$\mathbf{U}_{\text{nit-5}}$

Sets and Functions



Mathematics-10

Unit 5 - 5.1



Set

(K.B + U.B)

A well define collection of distinct objects is called Set. A set is represented by capital English alphabets.

For example: A= Set of integers,

 $B = \{1, 2, 3, ..., 10\}$ etc.

Presentation of Sets

(K.B + U.B)

A set is presented by 3 ways

- Tabular form (i)
- (ii) Descriptive Form
- Set builder notation (iii)

Some Important Sets

(K.B + U.B)

N =The set of natural numbers= $\{1,2,3,...\}$

W = The set of whole numbers= $\{0,1,2,3,...\}$

Z = The set of all integers= $\{0, \pm 1, \pm 2, \pm 3, ...\}$

 \mathbf{E} = The set of all even integers

$$= \{0, \pm 2, \pm 4, \pm 6, ...\}$$

O = The set of all odd integers

$$=\{\pm 1, \pm 3, \pm 5, ...\}$$

P =The set of prime numbers= $\{2,3,5,7,...\}$

 \mathbf{Q} = The set of rational numbers

$$= \left\{ x/x = \frac{m}{n}, \text{ where } m, n \in Z \land n \neq 0 \right\}$$

Q '= The set of irrational numbers

$$= \left\{ x/x \neq \frac{m}{n}, \text{ where } m, n \in \mathbb{Z} \land n \neq 0 \right\}$$

R = The set of real numbers = $Q \cup Q'$

Empty Set or Null Set (K.B + U.B)

A set having no element in it is called an empty set. It is represented by ϕ (phi) or $\{\}$.

Singleton Set

(K.B + U.B)

A set having only one element in it is called singleton set.

For example: $\{a\}$, $\{3\}$ etc.

Subset

(K.B + U.B)

(LHR 2016, D.G.K 2016)

"If A and B are two sets, such that every element of set A is present in set B, then set A is called subset of set B". It is represented as A⊂B.

For Example

If
$$A = \{1,2,3\}, B = \{1,2,3,4,5\}$$

Then $A \subset B$.

Note

(K.B + U.B)

- Number of all possible subsets = 2^n
- Number of all possible proper subsets =
- Number of improper subsets = 1

Proper Subset

(K.B + U.B)

"If A and B are two sets, such that every element of set A is present in set B and there is at least one element in set B which is not in set A, then set A is called proper subset of set B". It is represented as $A \subset B$.

For example:

If $A = \{1,2,3\}$ and $B = \{1,2,3,4,5\}$

Then set $A \subset B$.

Improper Subset

(K.B + U.B)

If A and B are two sets, such that every element of set A is present in set B and there is no more element in set B which is not in set A, then set A is called improper subset of set B.

For example: $A = \{1, 2, 3\}, B = \{3, 2, 1\}$

Then A is improper subset of set B.

Equal Sets

(K.B + U.B)

If A and B are two sets, such that $A \subseteq B$ and $B \subset A$ then A = B

For example

If
$$A = \{1,2,3\}, B = \{1,2,3\}$$

Then A = B

Unit-5 Sets and Functions

Power Set

(K.B + U.B)

A set consisting of all possible subsets of a set A is called power set of set A represented as P(A).

For example:

If $A = \{1,2\}$ then $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

Note

(K.B + U.B)

• Formula to find number of elements in the power set = 2ⁿ

Disjoint Sets

(K.B + U.B)

Two sets having no common elements are called disjoint sets.

i.e. If $A \cap B = \phi$ then A and B are called disjoint sets.

For example

If $A = \{1, 2\} B = \{a, b\}$, then A and B are disjoint sets.

Overlapping Sets

(K.B + U.B)

Two sets are called over lapping sets if they have:

- At least one element is common
- Neither set is subset of other.

For example

If $A = \{1,2,3\}, B = \{2,4,6,8,10\}$

Then A and B are over lapping sets.

Operation on Sets

Union of Two Sets

(BWP 2014) (K.B + U.B)

The union of two sets A and B written as $A \cup B$ (read as A union B) is a set consisting of all the elements which are either in A or in B or in both.

 $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B \text{ both}\}$ For example

If
$$A = \{1, 2, 3\}, B = \{2, 4, 6, 8, 10\}$$

Then $A \cup B = \{1, 2, 3, 4, 5, 8, 10\}$

Intersection of Two Sets

(LHR 2015) (K.B + U.B)

If A and B are two sets then a set consisting of common elements of set A and B is called A intersection B, represented by $A \cap B$.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

For example:

If
$$A = \{1, 2, 3\}, B = \{2, 4, 6, 8, 10\}$$

Then
$$A \cap B = \{2\}$$

Difference of Two Sets (K.B + U.B)

If A and B are two sets then their difference written as A - B or $A \setminus B$ is a set consisting of all the elements of A which are not in B.

$$A-B = \{x \mid x \in A \text{ and } x \notin B\}$$

$$B-A = \{x \mid x \in B \text{ and } x \notin A\}$$

For example:

If
$$A = \{1, 2, 3\}, B = \{2, 4, 6, 8, 10\}$$

Then
$$A - B = \{1, 3\}$$

Universal Set

(K.B + U.B)

A set consists of all the elements of the sets under consideration is called universal set. It is represented by capital English alphabet U or X.

For example:

U= Set of integers, $A = \{1,2,3\}$,

 $B=\{2,4,6,8,10\}$, then U is universal set.

Complement of a Set (K.B + U.B)

(GRW 2014, SWL 2015, BWP 2016)

If U is a universal set and A is its subset then U-A is called complement of A, represented as A' or A^c .

For example:

If
$$U = \{1, 2, 3, 4, 5\}, A = \{1, 2, 3\}$$

$$A' = U - A$$

= \{1, 2, 3, 4, 5\} - \{1, 2, 3\}

$$={4,5}$$

Exercise 5.1

Q.1 Given

$$X = \{1, 4, 7, 9\}$$

$$Y = \{2, 4, 5, 9\}$$

To Find

- (i) X∪Y
- (ii) $X \cap Y$
- (iii) YUX
- (iv) $Y \cap X$
- Solution:

(i)
$$X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$$

= $\{1, 2, 4, 5, 7, 9\}$

(A.B)

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Sets and Functions

(ii)
$$X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$$

= $\{4, 9\}$

(iii)
$$Y \cup X = \{2,4,5,9\} \cup \{1,4,7,9\}$$

= $\{1,2,4,5,7,9\}$

(iv)
$$Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$$

= $\{4, 9\}$

Q.2(A.B)

Given:

X = set of prime numbers less thanor equal to 17.

Y = set of first 12 natural number

To Find:

- (i) $X \cup Y$
- (ii) $Y \cup X$
- $X \cap Y$ (iii)
- $Y \cap X$ (iv)

Solution:

Here

X = Set of prime numbers less thanor equal to 17. $= \{2,3,5,7,11,13,17\}$

Y = Set of first 12 natural numbers ={1, 2, 3, ..., 12}

(i)
$$X \cup Y = \{2,3,5,7,11,13,17\} \cap \{1,2,3,...,12\}$$

= $\{1, 2, 3, ...,12, 13, 17\}$

(ii)
$$Y \cup X = \{1, 2, 3, ..., 12\} \cup \{2, 3, 5, 7, 11, 13, 17\}$$

= $\{1, 2, 3, ..., 12, 13, 17\}$

(iii)
$$X \cap Y = \{2,3,5,7,11,13,17\} \cap \{1,2,3,...,12\}$$

= $\{2,3,5,7,11\}$

(iv)
$$Y \cap X = \{1, 2, 3, ..., 12\} \cap \{2, 3, 5, 7, 11, 13, 17\}$$

= $\{2, 3, 5, 7, 11\}$

(A.B) Q.3

Given

$$X = \phi$$
, $Y = Z^+$, $T = O^+$

To Find

- $X \cup Y$ (i)
- (ii) $X \cup T$
- (iii) $Y \cup T$
- $X \cap Y$ (iv)
- $X \cap T$ (v)

(vi) $Y \cap T$ **Solution:**

(i)
$$X \cup Y = \phi \cup Z^+$$
$$= Z^+$$

(ii)
$$X \cup T = \phi \cup O^+$$

= O^+

(iii)
$$Y \cup T = Z^+ \cup O^+$$

= Z^+

(iv)
$$X \cap Y = \phi \cap Z^+$$

= ϕ

(v)
$$X \cap T = \phi \cap O^+$$

= ϕ

(vi)
$$Y \cap T = Z^+ \cap O^+$$

= O^+

 $U = \{x | x \in N \land 3 < x \le 25\}$

 $X = \{x \mid x \text{ is prime } \land 8 < x < 25\}$

$$Y = \{x \mid x \in W \land 4 \le x \le 17\}$$

To Find

(i)
$$(X \cup Y)'$$
 (ii) $X' \cap Y'$

(ii)
$$(X \cap Y)'$$
 (iv) $X' \cup Y'$

Solution: Here

$$U = \{x \mid x \in N \land 3 < x \le 25\}$$

= \{4,5,6,...,25\}

 $X = \{x \mid x \text{ is prime } \land 8 < x < 25\}$ = {11,13,17,19,23}

$$Y = \{x \mid x \in W \land 4 \le x \le 17\}$$

(i)
$$X \cup Y = \{11,13,17,19,23\} \cup \{4,5,6,...,17\}$$

$$(X \cup Y)' = U - (X \cup Y)$$

$$= \{4,5,6,...,25\} - \{4,5,6,...,17,19,23\}$$

$$= \{18, 20, 21, 22, 24, 25\}$$

(ii)
$$X' = U - X$$

$$= \{4, 5, 6, \dots, 25\} - \{11, 13, 17, 19, 23\}$$

=
$$\{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\}$$

 $\mathbf{Y}' = \mathbf{U} - \mathbf{Y}$

$$= \{18,19,20,21,22,23,24,25\}$$

$$X' \cap Y' = \{4,5,6,\dots,10,12,14,15,16,18,20,21,22,24,25\}$$

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$$= \{18, 20, 21, 22, 24, 25\}$$
(iii) $X \cap Y = \{11, 13, 17, 19, 23\} \cap \{4, 5, 6, ..., 17\}$

$$= \{11, 13, 17\}$$

$$(X \cap Y)' = U - (X \cap Y)$$

$$= \{4, 5, 6,, 25\} - \{11, 13, 17\}$$

$$= \{4, 5, 6,, 10, 12, 14, 15, 16, 18, 19, 20,, 25\}$$
(iv) $X' \cup Y'$

$$= \{4, 5, 6,, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$\cup \{18, 19, 20, 21, 22, 23, 24, 25\}$$

$$= \{4, 5, 6,, 10, 12, 14, 15, 16, 18, 19, 20,, 25\}$$
Q.5 Given (A.B)
$$X = \{2, 4, 6,, 20\} \quad \text{(LHR 2014)}$$

$$Y = \{4, 8, 12,, 24\} \quad \text{(FSD 2015)}$$
To Find
(i) $X - Y$
(ii) $Y - X$
Solution:
(i) $X - Y = \{2, 4, 6,, 20\} - \{4, 8, 12,, 24\}$

$$= \{2, 6, 10, 14, 18\}$$
(ii) $Y - X = \{4, 8, 12,, 24\} - \{2, 4, 6,, 20\}$

$$= \{24\}$$
Q.6 Given (BWP 2014) (A.B)
$$A = N, B = W \quad \text{(LHR 2015)}$$
To Find (D.G.K 2014)
(i) $A - B$
(ii) $B - A$
Solution:
(i) $A - B = N - W = \{1, 2, 3, ...\} - \{0, 1, 2, 3, ...\}$

$$= \phi$$
(ii) $B - A = W - N = \{0, 1, 2, 3, ...\} - \{1, 2, 3, ...\}$

$$= \{0\}$$