Mathematics-10
Unit 5 – 5.3

Venn Diagram **(K.B)**

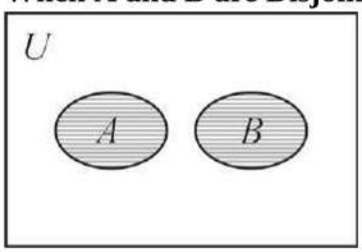
British mathematician John Venn introduced Venn Diagrams.

"A graphical method to represent sets in which Universal Set is represented by a rectangle and its subsets by a closed shaped (i.e. circle or oval) in it, is called Venn Diagram".

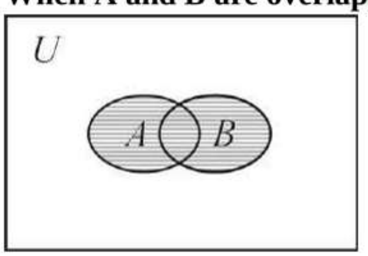
Operations on Sets Through Venn Diagrams **(K.B)**

Union of Two Sets

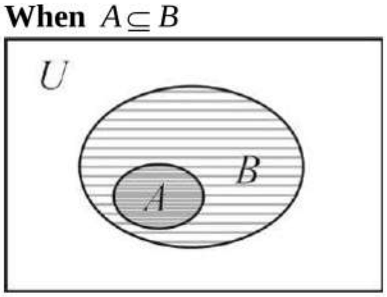
When A and B are Disjoint **(K.B)**



When A and B are overlapping **(K.B)**

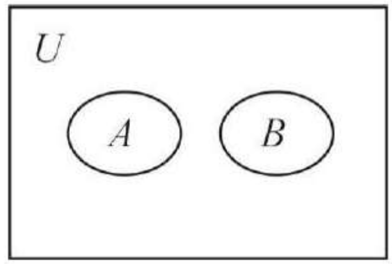


When $A \subseteq B$ **(K.B)**

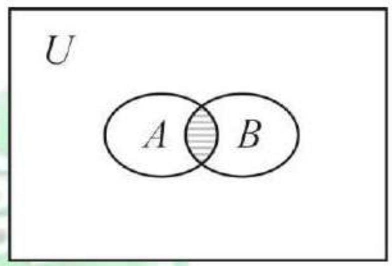


Intersection of Two Sets **(K.B)**

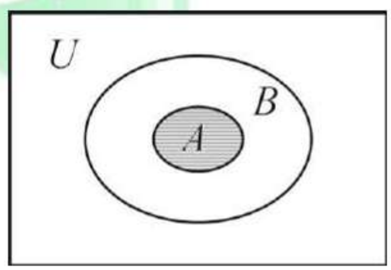
When A and B are Disjoint



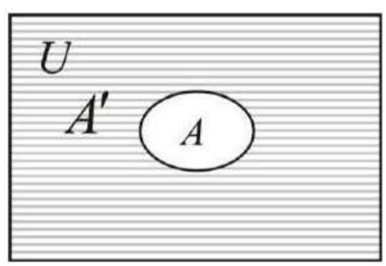
When A and B are overlapping **(K.B)**



When $A \subseteq B$



Compliment of a Set **(K.B)**



Exercise 5.3

Q.1 **Given** $U = \{1,2,3,4,...,10\}$

$A = \{1,3,5,7,9\}$ **(K.B)**

$B = \{1,4,7,10\}$ **(A.B)**

(LHR 2017, GRW 2016, FSD 2017, SWL 2017, RWP 2016, BWP 2016, D.G.K 2016)

To Prove

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

(vi) $(B - A)' = B' \cup A$

Proof

(i) $A - B = A \cap B'$

L.H.S = A – B

$= \{1,3,5,7,9\} - \{1,4,7,10\}$

$= \{3,5,9\} \rightarrow (i)$

R.H.S = $A \cap B'$

$B' = U - B$

$= \{1,2,3,....,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$A \cap B' = \{1,3,5,7,9\} \cap \{2,3,5,6,8,9\}$

$= \{3,5,9\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A - B = A \cap B'$

Hence Proved

(ii) $B - A = B \cap A'$ **(K.B + A.B)**

L.H.S = B – A

$= \{1,4,7,10\} - \{1,3,5,7,9\}$

$= \{4,10\} \rightarrow (i)$

R.H.S = $B \cap A'$

$A' = U - A = \{1,2,3,...,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B \cap A' = \{1,4,7,10\} \cap \{2,4,6,8,10\}$

$= \{4,10\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$B - A = B \cap A'$

Hence Proved

(iii) $(A \cup B)' = A' \cap B'$ **(K.B + A.B)**

$A \cup B = \{1,3,5,7,9\} \cup \{1,4,7,10\}$

$= \{1,3,4,5,7,9,10\}$

L.H.S = $(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,....,10\} - \{1,3,4,5,7,9,10\}$

$= \{2,6,8\} \rightarrow (i)$

R.H.S = $A' \cap B'$

$A' = U - A = \{1,2,3...,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = U - B = \{1,2,3,...,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

R.H.S = $A' \cap B'$

$= \{2,4,6,8,10\} \cap \{2,3,5,6,8,9\}$

$= \{2,6,8\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Hence Proved

(iv) $(A \cap B)' = A' \cup B'$ **(K.B + A.B)**

Proof

$A \cap B = \{1,3,5,7,9\} \cap \{1,4,7,10\}$

$= \{1,7\}$

L.H.S = $(A \cap B)' = U - (A \cap B)$

$= \{1,2,3,....,10\} - \{1,7\}$

$= \{2,3,4,5,6,8,9,10\} \rightarrow (i)$

R.H.S = $A' \cup B'$

$= \{2,4,6,8,10\} \cup \{2,3,5,6,8,9\}$

$= \{2,3,4,5,6,8,9,10\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A \cap B)' = A' \cup B'$

Hence Proved

(v) $(A-B)' = A' \cup B$ **(K.B + A.B)**

Proof

$\text{L.H.S} = (A-B)'$

$A-B = \{1,3,5,7,9,\} - \{1,4,7,10\}$

$= \{3,5,9\}$

$\text{L.H.S} = (A-B)' = U-(A-B)$

$= \{1,2,3,...,10\} - \{3,5,9\}$

$= \{1,2,4,6,7,8,10\} \rightarrow (\text{i})$

$\text{R.H.S} = A' \cup B$

$A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$A' \cup B = \{2,4,6,8,10\} \cup \{1,4,7,10\}$

$= \{1,2,4,6,7,8,10\} \rightarrow (\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$(A-B)' = A' \cup B$

Hence Proved

(vi) $(B-A)' = B' \cup A$ **(K.B + A.B)**

(FSD 2015)

Proof

$B-A = \{1,4,7,10\} - \{1,3,5,7,9\}$

$B-A = \{4,10\}$

$\text{L.H.S} = (B-A)' = U-(B-A)$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{4,10\}$

$= \{1,2,3,5,6,7,8,9\} \rightarrow (\text{i})$

$\text{R.H.S} = B' \cup A$

$B' = U - B$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$B' \cup A = \{2,3,5,6,8,9\} \cup \{1,3,5,7,9\}$

$= \{1,2,3,5,6,7,8,9\} \rightarrow (\text{ii})$

From (i) and (ii) we get

L.H.S=R.H.S

$(B-A)' = B' \cup A$

Hence Proved

Q.2 **If** $U=\{1,2,3,4,5,6,7,8,9,10\}$

$A=\{1,3,5,7,9\}$ **(K.B + A.B)**

$B=\{1,4,7,10\}$

$C=\{1,5,8,10\}$

Then Verify

(i) $(A\cup B)\cup C=A\cup(B\cup C)$

(ii) $(A\cap B)\cap C=A\cap(B\cap C)$

(iii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(iv) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(i) $(A\cup B)\cup C=A\cup(B\cup C)$

(K.B + A.B)

Proof

$A\cup B=\{1,3,5,7,9\}\cup\{1,4,7,10\}$

$=\{1,3,4,5,7,9,10\}$

L.H.S $=(A\cup B)\cup C$

$=\{1,3,4,5,7,9,10\}\cup\{1,5,8,10\}$

$=\{1,3,4,5,7,8,9,10\}\rightarrow$(i)

$B\cup C=\{1,4,7,10\}\cup\{1,5,8,10\}$

$=\{1,4,5,7,8,10\}$

R.H.S $=A\cup(B\cup C)$

$=\{1,3,5,7,9\}\cup\{1,4,5,7,8,10\}$

$=\{1,3,4,5,7,8,9,10\}\rightarrow$(ii)

From (i) and (ii), we get

L.H.S=R.H.S

$(A\cup B)\cup C=A\cup(B\cup C)$

Hence proved

(ii) $(A\cap B)\cap C=A\cap(B\cap C)$

(K.B + A.B)

Proof

L.H.S $=(A\cap B)\cap C$

$=(\{1,3,5,7,9\}\cap\{1,4,7,10\})\cap\{1,5,8,10\}$

$=\{1,7\}\cap\{1,5,8,10\}$

$=\{1\}\rightarrow$(i)

R.H.S $=A\cap(B\cap C)$

$=\{1,3,5,7,9\}\cap(\{1,4,7,10\}\cap\{1,5,8,10\})$

$=\{1,3,5,7,9\}\cap\{1,10\}$

$=\{1\}\rightarrow$(ii)

From (i) and (ii), we get

L.H.S=R.H.S

$(A\cup B)\cup C=A\cup(B\cup C)$

Hence proved

(iii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(K.B + A.B)

Proof

L.H.S $=A\cup(B\cap C)$

$=\{1,3,5,7,9\}\cup(\{1,4,7,10\}\cap\{1,5,8,10\})$

$=\{1,3,5,7,9\}\cup\{1,10\}$

$=\{1,3,5,7,9,10\}\rightarrow$(i)

$A\cup B=\{1,3,5,7,9\}\cup\{1,4,7,10\}$

$=\{1,3,4,5,7,9,10\}$

$A\cup C=\{1,3,5,7,9\}\cup\{1,5,8,10\}$

$=\{1,3,5,7,8,9,10\}$

R.H.S $=(A\cup B)\cap(A\cup C)$

$=\{1,3,4,5,7,9,10\}\cap\{1,3,5,7,8,9,10\}$

$=\{1,3,5,7,9,10\}\rightarrow$(ii)

From (i) and (ii), we get

L.H.S=R.H.S

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence Proved

(iv) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(K.B + A.B)

Proof

$B\cup C=\{1,4,5,7,8,10\}\cup\{1,5,8,10\}$

$=\{1,4,5,7,8,10\}$

L.H.S $=A\cap(B\cup C)$

$=\{1,3,5,7,9\}\cap\{1,4,5,7,8,10\}$

$=\{1,5,7\}\rightarrow$(i)

$A\cap B=\{1,3,5,7,9\}\cap\{1,4,7,10\}$

$=\{1,7\}$

$A\cap C=\{1,3,5,7,9\}\cap\{1,5,8,10\}$

$=\{1,5\}$

R.H.S $=(A\cap B)\cup(A\cap C)$

$=\{1,7\}\cup\{1,5\}$

$=\{1,5,7\}\rightarrow(\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence Proved

Q.3 Given $U=N,\quad A=\phi,\quad B=P$ **(K.B + A.B)**

(LHR 2016, GRW 2016, RWP 2015, 17, MTN 2016)

To prove:

De-Morgan's Laws

(i) $(A\cup B)'=A'\cap B'$

(ii) $(A\cap B)'=A'\cup B'$

(i) $(A\cup B)'=A'\cap B'$

Proof

$\text{L.H.S}=(A\cup B)'$

$A\cup B=\phi\cup P$

$=P$

$\text{L.H.S}=(A\cup B)'=U-(A\cup B)$

$=N-P\rightarrow(\text{i})$

$\text{R.H.S}=A'\cap B'$

$A'=U-A=N-\phi$

$=N$

$B'=U-B=N-P$

$\text{R.H.S}=A'\cap B'=N\cap(N-P)$

$=N-P\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)'=A'\cap B'$

Hence Proved

(ii) $(A\cap B)'=A'\cup B'$ **(K.B + A.B)**

Proof

$A\cap B=\phi\cap P$

$=\phi$

$\text{L.H.S}=(A\cap B)'$

$=U-(A\cap B)$

$=N-\phi$

$=N$

$\text{R.H.S}=A'\cup B'$

$=N\cup(N-P)$

$=N\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)'=A'\cup B'$

Hence Proved

Method-(II)

(i) $(A\cup B)'=A'\cap B'$ **(K.B + A.B)**

Proof

$A\cup B=\{\ \}\cup\{2,3,5,7.....\}$

$=\{2,3,5,7,....\}$

$\text{L.H.S}=(A\cup B)'=U-(A\cup B)$

$=\{1,2,3,....\}-\{2,3,5,7,....\}$

$=\{1,4,6,8,9,10,12,.....\}\rightarrow(\text{i})$

$A'=U-A$

$=\{1,2,3,....\}-\{\ \}$

$=\{1,2,3,4.....\}$

$B'=U-B$

$=\{1,2,3,4,.....\}-\{2,3,5,7....\}$

$=\{1,4,6,8,9,10,....\}$

Now

$\text{R.H.S}=A'\cap B'=\{1,2,3,...\}\cap\{1,4,6,8,9,10,...\}$

$=\{1,4,6,8,9,10,....\}\rightarrow(\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)'=A'\cap B'$

Hence Proved

Q.4 If $U=\{1,2,3,....,10\}$ **(K.B + A.B)**

$A=\{1,3,5,7,9\}$

$B=\{2,3,4,5,8\}$

then prove the following question by venn diagram.

(i) $A-B=A\cap B'$

(ii) $B-A=B\cap A'$

(iii) $(A\cup B)'=A'\cap B'$

(iv) $(A\cap B)'=A'\cup B'$

(v) $(A-B)' = A' \cup B$

(vi) $(B-A)' = B' \cup A$

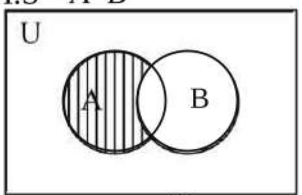
(Through Venn diagram)

(i) $A - B = A \cap B'$ **(K.B)**

Proof

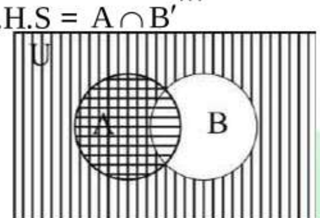
A and B are overlapping sets because $A \cap B = \{3,5\}$)

L.H.S $= A - B$



B-A |||

R.H.S = $A \cap B'$



A ≡, ⋕

B' |||, ⋕

$A \cap B'$ ⋕

Same regions represent that

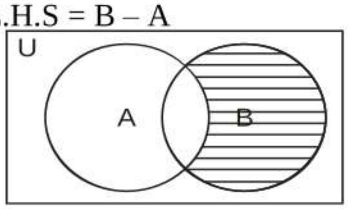
L.H.S = R.H.S

$A - B = A \cap B'$

Hence Proved

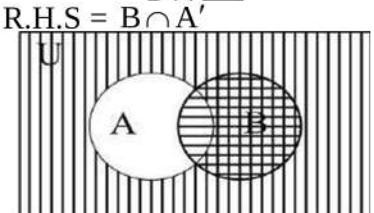
(ii) $B - A = B \cap A'$ **(K.B)**

L.H.S = B – A



B-A ≡

R.H.S = $B \cap A'$



B ≡, ⋕

A' |||, ⋕

$B \cap A'$ ⋕

Same regions represent that

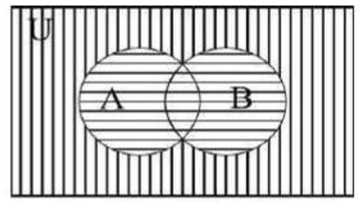
L.H.S = R.H.S

$B - A = B \cap A'$

Hence Proved

(iii) $(A \cup B)' = A' \cap B'$ **(K.B)**

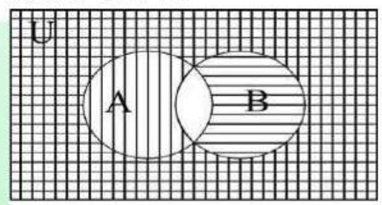
L.H.S=$(A \cup B)'$



$A \cup B$ ≡

$(A \cup B)'$ |||

R.H.S=$A' \cap B'$



A' ≡, ⋕

B' |||, ⋕

$A' \cap B'$ ⋕

Same regions represent that

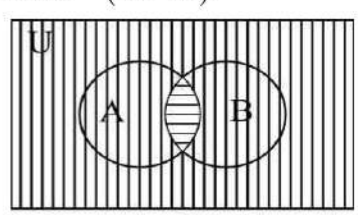
L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Hence Proved

(iv) $(A \cap B)' = A' \cup B'$ **(K.B)**

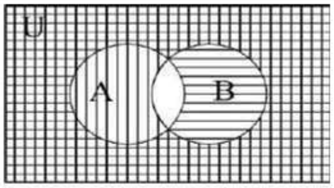
L.H.S = $(A \cap B)'$



$A \cap B$ ≡

$(A \cap B)'$ |||

R.H.S $= A' \cup B'$



A' ≡, #

B' |||, #

$A' \cup B'$ ≡, |||, #

Same regions represent that

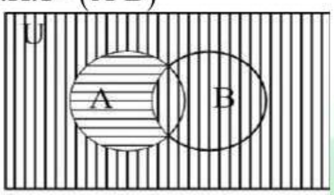
L.H.S = R.H.S

$(A \cap B)' = A' \cup B'$

Hence Proved

(v) $(A-B)' = A' \cup B$ **(K.B)**

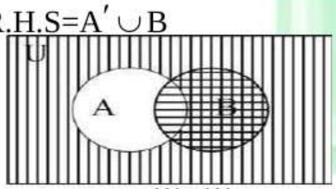
L.H.S=$(A\text{-}B)'$



$A-B$ ≡

$(A-B)'$ |||

R.H.S=$A' \cup B$



A' |||, #

B ≡, #

$A' \cup B$ ≡, |||, #

Same regions represent that

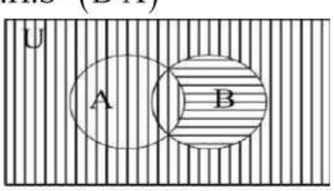
L.H.S = R.H.S

$(A-B)' = A' \cup B$

Hence Proved

(vi) $(B-A)' = B' \cup A$ **(K.B)**

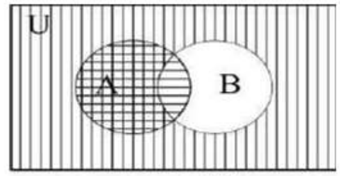
L.H.S=$(B\text{-}A)'$



$B-A$ ≡

$(B-A)'$ |||

R.H.S=$B' \cup A$



A ≡, #

B' |||, #

$B' \cup A$ ≡, |||, #

Same regions represent that

L.H.S = R.H.S

$(B-A)' = B' \cup A$

Hence Proved