Binary Relation **(K.B)**

(GRW 2017, RWP 2016, SWL 2016, MTN 2016)

If A and B are any two non-empty sets then a subset $R \subseteq A \times B$ is called a binary relation from set A into set B.

i.e $A = \{1,2\}, B = \{a\}$

Then $A \times B = \{(1,a),(2,a)\}$ and

$R_1 = \{(1,a)\}, \quad R_2 = \{(2,a)\}$

$R_3 = \{(1,a),(2,a)\}, \quad R_4 = \phi$

Are all possible relations.

Note **(K.B + U.B)**

Formula to find number of binary relations is $2^{m \times n}$ where $m =$ number of elements in set A and $n =$ number of elements in set B.

Domain of Relation **(K.B)**

Domain of a relation denoted by Dom(R) is a set consisting of all the first elements of each ordered pair in the relation.

For example:

If $R = \{(0,2),(2,3),(3,3),(3,4)\}$

Then $Dom(R) = \{0,2,3\}$

Range of a Relation **(K.B)**

Range of a relation is a set containing all the second elements of each ordered pair of the relation.

For Example:

If $R = \{(0,2),(2,3),(3,3),(3,4)\}$

Then Range (R) = $\{2,3,4\}$

Function **(K.B + U.B + A.B)**

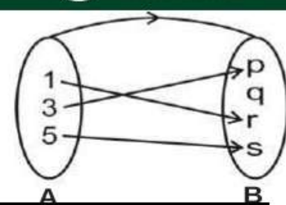
(LHR 2014)

Suppose A and B are two non-empty sets, then relation f : $A \to B$ is called a function if

(i) Dom $(f) = A$

(ii) Every $x \in A$ appears in one and only one ordered pair in *f*.

Example:



Types of Function **(K.B + U.B)**

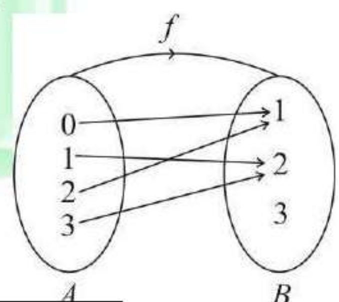
Into Function **(A.B)**

A function $f : A \to B$ is called an into function, if at least one element in set B is not an image of some element of set A i.e., Range of $f \subset B$.

For example, if $A = \{0,1,2,3\}$ and $B = \{1,2,3\}$, then $f : A \to B$ such that

$f = \{(0,1),(1,2),(2,1),(3,2)\}$.

$\because$ Range$(f) = \{1,2\} \subset B$. Thus *f* is an into function.



Onto Function **(K.B + U.B + A.B)**

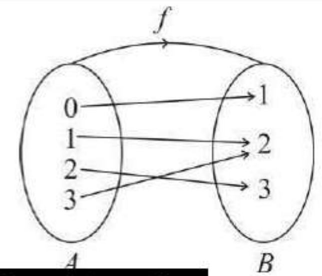
(SWL 2016, BWP 2014, 16, 17)

A function $f : A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e., Range of $f = B$.

For example, if $A = \{0,1,2,3\}$ and $B = \{1,2,3\}$, then $f : A \to B$ such that

$f = \{(0,1),(1,2),(2,3),(3,2)\}$.

$\because$ Range $(f) = \{1,2,3\} = B$. Thus f is an onto function.



One-to-One Function

(LHR 2015) **(K.B + U.B + A.B)**

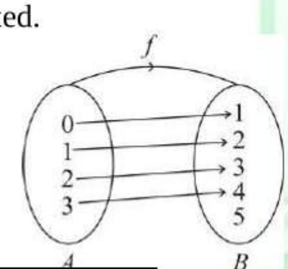
A function $f: A \to B$ is called one-one function if all distinct elements of A have distinct images in B, i.e., $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \in A$

or $x_1 \neq x_2 \in A \ \forall x_1 \in A \Rightarrow f(x_1) \neq f(x_2)$

For example, if $A = \{0,1,2,3\}$ and $B = \{1,2,3,4,5\}$, then we define a function $f: A \to B$ such that

$f = \{(0,1),(1,2),(2,3),(3,4)\}$

f is one-one function because no element in B is repeated.



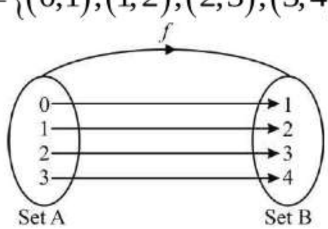
Bijective Function

(GRW 2014) **(K.B + U.B + A.B)**

A function $f: A \to B$ is called bijective function iff function f is one-one and onto.

For example, if $A = \{0,1,2,3,\}$ and $B = \{1,2,3,4\}$

Then $f = \{(0,1),(1,2),(2,3),(3,4)\}$



Note **(K.B)**

- Every function is ration but converse may not be true.
- Every function may not be one-one.
- Every function may not be onto.

Exercise 5.5

Q.1 Given (LHR 2014) **(A.B)**

$L = \{a,b,c\}$

$M = \{3,4\}$

To Find

Two binary relations of $L \times M$ and $M \times L$

Solution:

$L \times M = \{a,b,c\} \times \{3,4\}$

$= \{(a,3),(a,4),(b,3),(b,4),(c,3),(c,4)\}$

Two binary relations of $L \times M$ are:

$R_1 = \{(a,3),(a,4)\}$

$R_2 = \{(a,3),(b,3),(c,4)\}$

Now $M \times L = \{3,4\} \times \{a,b,c\}$

$= \{(3,a),(3,b),(3,c),(4,a),(4,b),(4,c)\}$

Two binary relations of $M \times L$ are:

$R_1 = \{(3,a)\}$

$R_2 = \{(3,a),(4,b)\}$

Q.2 Given **(A.B)**

(LHR 2015, GRW 2014)

$Y = \{-2,1,2\}$

Required:

Two binary relations of $Y \times Y$ and also find their domain and range.

Solution:

$Y \times Y = \{-2,1,2\} \times \{-2,1,2\}$

$= \{(-2,-2),(-2,1),(-2,2),(1,-2),$
$(1,1),(1,2),(2,-2),(2,1),(2,2)\}$

Two Binary relations for $Y \times Y$ are:

$R_1 = \{(-2,-2),(-2,1),(-2,2)\}$

$R_2 = \{(-2,-2),(1,1)\}$

Domain and Range

Dom $(R_1) = \{-2\}$

Range $(R_1) = \{-2,1,2\}$

Dom $(R_2) = \{-2,1\}$

Range $(R_2) = \{-2,1\}$

Q.3 Given **(A.B)**

$L = \{a,b,c\}$

$M = \{d, e, f, g\}$

Required

Two binary relation for

(i) $L \times L$ (ii) $L \times M$ (iii) $M \times M$

Solution:

$L \times L = \{a, b, c\} \times \{a, b, c\}$

$= \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),$
$(b,f),(b,g),(c,d),(c,e),(c,f),(c,g)\}$

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$= \{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),$
$(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary relations for $L \times L$ are:

$R_1 = \{(a,a),(b,b),(c,c)\}$

$R_2 = \{(a,b)\}$

Two Binary Relations for $L \times M$ are:

$R_1 = \{(a,d),(a,e)\}$

$R_2 = \{(a,d),(b,e),(c,f)\}$

Two Binary Relations for $M \times M$ are:

$R_1 = \{(d,d)\}$

$R_2 = \{(d,e),(f,g)\}$

Q.4 If set M has 5 elements, then find the number of binary relations in M. **(A.B)**

Solution:

No of binary relations in $M = 2^{mxn}$
$= 2^{5\times5}$
$= 2^{25}$

Q.5 Given $L = \{x \mid x \in N \wedge x \le 5\}$
$M = \{y \mid y \in P \wedge y < 10\}$ **(A.B + K.B)**

To Find

(i) $R_1 = \{(x, y) \mid y < x\}$

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Also find the domain and range of each relation.

Solution:

Here

$L = \{x \mid x \in N \wedge x \le 5\} = \{1, 2, 3, 4, 5\}$

And $M = \{y \mid y \in P \wedge y < 10\} = \{2, 3, 5, 7\}$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),$
$(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),$
$(4,5),(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) Relation **(A.B + K.B)**

$R_1 = \{(x, y) \mid y < x\}$

$= \{(3,2),(4,2),(4,3),(5,2),(5,3)\}$

Domain and Range

Dom $(R_1) = \{3, 4, 5\}$

Range $(R_1) = \{2, 3\}$

(ii) Relation **(A.B + K.B)**

$R_2 = \{(x, y) \mid y = x\}$

$= \{(2,2),(3,3),(5,5)\}$

Domain and Range

Dom $(R_2) = \{2, 3, 5\}$

Range $(R_2) = \{2, 3, 5\}$

(iii) Relation **(A.B + K.B)**

$R_3 = \{(x, y) \mid x + y = 6\}$

$= \{(1,5),(3,3),(4,2)\}$

Domain and Range of R_3

Dom $(R_3) = \{1, 3, 4\}$

Range $(R_3) = \{2, 3, 5\}$

(iv) Relation **(A.B + K.B)**

$R_4 = \{(x, y) \mid y - x = 2\}$

$= \{(1,3),(3,5),(5,7)\}$

Domain and Range of R_4

Dom $(R_4) = \{1, 3, 5\}$

Range $(R_4) = \{3, 5, 7\}$

Q.6 Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range. (In each part $(i) \rightarrow (vi)$ the co-domain set is equal to the range set) **(A.B + K.B + U.B)**

(i) $R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$

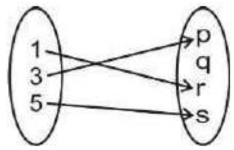
(ii) $R_2 = \{(1,2),(2,1),(3,4),(3,5)\}$

(iii) $R_3 = \{(b,a),(c,a),(d,a)\}$

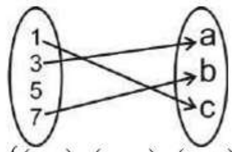
(iv) $R_4 = \{(1,1),(2,3),(3,4),(4,3),(5,4)\}$

(v) $R_5 = \{(a,b),(b,a),(c,d),(d,e)\}$

(vi) $R_6 = \{(1,2),(2,3),(1,3),(3,4)\}$

(vii) $R_7 =$ 

(viii) R_8 =



Solution:

(i) $\mathbf{R_1} = \{(1,1),(2,2),(3,3),(4,4)\}$

- Dom $(R_1) = \{1,2,3,4\}$

  There is no repetition in 1st element of any two ordered pairs, so R_1 is a function.

- Range $(R_1) = \{1,2,3,4\}$

  Also there is no repetition in 2nd element of any two ordered pairs.
  So R_1 is bijective function.

(ii) $\mathbf{R_2} = \{(1,2),(2,1),(3,4),(3,5)\}$

- Dom $(R_2) = \{1,2,3\}$

  There is a repetition in first element of last two ordered pairs.
  So R_2 is not a function.

(iii) $\mathbf{R_3} = \{(b,a),(c,a),(d,a)\}$

- Dom $(R_3) = \{b,c,d\}$

  There is no repetition in 1st element of any two pairs, so R_3 is a function.

- Range $(R_3) = \{a\}$

  There is a repetition in second element of all ordered pairs.
  So R_3 is on to function.

(iv) $\mathbf{R_4} = \{(1,1),(2,3),(3,4),(4,3),(5,4)\}$

- Dom $(R_4) = \{1,2,3,4,5\}$

  There is no repetition in 1st element of any two pairs, so R_4 is a function.

- Range $(R_4) = \{1,3,4\}$

  There is a repetition in second element of all ordered pairs.
  So R_4 is on to function.

(v) $\mathbf{R_5} = \{(a,b),(b,a),(c,d),(d,e)\}$

- Dom $(R_5) = \{a,b,c,d\}$

  There is no repetition in 1st element of any two ordered pairs, so R_5 is a function.

- Range $(R_5) = \{a,b,d,e\}$

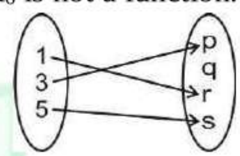
  Also there is no repetition in 2nd element of any two ordered pairs. So R_5 is bijective function.

(vi) $\mathbf{R_6} = \{(1,2),(2,3),(1,3),(3,4)\}$

- Dom $(R_6) = \{1,2,3\}$

  There is a repetition in first element of two ordered pairs.
  So R_6 is not a function.

(vii) $\mathbf{R_7}$=



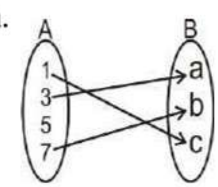
- Dom $(R_7) = \{1,3,5\} = A$

  Also there is no repetition in 1st element of any two ordered pairs. So R_7 is a function.

- Range $(R_7) = \{p,r,s\} \subset B$

  Also there is no repetition in 2nd element of any two ordered pairs.
  So R_7 is one-to-one (injective) function.

(viii) $\mathbf{R_8}$ =



- Dom $(R_8) = \{1,3,7\} \neq A$

  Therefore R_8 is not a function.