=2+4$ $\quad$ $b=1+2$

$\Rightarrow a=6$, $\quad$ $b=3$

(ii) **Given** $(2a+5,3)=(7,b-4)$ **(A.B)**

(SWL 2017, MTN 2017, RWP 2016, D.G.K 2015)

Required

Values of a and b

Solution:

Given that

$$(2a+5, 3)=(7,b-4)$$

By comparing, we get

$2a+5=7$ and $3=b-4$

$2a=7-5$ $\quad$ $3+4=b$

$2a=2$ $\quad$ $7=b$

$a=1$ $\quad$ $b=7$

(iii) **Given** $(3-2a,b-1)=(a-7,2b+5)$

Required

Values of a and b =?

Solution:

Given that

$$(3-2a,b-1)=(a-7,2b+5)$$

By comparing, we get

$3-2a=a-7$ and $b-1=2b+5$

$-2a-a=-7-3$ $\quad$ $b-2b=5+1$

$-3a=-10$ $\quad$ $-b=6$

$a=\dfrac{-10}{-3}$ $\quad$ $b=-6$

$\Rightarrow a=\dfrac{10}{3}$

Q.4 **Given**

$X \times Y = \{(a,a),(b,a),(c,a),(d,a)\}$

(A.B)

Required

Set X and Y

Solution:

Given that

$X \times Y = \{(a,a),(b,a),(c,a),(d,a)\}$

$X = \{a,b,c,d\}$

$Y = \{a\}$

Miscellaneous Exercise 5

(ii) Write all the subsets of the set {a, b}

Answer

Let $S = \{a, b\}$

All possible subset of set S are:

$\phi, \{a\}, \{b\}, \{a, b\}$

(vi) Define a function

Answer

Function **(K.B)**

Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if

(i) Dom $f = A$

(ii) Every $x \in A$ appears in one and only one ordered pair in f.

(vii) Define one-one function

Answer

One – One Function **(K.B)**

A function $f : A \to B$ is called one-one function if all distinct elements of A have distinct images in B, i.e., $f(x_1) = f(x_2)$

(viii) Define an Onto function or Surjective function.

Answer

Onto (Surjective) Function **(K.B)**

A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e., Range of $f = B$.

\

(x) Write De Morgan's Laws. **(K.B)**

Answer

De Morgan's Laws

For any two sets A and B belonging to universal set U,

(i) $(A \cap B)' = A' \cup B'$

(ii) $(A \cup B)' = A' \cap B'$ are called De Morgan's laws.

Unit-6

BASIC STATISTICS

Example 1: Find the modal size of shoe for the following data:

4, 4.5, 5, 6, 6, 6, 7, 7.5, 7.5, 8, 8, 8, 6, 5, 6.5, 7.

Solution: We note the most occurring observation in the given data and find that,

mode = 6.

Example 1: On 5 term tests in mathematics, a student has made marks of 82, 93, 86, 92 and 79. Find the median for the marks.

Solution: By arranging the grades in ascending order, the arranged data is

79 , 82 , 86 , 92 , 93

Since number of observations is odd *i.e.*, $n = 5$.

$\tilde{X}$ = size of $\left(\frac{5+1}{2}\right)$ th observation

$\tilde{X}$ = size of 3^{rd} observation

$\tilde{X} = 86$

Example 1: For the following data find the Harmonic mean.

X	12	5	8	4

Solution:

X	$^1/_X$
12	0.0833
5	0.2
8	0.125
4	0.25
Total	0.6583

$$\text{H.M.} = \frac{4}{0.6583} = 6.076$$

Exercise 6.2

Q.3 Find arithmetic mean by direct method for the following set of data: **(A.B)**

(i) 12, 14, 17, 20, 24, 29, 35, 45.

(ii) 200, 225, 350, 375, 270, 320, 290.

Ans:

(i) Given Data:

12, 14, 17, 20, 24, 29, 35, 45

(LHR 2014, 16, GRW 2017, FSD 2017, RWP 2015, 17, MTN 2016, D.G.K 2014, 16)

Required:

Arithmetic mean by direct method

Solution:

X	12	14	17	20	24	29	35	45	$\sum X = 196$

Formula:

$$\overline{X} = \frac{\Sigma X}{n}$$

$$= \frac{196}{8}$$

$$= 24.5$$

Result

$\overline{X} = 24.5$

(ii) Given Data:

200, 225, 350, 375, 270, 320, 290 **(GRW 2014, 16, D.G.K 2016)**

Required:

Arithmetic mean by direct method

Solution:

X	200	225	350	375	270	320	290	$\Sigma X = 2030$

We known that

$$\overline{X} = \frac{\Sigma X}{n} = \frac{2030}{7}$$

$$= 290$$

Result: $\overline{\mathbf{X}} = \mathbf{290}$

Q.4 The salaries of five teachers in Rupees are as follows. 11500, 12400, 15000, 14500, 14800. Find the range and standard deviation. (A.B)

Given Data:

11500, 12400, 15000, 14500, 14800

Required

(i) Range **(ii)** Standard Derivation

Solution:

X	X^2
11500	132250000
12400	153760000
15000	225000000
14500	210250000
14800	219040000
$\sum X = 68200$	$\sum X^2 = 940300000$

Range **(A.B)**

Max. value = X_m = 15,000

Min. value = X_o = 11,500

Range = $X_m - X_o$ = 15000 – 11500

= 3,500 Rs.

Arithmetic mean *Arithmetic mean* is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number.

Geometric mean *Geometric mean* of a variable X is the n^{th} positive root of the product of the $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ observations. In symbols we write,

Harmonic mean *Harmonic mean* refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ observations.

Mode: *Mode* is defined as the most frequent occurring observation of the variable or data.

Median: *Median* is the measure which determines the middlemost observation in a data set. $Median = L + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$

Dispersion: Statistically, *Dispersion* means the spread or scatterness of observations in a data set.

Range: *Range* measures the extent of variation between two extreme observations of a data set. It is given by the formula:

$$\text{Range} = X_{max} - X_{min} = X_m - X_0$$

Variance: *Variance* is defined as the mean of the squared deviations of x_i (i = 1, 2, ….., n) observations from their arithmetic mean. In symbols,

Exercise 7.1

Q.4 Express the following angles into radians.

(ii) 60° **(A.B)**

$$= 60 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{\pi}{3} \text{ radians}$$

(iii) 135° **(BWP 2014, D.G.K 2016)** **(A.B)**

$$= 135 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{3\pi}{4} \text{ radians}$$

(iv) 225° **(A.B)**

$$= 225 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{5\pi}{4} \text{ radians}$$

(v) -150° **(BWP 2014, D.G.K 2016)** **(A.B)**

$$= -150 \times \frac{\pi}{180} \text{ radians}$$

$$= -\frac{5\pi}{6} \text{ radians}$$

(vi) -225° **(A.B)**

$$= -225 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{-5\pi}{4} \text{ radians}$$

(vii) 300° **(SGD 2015)** **(A.B)**

$$= 300 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{5\pi}{3} \text{ radians}$$

(viii) 315° **(A.B)**

$$= 315 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{7\pi}{4} \text{ radians}$$

(ii) $\frac{5\pi}{6}$ **(A.B)**

(SWL 2014, SGD 2016, MTN 2015, 16)

$$=\left(\frac{5\pi}{6}\times\frac{180}{\pi}\right)^{o}$$

$$=150^{o}$$

(iii) $\frac{7\pi}{8}$ **(A.B)**

$$=\left(\frac{7\pi}{8}\times\frac{180}{\pi}\right)^{o}$$

$$=\frac{315^{o}}{2}$$

$$=157.5^{o}$$

(iv) $\frac{13\pi}{16}$ **(A.B)**

$$=\left(\frac{13\pi}{16}\times\frac{180}{\pi}\right)^{o}$$

$$=\left(\frac{585}{4}\right)^{o}$$

$$=146.25^{o}$$

(v) 3 **(A.B)**

$$=\left(3\times\frac{180}{\pi}\right)^{o}$$

$$=171.89^{o}$$

(vi) 4.5 **(A.B)**

$$=\left(4.5\times\frac{180}{\pi}\right)^{o}=257.83^{o}$$

(vii) $-\frac{7\pi}{8}$ **(A.B)**

(GRW 2017, RWP 2015, D.G.K 2017)

$$=-\left(\frac{7\pi}{8}\times\frac{180}{\pi}\right)^{o}=-157.5^{\circ}$$

(viii) $-\frac{13\pi}{16}$ **(A.B)**

$$=-\left(\frac{13\pi}{16}\times\frac{180}{\pi}\right)^{\circ}=-146.25^{\circ}$$

Exercise 7.2

Q.1

(i) Find θ, when $l = 2cm$, $r = 3.5cm$
(LHR 2015, GRW 2016, BWP 2014, MTN 2015, 16, 17, SGD 2015) **(A.B)**

Solution:

We know that

$l = r\theta$

or $\theta = \frac{l}{r}$

Putting the values

$= \frac{2cm}{3.5cm}$

$\theta = 0.57 \text{radians}$

(ii) Given: **(A.B)**

$l = 4.5m$, $r = 2.5m$

(FSD 2014, SGD 2014, 16, MTN 2016, D.G.K 2015, 17)

Required:

$\theta = ?$

Solution:

We know that

$\theta = \frac{l}{r}$

Putting the values

$\theta = \frac{4.5}{2.5}$

$\Rightarrow \theta = 1.8$ radians

Q.2

(i) Given $\theta = 180^o, r = 4.9cm$ **(A.B)**

(LHR 2014, GRW 2014, FSD 2015, SWL 2016, D.G.K 2016)

Required:

$l = ?$

Solution:

Here

$\theta = 180^o$

$= 180 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= 3.14$ radians

We know that

$l = r\theta$

Putting the values

$= 4.9 \text{ cm} \times 3.14$

$\Rightarrow l = 15.39\text{cm}$

(ii) Find *l*, when $\theta = 60^o 30'$, $r = 15mm$

(LHR 2014, SWL 2016, BWP 2016, MTN 2015)

Solution:

Here

$\theta = 60^o\ 30'$

$= 60^o + \left(\frac{30}{60}\right)^o$

$= 60.5^o$

$= 60.5 \times \frac{\pi}{180}$ radians

$= 1.0559$ radians

We know that

$l = r\theta$

putting the values

$= (15mm)(1.0559)$

$\Rightarrow l = 15.84mm$

Q.3 **(A.B)**

(i) Find *r*, when $l = 4cm, \theta = \frac{1}{4}$ **radian**

Solution:

We know that

$r = \frac{l}{\theta}$

Putting the values

$r = \frac{4cm}{\frac{1}{4}}$

$\Rightarrow r = 16cm$

(ii) Given: $l = 52cm, \theta = 45^o$ **(A.B)**

(LHR 2015, 17, GRW 2017, SWL 2014, 17, RWP 2010, 15, D.G.K 2017)

Required:

$r = ?$

Solution:

Here $\theta = 45^o$

$= 45 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= 0.785$ radians

We know that

$r = \frac{l}{\theta}$

$= \frac{52}{0.785} cm$

$\Rightarrow r = 66.21cm$

Q.4 Given: $r = 12m$, $\theta = 1.5$ radian

(SGD 2014) **(A.B)**

Required:

$l = ?$

Solution:

We know that

$l = r\theta$

Putting the values

$l = (12m)(1.5)$

Exercise 7.4

In problems 1-6, simplify each expression to a single trigonometric function.

Q.1 $\dfrac{\sin^2 x}{\cos^2 x}$ **(K.B + A.B)**

Solution:

$$\frac{\sin^2 x}{\cos^2 x} = \frac{(\sin x)^2}{(\cos x)^2}$$

$$= \left(\frac{\sin x}{\cos x}\right)^2 \quad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= (\tan x)^2$$

$$= \tan^2 x$$

Q.2 $\tan x \sin x \sec x$ **(K.B + A.B)**

Solution:

$$\tan x \sin x \sec x$$

$$= \frac{\sin x}{\cos x} \times \sin x \times \frac{1}{\cos x}$$

$$= \frac{\sin^2 x}{\cos^2 x} \quad \left(\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$= \left(\frac{\sin x}{\cos x}\right)^2 \quad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$$

$$= \tan^2 x$$

Q.3 $\dfrac{\tan x}{\sec x}$ **(K.B + A.B)**

Solution:

$$\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \qquad \because \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\because \sec\theta = \frac{1}{\cos\theta}$$

$$\frac{\tan x}{\sec x} = \frac{\sin x}{\cancel{\cos x}} \times \frac{\cancel{\cos x}}{1}$$

$$= \sin x$$

Q.4 $1-\cos^2 x$ **(K.B + A.B)**

Solution:

$$1-\cos^2 x$$
$$=1-\left(1-\sin^2 x\right) \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$$
$$=\not{1}-\not{1}+\sin^2 x$$
$$=\sin^2 x$$

Q.5 $\sec^2 x-1$ **(LHR 2014)** **(K.B + A.B)**

Solution: $\sec^2 x-1$

$$=\left(\frac{1}{\cos x}\right)^2-1$$
$$=\frac{1}{\cos^2 x}-1$$
$$=\frac{1-\cos^2 x}{\cos^2 x} \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$$
$$=\frac{\sin^2 x}{\cos^2 x}$$
$$=\left(\frac{\sin x}{\cos x}\right)^2 \quad \left(\because \frac{\sin\theta}{\cos\theta}=\tan\theta\right)$$
$$=\tan^2 x$$

Q.6 $\sin^2 x.\cot^2 x$ **(K.B + A.B)**

Solution: $\sin^2 x.\cot^2 x$

$$=\sin^2 x.\left(\frac{\cos x}{\sin x}\right)^2$$
$$=\cancel{\sin^2 x}.\frac{\cos^2 x}{\cancel{\sin^2 x}}$$
$$=\cos^2 x$$

In problems 7-12, verify the identities.

Q.7 $(1-\sin\theta)(1+\sin\theta)=\cos^2\theta$

(K.B + A.B)

(FSD 2015, SWL 2014, SGD 2014)

Proof:

$$\text{L.H.S}=(1-\sin\theta)(1+\sin\theta)$$
$$=(1)^2-(\sin\theta)^2 \quad \left[\because (a+b)(a-b)=a^2-b^2\right]$$
$$=1-\sin^2\theta$$
$$=\cos^2\theta \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$$
$$=\text{R.H.S}$$

Radian: The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle, is called one *radian.*

Circumference: $2\pi r$ is the *circumference* of a circle with radius r.

Circular area: πr^2 is the *circular area* of a circle of radius r.

Collinear points: The points lying on the same line are *collinear points* otherwise they are *non-collinear points*.

Circumcirlce: The circle passing through the vertices of a triangle is called its *circumcirlce* where ⊥ bisectors of sides of the triangle provides the centre.

Obtuse angle: An angle which is greater than 90° is called *obtuse angle.*

Right angle An angle which is equal to 90° is called *right angle.*

Acute: An angle which is less than 90° is called *acute angle.*

In-centre: *In-centre* of a triangle is the centre of a circle inscribed in a triangle.

Secant: A *secant* is a st line which cuts the circumference of a circle in two distinct points.

Tangent: A *tangent* to a circle is the St line which meets the circumference at one point only and being produced does not cut it at all. The point of tangency is also known as the point of contact. *AB* is the tangent line to the circle *C*.

Length of a tangent: The *length of a tangent* to a circle is measured from the given point to the point of contact.

Sector: The *sector* of a circle is an area bounded by any two radii and the arc intercepted between them.

Central angle: A *central angle* is subtended by two radii at the centre of the circle.

Circumangle: A *circumangle* is subtended between any two chords of a circle, having common point on its circumference.

Chord: The join of any two points on the circumference of the circle is called its chord.

Circumscribed circle: If a circle passes through all the vertices of a polygon the circle is said to be *circumscribed* about the polygon and the polygon is said to be *inscribed* in the circle.

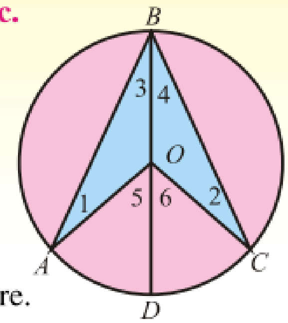
Escribed circle: If a circle touches one side of a triangle externally and the other two produced sides internally, is called *escribed* circle.

Circum circle: The circle passing through the vertices of triangle ABC is known as *circum circle*, its radius as *circum radius* and centre as *circum centre*.

In circle: A circle which touches the three sides of a triangle internally is known as *in-circle* its radius as *in-radius* and centre as *in-centre*.

THEOREM 1

12.1(*i*) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.



Given: $\widehat{AC}$ is an arc of a circle with centre O.

Whereas $\angle AOC$ is the central angle

and $\angle ABC$ is circum angle.

To prove: $m\angle AOC = 2m\angle ABC$

Construction: Join B with O and produce it to meet the circle at D.

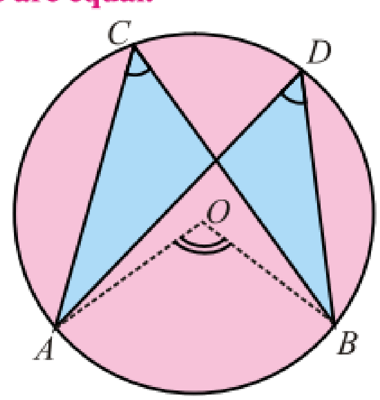
Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ $\angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in ΔOAB
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in ΔOBC
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
and $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
Then from figure	
$\Rightarrow$ $m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)
$\Rightarrow$ $m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$	

THEOREM 2

12.1(*ii*) Any two angles in the same segment of a circle are equal.



Given: $\angle ACB$ and $\angle ADB$ are the circum angles in the same segment of a circle with centre O.

To prove: $m\angle ACB = m\angle ADB$

Construction: Join O with A and O with B.

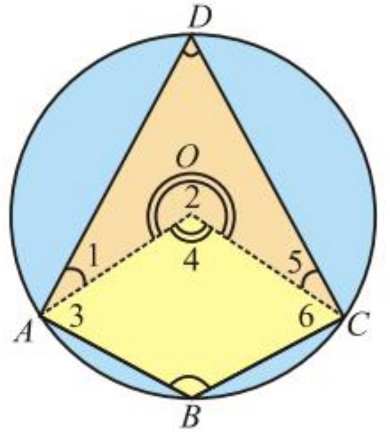
So that $\angle AOB$ is the central angle.

Proof:

Statements	Reasons
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas	Construction
$\angle ACB$ and $\angle ADB$ are circum angles	Given
$\therefore$ $m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem 1
$\Rightarrow$ $2m\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence, $m\angle ACB = m\angle ADB$	

THEOREM 4

12.1(*iv*) The opposite angles of any quadrilateral inscribed in a circle are supplementary.



Given: *ABCD* is a quadrilateral inscribed in a circle with centre *O*.

To prove: $\begin{cases} m\angle A + m\angle C = 2\ \angle rts \\ m\angle B + m\angle D = 2\ \angle rts \end{cases}$

Construction: Draw $\overline{OA}$ and $\overline{OC}$.

Write ∠1, ∠2, ∠3, ∠4, ∠5 and ∠6 as shown in the figure.

Proof:

Statements	Reasons
Standing on the same arc *ADC*, ∠2 is a central angle Whereas ∠*B* is the circum angle	Arc *ADC* of the circle with centre *O*.
$\therefore \quad m\angle B = \frac{1}{2}(m\angle 2)$ (i)	By theorem 1
Standing on the same arc *ABC*, ∠4 is a central angle whereas ∠*D* is the circum angle	Arc *ABC* of the circle with centre *O*.
$\therefore \quad m\angle D = \frac{1}{2}(m\angle 4)$ (ii)	By theorem 1
$\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$(Total central angle)	
i.e., $m\angle B + m\angle D = \frac{1}{2}(4\ \angle rt) = 2\angle rt$	
Similarly $m\angle A + m\angle C = 2\angle rt$	

Long Questions

Exercise 5.2

Q.1 **Given** $X = \{1,3,5,7,....,19\}$ **(A.B)**

$Y = \{0,2,4,6,....,20\}$

$Z = \{2,3,5,7,11,13,17,19,23\}$

To Find

(i) $X\cup(Y\cup Z)$

(ii) $(X\cup Y)\cup Z$

(iii) $X\cap(Y\cap Z)$

(iv) $(X\cap Y)\cap Z$

(v) $X\cup(Y\cap Z)$

(vi) $(X\cup Y)\cap(X\cup Z)$

(vii) $X\cap(Y\cup Z)$

(viii) $(X\cap Y)\cup(X\cap Z)$

Solution:

(i) $X\cup(Y\cup Z)$ **(RWP 2015)** **(A.B)**

$=\{1,3,5,7,......,19\}\cup$

$(\{0,2,4,6,.....,20\}\cup\{2,3,5,7,11,13,17,19,23\})$

$=\{1,3,5,.....,19\}\cup$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$ **(A.B)**

$X\cup Y=\{1,3,5,7,......,19\}\cup\{0,2,4,6,.....,20\}$

$=\{0,1,2,3,.....,20\}$

$(X\cup Y)\cup Z$

$=\{0,1,2,3,......,20\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,1,2,3,......,20,23\}$

(iii) $X\cap(Y\cap Z)$ **(A.B)**

$Y\cap Z = \{0,2,4,6,....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cap(Y\cap Z)=\{1,3,5,7,......,19\}\cap\{2\}$

$=\{\}$

(iv) $(X\cap Y)\cap Z$ **(A.B)**

$X\cap Y=\{1,3,5,7,.....,19\}\cap\{0,2,4,6,.....,20\}$

$=\{\}$

$(X\cap Y)\cap Z=\{\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{\}$

(v) $X\cup(Y\cap Z)$ **(A.B)**

$Y\cap Z=\{0,2,4,6,.....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cup(Y\cap Z)=\{1,3,5,7,.....,19\}\cup\{2\}$

$=\{1,2,3,5,7,.....,19\}$

(vi) $(X\cup Y)\cap(X\cup Z)$ **(A.B)**

$X\cup Y = \{1,3,5,....,19\}\cup\{0,2,4,6,.....,20\}$

$=\{0,1,2,3,.....,20\}$

$X\cup Z=\{1,3,5,....,19\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{1,2,3,5,7,.....,19,23\}$

$(X\cup Y)\cap(X\cup Z)=\{0,1,2,3,....,20\}$

$\cap\{1,2,3,5,7,......,19,23\}$

$=\{1,2,3,5,7,.....,19\}$

(vii) $X\cap(Y\cup Z)$ **(A.B)**

$Y\cup Z=\{0,2,4,6,....,20\}$

$\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$X\cap(Y\cup Z)=\{1,3,5,7,.....,19\}\cap$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{3,5,7,11,13,17,19\}$

(viii) $(X\cap Y)\cup(X\cap Z)$ **(A.B)**

$X\cap Y = \{1,3,5,7,.....,19\}\cap\{0,2,4,6,....,20\}$

$=\{\}$

$X\cap Z = \{1,3,5,7,.....,19\}$

$\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{3,5,7,11,13,17,19\}$

$(X\cap Y)\cup(X\cap Z)=\{\}\cup\{3,5,7,11,13,17,19\}$

$=\{3,5,7,11,13,17,19\}$

Q.2 **Given** A = {1,2,3,4,5,6} **(A.B + K.B)**

B = {2,4,6,8}

C = {1,4,8}

To Prove

(i) $A\cap B=B\cap A$

(ii) $A\cup B=B\cup A$

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proof

(i) $A\cap B=B\cap A$ **(A.B + K.B)**

L.H.S = $A\cap B$

$=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}\rightarrow(i)$

R.H.S = $B\cap A$

$=\{2,4,6,8\}\cap\{1,2,3,4,5,6\}$

$=\{2,4,6\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap B=B\cap A$

Hence Proved

(ii) $A\cup B=B\cup A$ **(A.B + K.B)**

Proof

L.H.S $=A\cup B$

$=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(i)$

R.H.S $=B\cup A$

$=\{2,4,6,8\}\cup\{1,2,3,4,5,6\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup B=B\cup A$

Hence Proved

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ **(A.B + K.B)**

Proof

L.H.S = $A\cap(B\cup C)$

$B\cup C=\{2,4,6,8\}\cup\{1,4,8\}$

$=\{1,2,4,6,8\}$

$A\cap(B\cup C)$

$=\{1,2,3,4,5,6\}\cap\{1,2,4,6,8\}$

$=\{1,2,4,6\}\rightarrow(i)$

R.H.S = $(A\cap B)\cup(A\cap C)$

$A\cap B=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}$

$A\cap C=\{1,2,3,4,5,6\}\cap\{1,4,8\}$

$=\{1,4\}$

$(A\cap B)\cup(A\cap C)=\{2,4,6\}\cup\{1,4\}$

$=\{1,2,4,6\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence Proved

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ **(A.B + K.B)**

Proof

L.H.S $=A\cup(B\cap C)$

$B\cap C=\{2,4,6,8\}\cap\{1,4,8\}$

$=\{4,8\}$

L.H.S $=A\cup(B\cap C)$

$=\{1,2,3,4,5,6\}\cup\{4,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(i)$

R.H.S $=(A\cup B)\cap(A\cup C)$

$A\cup B=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}$

$A\cup C=\{1,2,3,4,5,6\}\cup\{1,4,8\}$

$=\{1,2,3,4,5,6,8\}$

R.H.S $=(A\cup B)\cap(A\cup C)$

$=\{1,2,3,4,5,6,8\}\cap\{1,2,3,4,5,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence Proved

Q.3 **Given** $U=\{1,2,3,....,10\}$

(A.B + K.B)

$A=\{1,3,5,7,9\}$

$B=\{2,3,5,7\}$

To Prove

(i) $(A\cap B)'=A'\cup B'$

(ii) $(A\cup B)'=A'\cap B'$

(i) $(A\cap B)'=A'\cup B'$

Proof

L.H.S $=(A\cap B)'$

$A\cap B=\{1,3,5,7,9\}\cap\{2,3,5,7\}$

$=\{3,5,7\}$

$(A\cap B)'=U-(A\cap B)$

$=\{1,2,3,....,10\}-\{3,5,7\}$

$=\{1,2,4,6,8,9,10\}\to(i)$

R.H.S $=A'\cup B'$

$A'=U-A$

$=\{1,2,3,...,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$B'=U-B$

$=\{1,2,3,....,10\}-\{2,3,5,7\}$

$=\{1,4,6,8,9,10\}$

$A'\cup B'=\{2,4,6,8,10\}\cup\{1,4,6,8,9,10\}$

$=\{1,2,4,6,8,9,10\}\to(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)'=A'\cup B'$

Hence Proved

(ii) $(A\cup B)' = A'\cap B'$ **(A.B + K.B)**

Proof

L.H.S $=(A\cup B)'$

$A\cup B = \{1,3,5,7,9\}\cup\{2,3,5,7\}$

$=\{1,2,3,5,7,9\}$

L.H.S $=(A\cup B)' = U-(A\cup B)$

$=\{1,2,3,\ldots,10\}-\{1,2,3,5,7,9\}$

$=\{4,6,8,10\}\rightarrow(\text{i})$

R.H.S $= A'\cap B'$

$=\{2,4,6,8,10\}\cap\{1,4,6,8,9,10\}$

$=\{4,6,8,10\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)' = A'\cap B'$

Hence Proved

Q.4 Given **(A.B + K.B)**

$U = \{1,2,3,\ldots,20\}$

$X = \{1,3,7,9,15,18,20\}$

$Y = \{1,3,5,\ldots,17\}$

To Prove

(i) $X-Y = X\cap Y'$

(ii) $Y-X = Y\cap X'$

Proof

(i) $X-Y = X\cap Y'$ **(A.B + K.B)**

L.H.S = X – Y

$=\{1,3,7,9,15,18,20\}-\{1,3,5,\ldots,17\}$

$=\{18,20\}\rightarrow(\text{i})$

$Y' = U - Y$

$=\{1,2,3,\ldots,20\}-\{1,3,5,\ldots,17\}$

$=\{2,4,6,\ldots,18,19,20\}$

R.H.S $= X\cap Y'$

$=\{1,3,7,9,15,18,20\}\cap\{2,4,6,\ldots,18,19,20\}$

$=\{18,20\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$X-Y = X\cap Y'$

Hence Proved

(ii) $Y-X = Y\cap X'$ **(A.B + K.B)**

Proof

L.H.S = Y – X

$=\{1,3,5,\ldots,17\}-\{1,3,7,9,15,18,20\}$

$=\{5,11,13,17\}\rightarrow(\text{i})$

L.H.S $= Y\cap X'$

$X' = U - X$

$=\{1,2,3,\ldots,20\}-\{1,3,7,9,15,18,20\}$

$=\{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$Y\cap X' = \{1,3,5,\ldots,17\}$

$\cap\{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$=\{5,11,13,17\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$Y-X = Y\cap X'$

Hence Proved

Exercise 5.3

Q.1 **Given** $U=\{1,2,3,4,...,10\}$

$A=\{1,3,5,7,9\}$ **(K.B)**

$B=\{1,4,7,10\}$ **(A.B)**

(LHR 2017, GRW 2016, FSD 2017, SWL 2017, RWP 2016, BWP 2016, D.G.K 2016)

To Prove

(i) $A-B=A\cap B'$

(ii) $B-A=B\cap A'$

(iii) $(A\cup B)'=A'\cap B'$

(iv) $(A\cap B)'=A'\cup B'$

(v) $(A-B)'=A'\cup B$

(vi) $(B-A)'=B'\cup A$

Proof

(i) $A-B=A\cap B'$

L.H.S = A – B

$=\{1,3,5,7,9\}-\{1,4,7,10\}$

$=\{3,5,9\}\rightarrow(\text{i})$

R.H.S = $A\cap B'$

$B'=U-B$

$=\{1,2,3,....,10\}-\{1,4,7,10\}$

$=\{2,3,5,6,8,9\}$

$A\cap B'=\{1,3,5,7,9\}\cap\{2,3,5,6,8,9\}$

$=\{3,5,9\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$A-B=A\cap B'$

Hence Proved

(ii) $B-A=B\cap A'$ **(K.B + A.B)**

L.H.S = B – A

$=\{1,4,7,10\}-\{1,3,5,7,9\}$

$=\{4,10\}\rightarrow(\text{i})$

R.H.S $=B\cap A'$

$A'=U-A=\{1,2,3,...,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$B\cap A'=\{1,4,7,10\}\cap\{2,4,6,8,10\}$

$=\{4,10\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$B-A=B\cap A'$

Hence Proved

(iii) $(A\cup B)'=A'\cap B'$ **(K.B + A.B)**

$A\cup B=\{1,3,5,7,9\}\cup\{1,4,7,10\}$

$=\{1,3,4,5,7,9,10\}$

L.H.S $=(A\cup B)'=U-(A\cup B)$

$=\{1,2,3,....,10\}-\{1,3,4,5,7,9,10\}$

$=\{2,6,8\}\rightarrow(\text{i})$

R.H.S = $A'\cap B'$

$A'=U-A=\{1,2,3...,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$B'=U-B=\{1,2,3,...,10\}-\{1,4,7,10\}$

$=\{2,3,5,6,8,9\}$

R.H.S $=A'\cap B'$

$=\{2,4,6,8,10\}\cap\{2,3,5,6,8,9\}$

$=\{2,6,8\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)'=A'\cap B'$

Hence Proved

(iv) $(A\cap B)'=A'\cup B'$ **(K.B + A.B)**

Proof

$A\cap B=\{1,3,5,7,9\}\cap\{1,4,7,10\}$

$=\{1,7\}$

L.H.S $=(A\cap B)' = U-(A\cap B)$

$=\{1,2,3,\ldots,10\}-\{1,7\}$

$=\{2,3,4,5,6,8,9,10\}\rightarrow(\text{i})$

R.H.S $= A'\cup B'$

$=\{2,4,6,8,10\}\cup\{2,3,5,6,8,9\}$

$=\{2,3,4,5,6,8,9,10\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)' = A'\cup B'$

Hence Proved

(v) $(A-B)' = A'\cup B$ **(K.B + A.B)**

Proof

L.H.S $=(A-B)'$

$A-B=\{1,3,5,7,9,\}-\{1,4,7,10\}$

$=\{3,5,9\}$

L.H.S $=(A-B)' = U-(A-B)$

$=\{1,2,3,\ldots,10\}-\{3,5,9\}$

$=\{1,2,4,6,7,8,10\}\rightarrow(\text{i})$

R.H.S $=A'\cup B$

$A' = U-A$

$=\{1,2,3,4,5,6,7,8,9,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$A'\cup B=\{2,4,6,8,10\}\cup\{1,4,7,10\}$

$=\{1,2,4,6,7,8,10\}\rightarrow(\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$(A-B)' = A'\cup B$

Hence Proved

(vi) $(B-A)' = B'\cup A$ **(K.B + A.B)**

(FSD 2015)

Proof

$B-A=\{1,4,7,10\}-\{1,3,5,7,9\}$

$B-A=\{4,10\}$

L.H.S $=(B-A)' = U-(B-A)$

$=\{1,2,3,4,5,6,7,8,9,10\}-\{4,10\}$

$=\{1,2,3,5,6,7,8,9\}\rightarrow(\text{i})$

R.H.S $=B'\cup A$

$B' = U-B$

$=\{1,2,3,4,5,6,7,8,9,10\}-\{1,4,7,10\}$

$=\{2,3,5,6,8,9\}$

$B'\cup A=\{2,3,5,6,8,9\}\cup\{1,3,5,7,9\}$

$=\{1,2,3,5,6,7,8,9\}\rightarrow(\text{ii})$

From (i) and (ii) we get

L.H.S=R.H.S

$(B-A)' = B'\cup A$

Hence Proved

MISCELLANEOUS EXERCISE - 1

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is

(a) $bx + c = 0,\ b \neq 0$ ● $ax^2 + bx + c = 0,\ a \neq 0$

(c) $ax^2 = bx,\ a \neq 0$ (d) $ax^2 = 0,\ a \neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ is

(a) 1 (b) 2 ● 3 (d) 4

(iii) The number of methods to solve a quadratic equation is

(a) 1 (b) 2 ● 3 (d) 4

(iv) The quadratic formula is

● $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

(c) $x = \dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ (d) $x = \dfrac{b \pm \sqrt{b^2 + 4ac}}{2a}$

(v) Two linear factors of $x^2 - 15x + 56$ are

(a) $(x - 7)$ and $(x + 8)$ (b) $(x + 7)$ and $(x - 8)$

● $(x - 7)$ and $(x - 8)$ (d) $(x + 7)$ and $(x + 8)$

(vi) An equation, which remains unchanged when x is replaced by $\dfrac{1}{x}$ is called a/an

(a) Exponential equation ● Reciprocal equation

(c) Radical equation (d) None of these

(vii) An equation of the type $3^x + 3^{2-x} + 6 = 0$ is a/an

● Exponential equation (b) Radical equation

(c) Reciprocal equation (d) None of these

(viii) The solution set of equation $4x^2 - 16 = 0$ is

(a) $\{\pm 4\}$ (b) $\{4\}$ ● $\{\pm 2\}$ (d) ± 2

(ix) An equation of the form $2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$ is called a/an

● Reciprocal equation (b) Radical equation

(c) Exponential equation (d) None of these

MISCELLANEOUS EXERCISE - 2

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{-5}{3}$ (d) $\frac{-2}{3}$

(ii) If α, β are the roots of $7x^2 - x + 4 = 0$, then $\alpha\beta$ is

(a) $\frac{-1}{7}$ (b) $\frac{4}{7}$ (c) $\frac{7}{4}$ (d) $\frac{-4}{7}$

(iii) Roots of the equation $4x^2 - 5x + 2 = 0$ are

(a) irrational (b) imaginary (c) rational (d) none of these

(iv) Cube roots of -1 are

(a) $-1, -\omega, -\omega^2$ (b) $-1, \omega, -\omega^2$ (c) $-1, -\omega, \omega^2$ (d) $1, -\omega, -\omega^2$

(v) Sum of the cube roots of unity is

(a) 0 (b) 1 (c) -1 (d) 3

(vi) Product of cube roots of unity is

(a) 0 (b) 1 (c) -1 (d) 3

(vii) If $b^2 - 4ac < 0$, then the roots of $ax^2 + bx + c = 0$ are

(a) irrational (b) rational (c) imaginary (d) none of these

(viii) If $b^2 - 4ac > 0$, but not a perfect square then roots of $ax^2 + bx + c = 0$ are

(a) imaginary (b) rational (c) irrational (d) none of these

(ix) $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha} - \frac{1}{\beta}$ (c) $\frac{\alpha - \beta}{\alpha\beta}$ (d) $\frac{\alpha + \beta}{\alpha\beta}$

(x) $\alpha^2 + \beta^2$ is equal to

(a) $\alpha^2 - \beta^2$ (b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(c) $(\alpha + \beta)^2 - 2\alpha\beta$ (d) $\alpha + \beta$

(xi) Two square roots of unity are

(a) $1, -1$ (b) $1, \omega$ (c) $1, -\omega$ (d) ω, ω^2

(xii) Roots of the equation $4x^2 - 4x + 1 = 0$ are

(a) real, equal (b) real, unequal (c) imaginary (d) irrational

(xiii) If α, β are the roots of $px^2 + qx + r = 0$, then sum of the roots 2α and 2β is

(a) $\frac{-q}{p}$ (b) $\frac{r}{p}$ (c) $\frac{-2q}{p}$ (d) $-\frac{q}{2p}$

(xiv) If α, β are the roots of $x^2 - x - 1 = 0$, then product of the roots 2α and 2β is

(a) -2 (b) 2 (c) 4 (d) -4

(xv) The nature of the roots of equation $ax^2 + bx + c = 0$ is determined by

(a) sum of the roots (b) product of the roots

(c) synthetic division (d) discriminant

(xvi) The discriminant of $ax^2 + bx + c = 0$ is

(a) $b^2 - 4ac$ (b) $b^2 + 4ac$ (c) $-b^2 + 4ac$ (d) $-b^2 - 4ac$

MISCELLANEOUS EXERCISE - 3

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) In a ratio $a : b$, a is called
- (a) relation
- (b) antecedent
- (c) consequent
- (d) None of these

(ii) In a ratio $x : y$, y is called
- (a) relation
- (b) antecedent
- (c) consequent
- (d) None of these

(iii) In a proportion $a : b : : c : d$, a and d are called,
- (a) means
- (b) extremes
- (c) third proportional
- (d) None of these

(iv) In a proportion $a : b : : c : d$, b and c are called
- (a) means
- (b) extremes
- (c) fourth proportional
- (d) None of these

(v) In continued proportion $a : b = b : c$, $ac = b^2$, b is said to be _______ proportional between a and c.
- (a) third
- (b) fourth
- (c) means
- (d) None of these

(vi) In continued proportion $a : b = b : c$, c is said to be ______ proportional to a and b.
- (a) third
- (b) fourth
- (c) means
- (d) None of these

(vii) Find x in proportion $4 : x : : 5 : 15$
- (a) $\frac{75}{4}$
- (b) $\frac{4}{3}$
- (c) $\frac{3}{4}$
- (d) 12

(viii) If $u \propto v^2$, then
- (a) $u = v^2$
- (b) $u = kv^2$
- (c) $uv^2 = k$
- (d) $uv^2 = 1$

(ix) If $y^2 \propto \frac{1}{x^3}$, then
- (a) $y^2 = \frac{k}{x^3}$
- (b) $y^2 = \frac{1}{x^3}$
- (c) $y^2 = x^2$
- (d) $y^2 = kx^3$

(x) If $\frac{u}{v} = \frac{v}{w} = k$, then
- (a) $u = wk^2$
- (b) $u = vk^2$
- (c) $u = w^2k$
- (d) $u = v^2k$

(xi) The third proportional of x^2 and y^2 is
- (a) $\frac{y^2}{x^2}$
- (b) x^2y^2
- (c) $\frac{y^4}{x^2}$
- (d) $\frac{y^2}{x^4}$

(xii) The fourth proportional w of $x : y :: v : w$ is

(a) $\frac{xy}{v}$ (b) $\frac{vy}{x}$

(c) xyv (d) $\frac{x}{vy}$

(xiii) If $a : b = x : y$, then alternando property is

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{b} = \frac{x}{y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$ (d) $\frac{a-b}{x} = \frac{x-y}{y}$

(xiv) If $a : b = x : y$, then invertendo property is

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{a-b} = \frac{x}{x-y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$ (d) $\frac{b}{a} = \frac{y}{x}$

(xv) If $\frac{a}{b} = \frac{c}{d}$, then componendo property is

(a) $\frac{a}{a+b} = \frac{c}{c+d}$ (b) $\frac{a}{a-b} = \frac{c}{c-d}$

(c) $\frac{ad}{bc}$ (d) $\frac{a-b}{b} = \frac{c-d}{d}$

MISCELLANEOUS EXERCISE - 4

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The identity $(5x + 4)^2 = 25x^2 + 40x + 16$ is true for

(a) one value of x (b) two values of x
(c) all values of x (d) none of these

(ii) A function of the form $f(x) = \frac{N(x)}{D(x)}$, with $D(x) \neq 0$, where $N(x)$ and $D(x)$ are polynomials in x is called

(a) an identity (b) an equation
(c) a fraction (d) none of these

(iii) A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called

(a) a proper fraction (b) an improper fraction
(c) an equation (d) algebraic relation

(iv) A fraction in which the degree of numerator is less than the degree of the denominator is called

(a) an equation (b) an improper fraction
(c) an identity (d) a proper fraction

(v) $\frac{2x+1}{(x+1)(x-1)}$ is:

(a) an improper fraction (b) an equation
(c) a proper fraction (d) none of these

(vi) $(x+3)^2 = x^2 + 6x + 9$ is

(a) a linear equation (b) an equation
(c) an identity (d) none of these

(vii) $\frac{x^3+1}{(x-1)(x+2)}$ is

(a) a proper fraction (b) an improper fraction
(c) an identity (d) a constant term

(viii) Partial fractions of $\frac{x-2}{(x-1)(x+2)}$ are of the form

(a) $\frac{A}{x-1} + \frac{B}{x+2}$ (b) $\frac{Ax}{x-1} + \frac{B}{x+2}$
(c) $\frac{A}{x-1} + \frac{Bx+C}{x+2}$ (d) $\frac{Ax+B}{x-1} + \frac{C}{x+2}$

(ix) Partial fractions of $\frac{x+2}{(x+1)(x^2+2)}$ are of the form

(a) $\frac{A}{x+1} + \frac{B}{x^2+2}$ (b) $\frac{A}{x+1} + \frac{Bx+C}{x^2+2}$
(c) $\frac{Ax+B}{x+1} + \frac{C}{x^2+2}$ (d) $\frac{A}{x+1} + \frac{Bx}{x^2+2}$

(x) Partial fractions of $\frac{x^2+1}{(x+1)(x-1)}$ are of the form

(a) $\frac{A}{x+1} + \frac{B}{x-1}$ (b) $1 + \frac{A}{x+1} + \frac{Bx+C}{x-1}$
(c) $1 + \frac{A}{x+1} + \frac{B}{x-1}$ (d) $\frac{Ax+B}{(x+1)} + \frac{C}{x-1}$

MISCELLANEOUS EXERCISE - 5

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick mark (✓) the correct answer.

(i) A collection of well-defined objects is called
- (a) subset
- (b) power set
- (c) set
- (d) none of these

(ii) A set $Q = \left\{ \frac{a}{b} \mid a, b \in Z \wedge b \neq 0 \right\}$ is called a set of
- (a) Whole numbers
- (b) Natural numbers
- (c) Irrational numbers
- (d) Rational numbers

(iii) The different number of ways to describe a set are
- (a) 1
- (b) 2
- (c) 3
- (d) 4

(iv) A set with no element is called
- (a) Subset
- (b) Empty set
- (c) Singleton set
- (d) Super set

(v) The set $\{x \mid x \in W \wedge x \leq 101\}$ is
- (a) Infinite set
- (b) Subset
- (c) Null set
- (d) Finite set

(vi) The set having only one element is called
- (a) Null set
- (b) Power set
- (c) Singleton set
- (d) Subset

(vii) Power set of an empty set is
- (a) ϕ
- (b) $\{a\}$
- (c) $\{\phi, \{a\}\}$
- (d) $\{\phi\}$

(viii) The number of elements in power set $\{1, 2, 3\}$ is
- (a) 4
- (b) 6
- (c) 8
- (d) 9

(ix) If $A \subseteq B$, then $A \cup B$ is equal to
- (a) A
- (b) B
- (c) ϕ
- (d) none of these

(x) If $A \subseteq B$, then $A \cap B$ is equal to
- (a) A
- (b) B
- (c) ϕ
- (d) none of these

(xi) If $A \subseteq B$, then $A - B$ is equal to
- (a) A
- (b) B
- (c) ϕ
- (d) $B - A$

(xii) $(A \cup B) \cup C$ is equal to
- (a) $A \cap (B \cup C)$
- (b) $(A \cup B) \cap C$
- (c) $A \cup (B \cup C)$
- (d) $A \cap (B \cap C)$

(xiii) $A \cup (B \cap C)$ is equal to
- (a) $(A \cup B) \cap (A \cup C)$
- (b) $A \cap (B \cap C)$
- (c) $(A \cap B) \cup (A \cap C)$
- (d) $A \cup (B \cup C)$

(xiv) If A and B are disjoint sets, then $A \cup B$ is equal to
(a) A (b) B
(c) ϕ (d) $B \cup A$

(xv) If number of elements in set A is 3 and in set B is 4, then number of elements in $A \times B$ is
(a) 3 (b) 4
(c) 12 (d) 7

(xvi) If number of elements in set A is 3 and in set B is 2, then number of binary relations in $A \times B$ is
(a) 2^3 (b) 2^6
(c) 2^8 (d) 2^2

(xvii) The domain of $R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$ is
(a) $\{0, 3, 4\}$ (b) $\{0, 2, 3\}$
(c) $\{0, 2, 4\}$ (d) $\{2, 3, 4\}$

(xviii) The range of $R = \{(1, 3), (2, 2), (3, 1), (4, 4)\}$ is
(a) $\{1, 2, 4\}$ (b) $\{3, 2, 4\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{1, 3, 4\}$

(xix) Point $(-1, 4)$ lies in the quadrant
(a) I (b) II
(c) III (d) IV

(xx) The relation $\{(1, 2), (2, 3), (3, 3), (3, 4)\}$ is
(a) onto function (b) into function
(c) not a function (d) one-one function

MISCELLANEOUS EXERCISES

1. Multiple Choice Questions

Three possible answers are given for the following questions. Tick (✓) the correct answer.

(i) A grouped frequency table is also called
(a) data (b) frequency distribution
(c) frequency polygon

(ii) A histogram is a set of adjacent
(a) squares (b) rectangles
(c) circles

(iii) A frequency polygon is a many sided
(a) closed figure (b) rectangle
(c) square

(iv) A cumulative frequency table is also called
(a) frequency distribution (b) data
(c) less than cumulative frequency distribution

(v) In a cumulative frequency polygon frequencies are plotted against
(a) midpoints (b) upper class boundaries
(c) class limits

(vi) Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their
(a) number (b) group
(c) denominator

(vii) A Deviation is defined as a difference of any value of the variable from a
(a) constant (b) histogram
(c) sum

(viii) A data in the form of frequency distribution is called
(a) Grouped data (b) Ungrouped data
(c) Histogram

(ix) Mean of a variable with similar observations say constant k is
(a) negative (b) k itself
(c) zero

(x) Mean is affected by change in
(a) value (b) ratio
(c) origin

(xi) Mean is affected by change in
(a) place (b) scale
(c) rate

(xii) Sum of the deviations of the variable X from its mean is always
(a) zero (b) one
(c) same

(xiii) The n[th] positive root of the product of the $x_1, x_2, x_3, ..., x_n$ observations is called

(a) Mode (b) Mean

● Geometric mean

(xiv) The value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, ..., x_n$ observations is called

(a) Geometric mean (b) Median

● Harmonic mean

(xv) The most frequent occurring observation in a data set is called

● mode (b) median

(c) harmonic mean

(xvi) The measure which determines the middlemost observation in a data set is called

● median (b) mode

(c) mean

(xvii) The observations that divide a data set into four equal parts are called

(a) deciles ● quartiles

(c) percentiles

(xviii) The spread or scatterness of observations in a data set is called

(a) average ● dispersion

(c) central tendency

(xix) The measures that are used to determine the degree or extent of variation in a data set are called measures of

● dispersion (b) central tendency

(c) average

(xx) The extent of variation between two extreme observations of a data set is measured by

(a) average ● range

(c) quartiles

(xxi) The mean of the squared deviations of x_i (i = 1, 2,.....,n) observations from their arithmetic mean is called

● variance (b) standard deviation

(c) range

(xxii) The positive square root of mean of the squared deviations of X_i (i = 1, 2, ..., n) observations from their arithmetic mean is called

(a) harmonic mean (b) range

● standard deviation

MISCELLANEOUS EXERCISE - 7

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The union of two non-collinear rays, which have common end point is called

an angle (b) a degree (c) a minute (d) a radian

(ii) The system of measurement in which the angle is measured in radians is called

(a) CGS system (b) sexagesimal system

(c) MKS system circular system

(iii) $20^\circ =$

(a) $360'$ (b) $630'$ $1200'$ (d) $3600'$

(iv) $\frac{3\pi}{4}$ radians =

(a) 115° 135° (c) 150° (d) 30°

(v) If $\tan\theta = \sqrt{3}$, then θ is equal to

(a) 90° (b) 45° 60° (d) 30°

(vi) $\sec^2\theta =$

(a) $1-\sin^2\theta$ $1+\tan^2\theta$ (c) $1+\cos^2\theta$ (d) $1-\tan^2\theta$

(vii) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=$

$2\sec^2\theta$ (b) $2\cos^2\theta$ (c) $\sec^2\theta$ (d) $\cos\theta$

(viii) $\frac{1}{2}\operatorname{cosec}45^\circ =$

(a) $\frac{1}{2\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$

(ix) $\sec\theta\cot\theta =$

(a) $\sin\theta$ (b) $\frac{1}{\cos\theta}$ $\frac{1}{\sin\theta}$ (d) $\frac{\sin\theta}{\cos\theta}$

(x) $\operatorname{cosec}^2\theta - \cot^2\theta =$

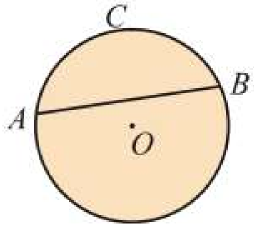
(a) -1 1 (c) 0 (d) $\tan\theta$

MISCELLANEOUS EXERCISE 9

Multiple Choice Questions

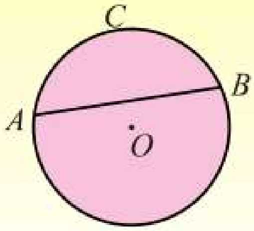
1. Four possible answers are given for the following questions.
Tick (✓) the correct answer.

(*i*) In the circular figure, *ADB* is called



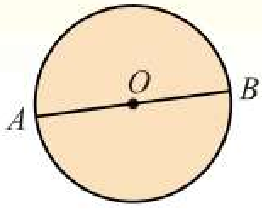
(*a*) an arc (*b*) a secant
● a chord (*d*) a diameter

(*ii*) In the circular figure, $\overset{\frown}{ACB}$ is called



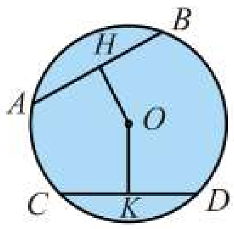
● an arc (*b*) a secant
(*c*) a chord (*d*) a diameter

(*iii*) In the circular figure, *AOB* is called



(*a*) an arc (*b*) a secant
(*c*) a chord ● a diameter

(*iv*) In a circular figure, two chords $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre. They will be



(*a*) parallel (*b*) non congruent
● congruent (*d*) perpendicular

(*v*) Radii of a circle are
● all equal (*b*) double of the diameter
(*c*) all unequal (*d*) half of any chord

(*vi*) A chord passing through the centre of a circle is called
(*a*) radius ● diameter
(*c*) circumference (*d*) secant

(*vii*) Right bisector of the chord of a circle always passes through the
(*a*) radius (*b*) circumference
● centre (*d*) diameter

(*viii*) The circular region bounded by two radii and the corresponding arc is called
(*a*) circumference of a circle ● sector of a circle
(*c*) diameter of a circle (*d*) segment of a circle

(*ix*) The distance of any point of the circle to its centre is called
● radius (*b*) diameter (*c*) a chord (*d*) an arc

(*x*) Line segment joining any point of the circle to the centre is called
(*a*) circumference (*b*) diameter
● radial segment (*d*) perimeter

(*xi*) Locus of a point in a plane equidistant from a fixed point is called
(*a*) radius ● circle (*c*) circumference (*d*) diameter

(*xii*) The symbol for a triangle is denoted by
(*a*) ∠ ● Δ (*c*) ⊥ (*d*) ⊙

(*xiii*) A complete circle is divided into
(*a*) 90 degrees (*b*) 180 degrees (*c*) 270 degrees ● 360 degrees

(*xiv*) Through how many non collinear points, can a circle pass?
(*a*) one (*b*) two ● three (*d*) none

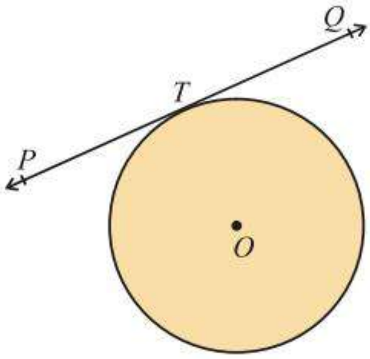
MISCELLANEOUS EXERCISE 10

Multiple Choice Questions

1. Four possible answers are given for the following questions. Tick (✓) the correct answer.

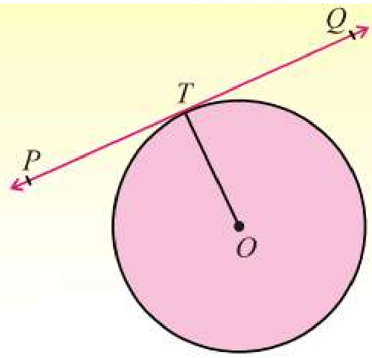
(i) In the adjacent figure of the circle, the line $\overleftrightarrow{PTQ}$ is named as

(a) an arc (b) a chord
● a tangent (d) a secant



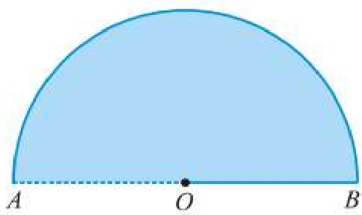
(ii) In a circle with centre O, if $\overline{OT}$ is the radial segment and $\overleftrightarrow{PTQ}$ is the tangent line, then

● $\overline{OT} \perp \overleftrightarrow{PQ}$ (b) $\overline{OT} \not\perp \overleftrightarrow{PQ}$
(c) $\overline{OT} \parallel \overleftrightarrow{PQ}$
(d) $\overline{OT}$ is right bisector of $\overleftrightarrow{PQ}$



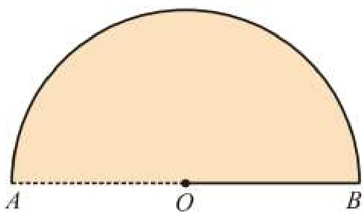
(iii) In the adjacent figure, find semicircular area if $\pi \simeq 3.1416$ and $m\overline{OA} = 20$cm.

(a) 62.83sq cm (b) 314.16sq cm
(c) 436.20sq cm ● 628.32sq cm



(iv) In the adjacent figure, find half the perimeter of circle with centre O if $\pi \simeq 3.1416$ and $m\overline{OA} = 20$cm.

(a) 31.42 cm ● 62.832 cm
(c) 125.65 cm (d) 188.50 cm



(v) A line which has two points in common with a circle is called:

(a) sine of a circle (b) cosine of a circle
(c) tangent of a circle ● secant of a circle

(vi) A line which has only one point in common with a circle is called:

(a) sine of a circle (b) cosine of a circle
● tangent of a circle (d) secant of a circle

(vii) Two tangents drawn to a circle from a point outside it are of .... in length.

(a) half ● equal (c) double (d) triple

(viii) A circle has only one:

(a) secant (b) chord (c) diameter ● centre

(ix) A tangent line intersects the circle at:

(a) three points (b) two points ● single point (d) no point at all

(x) Tangents drawn at the ends of diameter of a circle are .... to each other.

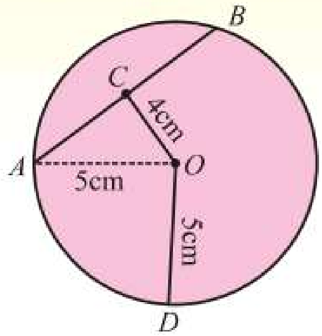
● parallel (*b*) non-parallel (*c*) collinear (*d*) perpendicular

(xi) The distance between the centres of two congruent touching circles externally is:

(*a*) of zero length (*b*) the radius of each circle

● the diameter of each circle (*d*) twice the diameter of each circle

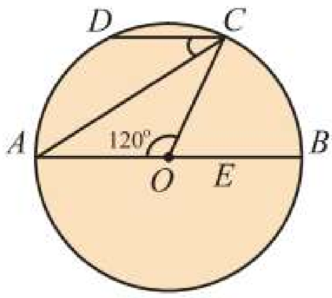
(xii) In the adjacent circular figure with centre O and radius 5cm, the length of the chord intercepted at 4cm away from the centre of this circle is:



(*a*) 4cm ● 6cm

(*c*) 7cm (*d*) 9cm

(xiii) In the adjoining figure, there is a circle with centre O. If $\overline{DC}$ // diameter $\overline{AB}$ and $m\angle AOC = 120^\circ$, then $m\angle ACD$ is:



(*a*) 40° ● 30°

(*c*) 50° (*d*) 60°

MISCELLANEOUS EXERCISE 11

1. Multiple Choice Questions

Four possible answers are given for the following questions.

Tick (✓) the correct answer.

(*i*) A 4 cm long chord subtends a central angle of 60°. The radial segment of this circle is:

(*a*) 1 (*b*) 2 (*c*) 3 ● 4

(*ii*) The length of a chord and the radial segment of a circle are congruent, the central angle made by the chord will be:

(*a*) 30° (*b*) 45° ● 60° (*d*) 75°

(*iii*) Out of two congruent arcs of a circle, if one arc makes a central angle of 30° then the other arc will subtend the central angle of:

(*a*) 15° ● 30° (*c*) 45° (*d*) 60°

(*iv*) An arc subtends a central angle of 40° then the corresponding chord will subtend a central angle of:

(*a*) 20° ● 40° (*c*) 60° (*d*) 80°

(*v*) A pair of chords of a circle subtending two congruent central angles is:

● congruent (*b*) incongruent (*c*) over lapping (*d*) parallel

(*vi*) If an arc of a circle subtends a central angle of 60°, then the corresponding chord of the arc will make the central angle of:

(*a*) 20° (*b*) 40° ● 60° (*d*) 80°

(*vii*) The semi circumference and the diameter of a circle both subtend a central angle of:

(*a*) 90° ● 180° (*c*) 270° (*d*) 360°

(*viii*) The chord length of a circle subtending a central angle of 180° is always:

(*a*) less than radial segment (*b*) equal to the radial segment

● double of the radial segment (*d*) none of these

(*ix*) If a chord of a circle subtends a central angle of 60°, then the length of the chord and the radial segment are:

● congruent (*b*) incongruent (*c*) parallel (*d*) perpendicular

(*x*) The arcs opposite to incongruent central angles of a circle arc always:

(*a*) congruent ● incongruent (*c*) parallel (*d*) perpendicular

MISCELLANEOUS EXERCISE 12

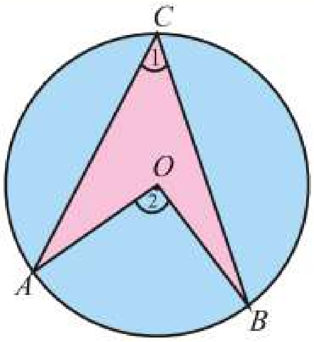
1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(*i*) A circle passes through the vertices of a right angled ΔABC with $m\overline{AC} = 3\text{cm}$ and $m\overline{BC} = 4$ cm, $m\angle C = 90^\circ$. Radius of the circle is:

(*a*) 1.5 cm (*b*) 2.0 cm (*c*) ● 2.5 cm (*d*) 3.5 cm

(*ii*) In the adjacent circular figure, central and inscribed angles stand on the same arc AB. Then



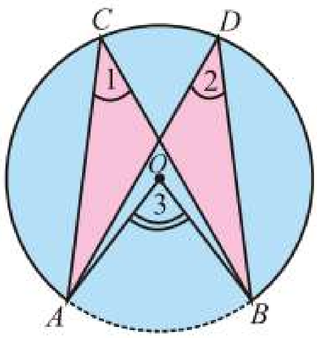
(*a*) $m\angle 1 = m\angle 2$ (*b*) $m\angle 1 = 2m\angle 2$

(*c*) $m\angle 2 = 3m\angle 1$ (*d*) ● $m\angle 2 = 2m\angle 1$

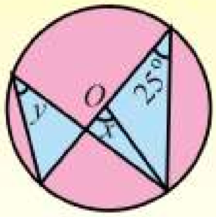
(*iii*) In the adjacent figure if $m\angle 3 = 75^\circ$, then find $m\angle 1$ and $m\angle 2$.

(*a*) ● $37\frac{1}{2}^\circ, 37\frac{1}{2}^\circ$ (*b*) $37\frac{1}{2}^\circ, 75^\circ$

(*c*) $75^\circ, 37\frac{1}{2}^\circ$ (*d*) $75^\circ, 75^\circ$

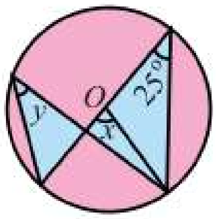


(*iv*) Given that O is the centre of the circle. The angle marked x will be:



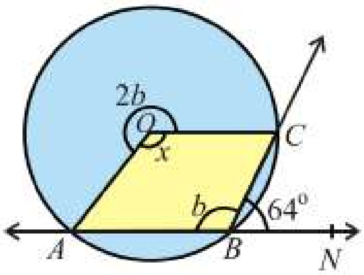
(*a*) $12\frac{1}{2}^{\circ}$ (*b*) 25° 50° (*d*) 75°

(*v*) Given that O is the centre of the circle the angle marked y will be:



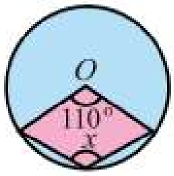
(*a*) $12\frac{1}{2}^{\circ}$ 25° (*c*) 50° (*d*) 75°

(*vi*) In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line.
The obtuse angle $AOC = x$ is:



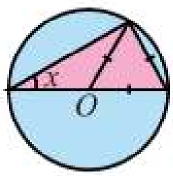
(*a*) 32° (*b*) 64° (*c*) 96° 128°

(*vii*) In the figure, O is the centre of the circle, then the angle x is:



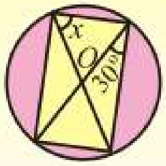
(*a*) 55° (*b*) 110° (*c*) 220° 125°

(*viii*) In the figure, O is the centre of the circle then angle x is:



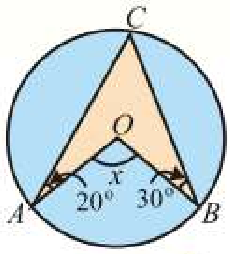
(*a*) 15° 30° (*c*) 45° (*d*) 60°

(*ix*) In the figure, O is the centre of the circle then the angle x is:



(*a*) 15° (*b*) 30° (*c*) 45° 60°

(*x*) In the figure, O is the centre of the circle then the angle x is:



(*a*) 50° (*b*) 75° 100° (*d*) 125°

MISCELLANEOUS EXERCISE - 13

1. Multiple Choice Questions

Three possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The circumference of a circle is called
(a) chord (b) segment boundary

(ii) A line intersecting a circle is called
(a) tangent secant (c) chord

(iii) The portion of a circle between two radii and an arc is called
sector (b) segment (c) chord

(iv) Angle inscribed in a semi-circle is
$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$

(v) The length of the diameter of a circle is how many times the radius of the circle
(a) 1 2 (c) 3

(vi) The tangent and radius of a circle at the point of contact are
(a) parallel (b) not perpendicular perpendicular

(vii) Circles having three points in common
over lapping (b) collinear (c) not coincide

(viii) If two circles touch each other, their centres and point of contact are
(a) coincident (b) non-collinear collinear

(ix) The measure of the external angle of a regular hexagon is
$\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$

(x) If the incentre and circumcentre of a triangle coincide, the triangle is
(a) an isoscenes (b) a right triangle an equilateral

(xi) The measure of the external angle of a regular octagon is
$\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$

(xii) Tangents drawn at the end points of the diameter of a circle are
parallel (b) perpendicular (c) Intersecting

(xiii) The lengths of two transverse tangents to a pair of circles are
(a) unequal equal (c) overlapping

(xiv) How many tangents can be drawn from a point outside the circle?
(a) 1 2 (c) 3

(xv) If the distance between the centers of two circles is equal to the sum of their radii, then the circles will
(a) intersect (b) do not intersect
touch each other externaly

(xvi) If the two circles touches externally, then the distance between their centers is equal to the
(a) difference of their radii sum of their radii
(c) product of their radii

(xvii) How many common tangents can be drawn for two touching circles?
(a) 2 3 (c) 4

(xviii) How many common tangents can be drawn for two disjoint circles?
(a) 2 (b) 3 4