Properties of Union and Intersection of Sets (K.B)

- **Commutative property of Union**
  $A \cup B = B \cup A$
- **Commutative property of intersection**
  $A \cap B = B \cap A$
- **Associative property of union**
  $A \cup (B \cup C) = (A \cup B) \cup C$
- **Associative property of intersection**
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Distributive property of union over intersection**
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- **Distributive property of intersection over union**
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **De-Morgan's laws**
  $(A \cup B)' = A' \cap B'$
  $(A \cap B)' = A' \cup B'$

Commutative Property of Union

(K.B + U.B)

For any two sets A and B, prove that $A \cup B = B \cup A$.

Proof

Let $x \in A \cup B$

$\Rightarrow x \in A \text{ or } x \in B$ (by definition of union of sets)

$\Rightarrow x \in B \text{ or } x \in A$

$\Rightarrow x \in (B \cup A)$

$\Rightarrow (A \cup B) \subseteq (B \cup A) \rightarrow \text{(i)}$

Now let $y \in (B \cup A)$

$\Rightarrow y \in B \text{ or } y \in A$ (by definition of union of sets)

$\Rightarrow y \in A \text{ or } y \in B$

$\Rightarrow y \in (A \cup B)$

$\Rightarrow (B \cup A) \subseteq (A \cup B) \rightarrow \text{(ii)}$

From (i) and (ii), we have $A \cup B = B \cup A$.
(by definition of equal sets)

Commutative Property of Intersection

(MTN 2014) **(K.B + U.B)**

For any two sets A and B, prove that $A \cap B = B \cap A$

Proof

Let $x \in (A \cap B)$

$\Rightarrow x \in A$ and $x \in B$
(by definition of intersection of sets)

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in (B \cap A)$

$\therefore (A \cap B) \subseteq (B \cap A) \rightarrow \text{(i)}$

Now let $y \in (B \cap A)$

$\Rightarrow y \in B$ and $y \in A$
(by definition of intersection of sets)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in (A \cap B)$

Therefore, $(B \cap A) \subseteq (A \cap B) \rightarrow \text{(ii)}$

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)

Associative Property of Union

(K.B + U.B)

For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$

Proof

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in c$

(Associative property)

$\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Rightarrow x \in A \text{ or } x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$\Rightarrow (A \cup B) \cup C \subseteq A \cup (B \cup C) \rightarrow \text{(i)}$

Now let $y \in A \cup (B \cup C)$

$\Rightarrow y \in A \text{ or } y \in (B \cup C)$

$\Rightarrow y \in A \text{ or } (y \in B \text{ or } y \in C)$

$\Rightarrow y \in (A \text{ or } y \in B) \text{ or } y \in C$

$\Rightarrow y \in (A \cup B) \text{ or } y \in C$

$\Rightarrow y \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C \rightarrow \text{(ii)}$

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

Associative Property of Intersection

(K.B + U.B)

For any three sets A, B and C, prove that $(A \cap B) \cap C = A \cap (B \cap C)$

Proof

Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B) \text{ and } x \in C$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C$

$\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \in C)$

$\Rightarrow x \in A \text{ and } x \in (B \cap C)$

$\Rightarrow x \in A \cap (B \cap C)$

$\Rightarrow (A \cap B) \cap C \subseteq A \cap (B \cap C) \rightarrow \text{(i)}$

Now let $y \in A \cap (B \cap C)$

$\Rightarrow y \in A \text{ and } y \in (B \cap C)$

$\Rightarrow y \in A \text{ and } (y \in B \text{ and } y \in C)$

$\Rightarrow (y \in A \text{ and } y \in B) \text{ and } y \in C$

$\Rightarrow y \in (A \cap B) \text{ and } y \in C$

$\Rightarrow y \in (A \cap B) \cap C$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C \rightarrow \text{(ii)}$

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Property of Union over Intersection

(K.B + U.B)

For any three sets A, B and C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof

Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A \text{ or } x \in (B \cap C)$

$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \rightarrow \text{(i)}$

Similarly, now let y $\in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B) \text{ and } y \in (A \cup C)$

$\Rightarrow (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \in C)$

$\Rightarrow y \in A \text{ or } (y \in B \text{ and } y \in C)$

$\Rightarrow y \in A \text{ or } y \in (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \rightarrow \text{(ii)}$

From (i) and (ii), we have

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributive Property of Intersection over Union

(K.B + U.B)

For any three sets A, B and C, prove that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof

Let $x\in A\cap(B\cup C)$

$\Rightarrow x\in A \text{ and } x\in(B\cup C)$

$\Rightarrow x\in A \text{ and } (x\in B \text{ or } x\in C)$

$\Rightarrow (x\in A \text{ and } x\in B)$
or $(x\in A \text{ and } x\in C)$

$\Rightarrow x\in(A\cap B) \text{ or } x\in(A\cap C)$

$\Rightarrow x\in(A\cap B)\cup(A\cap C)$

$A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)\rightarrow(\text{i})$

Now let

$\Rightarrow y\in(A\cap B)\cup(A\cap C)$

$\Rightarrow y\in(A\cap B) \text{ or } y\in(A\cap C)$

or $(y\in A \text{ and } y\in C)$

$\Rightarrow (y\in A \text{ and } y\in B)$

$\Rightarrow y\in A \text{ and } (y\in B \text{ or } y\in C)$

$\Rightarrow y\in A \text{ and } y\in(B\cup C)$

$y\in A\cap(B\cup C)$

$(A\cap B)\cup(A\cap C)\subseteq A\cap(B\cup C)\rightarrow(\text{ii})$

From (i) and (ii), we have

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

De-Morgan's Laws **(K.B + U.B)**

(LHR 2016, MTN 2016, SGD 2015, BWP 2016, D.G.K 2016)

For any two sets A and B belonging from universal set U.

$(A\cup B)'=A'\cap B'\rightarrow(\text{i})$

$(A\cap B)'=A'\cup B'\rightarrow(\text{ii})$

(i) Proof

Let $x\in(A\cup B)'$

$\Rightarrow x\notin(A\cup B)$ (by definition of complement of set)

$\Rightarrow x\notin A \text{ and } x\notin B$

$\Rightarrow x\in A' \text{ and } x\in B'$

$\Rightarrow x\in A'\cap B'$ (by definition of intersection of sets)

$\Rightarrow (A\cup B)'\subseteq A'\cap B'\rightarrow(\text{i})$

Let $y\in A'\cap B'$

$\Rightarrow y\in A' \text{ and } y\in B'$

$\Rightarrow y\notin A \text{ and } y\notin B$

$\Rightarrow y\notin(A\cup B)$ (by definition of union of set)

$y\in(A\cup B)'$

$A'\cap B'\subseteq(A\cup B)'\rightarrow(\text{ii})$

Using (i) and (ii), we have

$(A\cup B)'=A'\cap B'$

(ii) Proof

Let $x\in(A\cap B)'$

$\Rightarrow x\notin A\cap B$ (by definition of complement of set)

$\Rightarrow x\notin A \text{ or } x\notin B$

$\Rightarrow x\in A' \text{ or } x\in B'$

$\Rightarrow x\in A'\cup B'$ (by definition of union of set)

$\Rightarrow (A\cap B)'\subseteq A'\cup B'\rightarrow(\text{i})$

Let $y\in A'\cup B'$

$\Rightarrow y\in A' \text{ or } y\in B'$

$\Rightarrow y\notin A \text{ or } y\notin B$

$\Rightarrow y\notin A\cap B$ (by definition of intersection of sets)

$\Rightarrow y\in(A\cap B)'$

$\Rightarrow A'\cup B'\subseteq(A\cap B)'\rightarrow(\text{ii})$

From (i) and (ii), we have proved that

$(A\cap B)'=A'\cup B'$

Exercise 5.2

Q.1 Given $X=\{1,3,5,7,....,19\}$ **(A.B)**

$Y=\{0,2,4,6,....,20\}$

$Z=\{2,3,5,7,11,13,17,19,23\}$

To Find

(i) $X\cup(Y\cup Z)$

(ii) $(X\cup Y)\cup Z$

(iii) $X\cap(Y\cap Z)$

(iv) $(X\cap Y)\cap Z$

(v) $X\cup(Y\cap Z)$

(vi) $(X\cup Y)\cap(X\cup Z)$

(vii) $X\cap(Y\cup Z)$

(viii) $(X\cap Y)\cup(X\cap Z)$

Solution:

(i) $X\cup(Y\cup Z)$ **(RWP 2015)** **(A.B)**

$=\{1,3,5,7,.....,19\}\cup$

$(\{0,2,4,6,....,20\}\cup\{2,3,5,7,11,13,17,19,23\})$

$=\{1,3,5,....,19\}\cup$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$ **(A.B)**

$X\cup Y=\{1,3,5,7,.....,19\}\cup\{0,2,4,6,....,20\}$

$=\{0,1,2,3,....,20\}$

$(X\cup Y)\cup Z$

$=\{0,1,2,3,.....,20\}\bigcup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,1,2,3,.....,20,23\}$

(iii) $X\cap(Y\cap Z)$ **(A.B)**

$Y\cap Z=\{0,2,4,6,....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cap(Y\cap Z)=\{1,3,5,7,.....,19\}\cap\{2\}$

$=\{\}$

(iv) $(X\cap Y)\cap Z$ **(A.B)**

$X\cap Y=\{1,3,5,7,....,19\}\cap\{0,2,4,6,....,20\}$

$=\{\}$

$(X\cap Y)\cap Z=\{\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{\}$

(v) $X\cup(Y\cap Z)$ **(A.B)**

$Y\cap Z=\{0,2,4,6,....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cup(Y\cap Z)=\{1,3,5,7,....,19\}\cup\{2\}$

$=\{1,2,3,5,7,....,19\}$

(vi) $(X\cup Y)\cap(X\cup Z)$ **(A.B)**

$X\cup Y=\{1,3,5,....,19\}\cup\{0,2,4,6,....,20\}$

$=\{0,1,2,3,....,20\}$

$X\cup Z=\{1,3,5,...,19\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{1,2,3,5,7,....,19,23\}$

$(X\cup Y)\cap(X\cup Z)=\{0,1,2,3,...,20\}$

$\cap\{1,2,3,5,7,.....,19,23\}$

$=\{1,2,3,5,7,....,19\}$

(vii) $X\cap(Y\cup Z)$ **(A.B)**

$Y\cup Z=\{0,2,4,6,...,20\}$

$\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$X\cap(Y\cup Z)=\{1,3,5,7,....,19\}\cap$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{3,5,7,11,13,17,19\}$

(viii) $(X\cap Y)\cup(X\cap Z)$ **(A.B)**

$X\cap Y=\{1,3,5,7,....,19\}\bigcap\{0,2,4,6,...,20\}$

$=\{\}$

$X\cap Z=\{1,3,5,7,....,19\}$

$\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{3,5,7,11,13,17,19\}$

$(X\cap Y)\cup(X\cap Z)=\{\}\cup\{3,5,7,11,13,17,19\}$

$=\{3,5,7,11,13,17,19\}$

Q.2 **Given** $A=\{1,2,3,4,5,6\}$ **(A.B + K.B)**

$B=\{2,4,6,8\}$

$C=\{1,4,8\}$

To Prove

(i) $A\cap B=B\cap A$

(ii) $A\cup B=B\cup A$

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proof

(i) $A\cap B=B\cap A$ **(A.B + K.B)**

L.H.S = $A\cap B$

$=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}\rightarrow(i)$

R.H.S = $B\cap A$

$=\{2,4,6,8\}\cap\{1,2,3,4,5,6\}$

$=\{2,4,6\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap B=B\cap A$

Hence Proved

(ii) $A\cup B=B\cup A$ **(A.B + K.B)**

Proof

L.H.S $=A\cup B$

$=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(i)$

R.H.S $=B\cup A$

$=\{2,4,6,8\}\cup\{1,2,3,4,5,6\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup B=B\cup A$

Hence Proved

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(A.B + K.B)

Proof

L.H.S $= A\cap(B\cup C)$

$B\cup C=\{2,4,6,8\}\cup\{1,4,8\}$

$=\{1,2,4,6,8\}$

$A\cap(B\cup C)$

$=\{1,2,3,4,5,6\}\cap\{1,2,4,6,8\}$

$=\{1,2,4,6\}\rightarrow(\text{i})$

R.H.S $=(A\cap B)\cup(A\cap C)$

$A\cap B=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}$

$A\cap C=\{1,2,3,4,5,6\}\cap\{1,4,8\}$

$=\{1,4\}$

$(A\cap B)\cup(A\cap C)=\{2,4,6\}\cup\{1,4\}$

$=\{1,2,4,6\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence Proved

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(A.B + K.B)

Proof

L.H.S $=A\cup(B\cap C)$

$B\cap C=\{2,4,6,8\}\cap\{1,4,8\}$

$=\{4,8\}$

L.H.S $=A\cup(B\cap C)$

$=\{1,2,3,4,5,6\}\cup\{4,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(\text{i})$

R.H.S $=(A\cup B)\cap(A\cup C)$

$A\cup B=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}$

$A\cup C=\{1,2,3,4,5,6\}\cup\{1,4,8\}$

$=\{1,2,3,4,5,6,8\}$

R.H.S $=(A\cup B)\cap(A\cup C)$

$=\{1,2,3,4,5,6,8\}\cap\{1,2,3,4,5,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence Proved

Q.3 **Given** $U=\{1,2,3,....,10\}$

(A.B + K.B)

$A=\{1,3,5,7,9\}$

$B=\{2,3,5,7\}$

To Prove

(i) $(A\cap B)'=A'\cup B'$

(ii) $(A\cup B)'=A'\cap B'$

(i) $(A\cap B)'=A'\cup B'$

Proof

L.H.S $=(A\cap B)'$

$A\cap B=\{1,3,5,7,9\}\cap\{2,3,5,7\}$

$=\{3,5,7\}$

$(A\cap B)'=U-(A\cap B)$

$=\{1,2,3,....,10\}-\{3,5,7\}$

$=\{1,2,4,6,8,9,10\}\rightarrow(\text{i})$

R.H.S $=A'\cup B'$

$A'=U-A$

$=\{1,2,3,...,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$B'=U-B$

$=\{1,2,3,....,10\}-\{2,3,5,7\}$

$=\{1,4,6,8,9,10\}$

$A'\cup B'=\{2,4,6,8,10\}\cup\{1,4,6,8,9,10\}$

$=\{1,2,4,6,8,9,10\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)'=A'\cup B'$

Hence Proved

(ii) $(A \cup B)' = A' \cap B'$ **(A.B + K.B)**

Proof

L.H.S $= (A \cup B)'$

$A \cup B = \{1,3,5,7,9\} \cup \{2,3,5,7\}$

$= \{1,2,3,5,7,9\}$

L.H.S $= (A \cup B)' = U - (A \cup B)$

$= \{1,2,3,....,10\} - \{1,2,3,5,7,9\}$

$= \{4,6,8,10\} \rightarrow (i)$

R.H.S $= A' \cap B'$

$= \{2,4,6,8,10\} \cap \{1,4,6,8,9,10\}$

$= \{4,6,8,10\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Hence Proved

Q.4 Given **(A.B + K.B)**

$U = \{1,2,3,....,20\}$

$X = \{1,3,7,9,15,18,20\}$

$Y = \{1,3,5,...,17\}$

To Prove

(i) $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

Proof

(i) $X - Y = X \cap Y'$ **(A.B + K.B)**

L.H.S = X – Y

$= \{1,3,7,9,15,18,20\} - \{1,3,5,....,17\}$

$= \{18,20\} \rightarrow (i)$

$Y' = U - Y$

$= \{1,2,3,....,20\} - \{1,3,5,...,17\}$

$= \{2,4,6,.....,18,19,20\}$

R.H.S $= X \cap Y'$

$= \{1,3,7,9,15,18,20\} \cap \{2,4,6,.....,18,19,20\}$

$= \{18,20\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$X - Y = X \cap Y'$

Hence Proved

(ii) $Y - X = Y \cap X'$ **(A.B + K.B)**

Proof

L.H.S = Y – X

$= \{1,3,5,....,17\} - \{1,3,7,9,15,18,20\}$

$= \{5,11,13,17\} \rightarrow (i)$

L.H.S $= Y \cap X'$

$X' = U - X$

$= \{1,2,3,.....,20\} - \{1,3,7,9,15,18,20\}$

$= \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$Y \cap X' = \{1,3,5,....,17\}$

$\cap \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$= \{5,11,13,17\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$Y - X = Y \cap X'$

Hence Proved