Q.1 Which of the following sets have closure property w.r.t addition and multiplications?

(i) $\{0\}$

Solution:

Let $G = \{0\}$

As $0+0=0 \in G$

Hence, G possess closure property w.r.t addition

As $0\times 0=0 \in G$

Hence, G possess closure property w.r.t multiplications.

(ii) $\{1\}$ ***FSD* 2023**

Solution:

Let $G=\{1\}$

As $1+1=2 \notin G$

Hence, G does not possess closure property w.r.t addition

As $1\times 1=1 \in G$

Hence, G possess closure property w.r.t multiplication.

(iii) $\{0,-1\}$

Solution:

Let $G = \{0, -1\}$

$0 + 0 = 0$

$0+(-1)=(-1)+0=-1$

$-1+(-1)=-2 \notin G$

Hence, G does not possess a closure property w.r.t addition because all the sums do not belong to the set G

$0\times 0=0$

$0\times(-1)=(-1)\times 0=0$

$(-1)\times(-1)=1 \notin G$

Hence, G does not possess a closure property w.r.t multiplication because all the multiplications do not belong to the set G

(iv) $\{1,-1\}$

***LHR* 2022, *GRW* 2021-23, *DGK* 2022, *RWP* 2022, *FSD* 2021**

Solution:

Let $G=\{1,-1\}$

$1+1=2 \notin G$

$1+(-1)=(-1)+1=0 \notin G$

$-1+(-1)=-2 \notin G$

Hence, G does not possess a closure property w.r.t addition, because all the sums do not belongs to the set G.

As $1\times 1=1 \in G$

$1\times(-1)=(-1)\times 1=-1 \in G$

$(-1)\times(-1)=1 \in G$

Hence, G possess a closure property w.r.t multiplications because all the products belong to the set G.

Q.2 Name the properties used in the following equations.
(Letters, where used, represent real numbers).

(i) $4+9=9+4$ ***FSD* 2019**

Solution:

[Commutative Property w.r.t addition]

(ii) $(a+1)+\frac{3}{4}=a+\left(1+\frac{3}{4}\right)$

Solution:

[Associative Property w.r.t addition.]

(iii) $\left(\sqrt{3}+\sqrt{5}\right)+\sqrt{7}=\sqrt{3}+\left(\sqrt{5}+\sqrt{7}\right)$

Solution:

[Associative Property w.r.t addition.]

(iv) $100+0=100$

Solution:

[Additive Identity]

(v) $1000\times1=1000$ ***FSD*2019, *RWP*2023**

Solution:

[Multiplicative Identity]

(v) $4.1+(-4.1)=0$

Solution:

[Additive Inverse]

(vi) $a-a=0$

Solution:

[Additive Inverse]

(vii) $\sqrt{2}\times\sqrt{5}=\sqrt{5}\times\sqrt{2}$

Solution:

[Commutative property w.r.t multiplication.]

(viii) $a(b-c)=ab-ac$

Solution:

[Distributivity of multiplication over subtraction

(ix) $(x-y)z=xz-yz$

Solution:

[Distributivity of multiplication over subtraction]

(x) $4\times(5\times8)=(4\times5)\times8$

Solution:

[Associative property w.r.t multiplication.]

(xi) $a(b+c-d)=ab+ac-ad$

Solution:

[Distributivity of multiplication over addition and subtraction.]

Q.3 Name the properties used in the following Inequalities:

(i) $-3<-2\Rightarrow0<1$

Solution:

$-3<-2$

By adding 3 on both sides

$-3+(3)<-2+3$

$0<1$

Additive property of inequalities

(ii) $-5<-4\Rightarrow20>16$

Solution:

$-5<-4$

By multiplying –4 on both sides

$(-4)(-5)>(-4)(-4)$

$20>16$

Multiplicative property of inequalities

(iii) $1>-1\Rightarrow-3>-5$

Solution:

$1>-1$

By adding (– 4) on both sides

$1+(-4)>-1+(-4)$

$-3>-5$

Additive property of inequalities

(iv) $a<0\Rightarrow-a>0$

Solution:

$a<0$

By multiplying -1 on both sides

$(-1)a>(-1)0$

$-a>0$

Multiplicative property of inequalities

(v) $a>b\Rightarrow\frac{1}{a}<\frac{1}{b}$

Solution:

$a>b$

Multiplying both sides by $\frac{1}{ab}$

$\frac{1}{ab}a>\frac{1}{ab}b$

$\frac{1}{b}\left(\frac{1}{a}a\right)>\frac{1}{a}\left(\frac{1}{b}b\right)$

$\frac{1}{b}(1)>\frac{1}{a}.(1)$

By multiplicative Inverse Law

$\frac{1}{b}>\frac{1}{a}$

$\Rightarrow\frac{1}{a}<\frac{1}{b}$

Multiplicative property of inequalities

(vi) $a>b\Rightarrow-a<-b$

Solution:

$a>b$

By multiplying both sides by -1

$(-1)a<(-1)b$

$-a<-b$

Q.4 Prove the following rules of addition.

(i) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

LHR 2019, *GRW* 2021, *RWP* 2021

Proof:

$$\text{L.H.S} = \frac{a}{c} + \frac{b}{c}$$

$$= a.\frac{1}{c} + b.\frac{1}{c}$$ **[Rule for product of fractions]**

$$= (a+b)\frac{1}{c}$$ **[Distributive Law]**

$$= \frac{a+b}{c}$$ **[Rule for product of fractions]**

$$= \text{R.H.S}$$

(ii) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

Proof:

$$\text{L.H.S} = \frac{a}{b} + \frac{c}{d}$$

$$= \frac{ad}{bd} + \frac{bc}{bd}$$ **[By Golden rule of fraction]**

$$= ad.\frac{1}{bd} + bc.\frac{1}{bd}$$

[Rule for product of fractions]

$$= (ad+bc).\frac{1}{bd}$$ **[Distributive Law]**

$$= \frac{ad+bc}{bd}$$ **[Rule for product of fractions]**

$$= \text{R.H.S}$$

Q.5 Prove that $-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$

SGD 2021

Proof:

$$\text{L.H.S} = -\frac{7}{12} - \frac{5}{18}$$

$$= -\frac{7\times 3}{12\times 3} - \frac{5\times 2}{18\times 2}$$

[Golden rule of fraction]

$$= -\frac{21}{36} - \frac{10}{36}$$

$$= -21\times\frac{1}{36} - 10\times\frac{1}{36}$$

[Rule for product of fraction]

$$= (-21-10)\times\frac{1}{36}$$ **[Distributive Law]**

$$= \frac{-21-10}{36}$$ **[Rule for product of Fraction]**

$$= \text{R. H.S}$$

Q.6 Simplify by justifying each step:

(i) $\frac{4+16x}{4}$

Solution:

$$\frac{4+16x}{4}$$

$$= \frac{1}{4}.(4+16x)$$

[Rule for product of Fraction]

$$= \frac{1}{4}.\left[(4)(1)+(4)(4x)\right]$$

$$= \frac{1}{4}.4\left[1+4x\right]$$ **[Distributive Law]**

$$= 1\left[1+4x\right]$$ **[Multiplicative inverse]**

$$= 1+4x$$ **[Multiplicative Identity]**

(ii) $\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$ **(GRW 2018, RWP 2019, SHW, 2022)**

Solution:

$$\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$$

$$= \frac{\frac{1\times 5}{4\times 5} + \frac{1\times 4}{5\times 4}}{\frac{1\times 5}{4\times 5} - \frac{1\times 4}{5\times 4}}$$ **[Golden rule of fraction]**

$$= \frac{\frac{5}{20}+\frac{4}{20}}{\frac{5}{20}-\frac{4}{20}}$$ **[Closure Law]**

$$= \frac{5\times\frac{1}{20}+4\times\frac{1}{20}}{5\times\frac{1}{20}-4\times\frac{1}{20}}$$

[Rule for product of fractions]

$$= \frac{(5+4)\frac{1}{20}}{(5-4)\frac{1}{20}}$$ **[Distributive law]**

$$= \frac{(5+4)\ \frac{1}{20}\cdot 20}{(5-4)\ \frac{1}{20}\cdot 20}$$ [Golden rule of Fraction]

$$= \frac{(5+4).1}{(5-4).1}$$ [Multiplicative Inverse]

$$= \frac{9}{1} = 9$$ [Closure Law]

(iii) $\dfrac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$ ***FSD* 2018, *DGK* 2022, *SGD* 2023**

Solution:

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$$

$$= \frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}-\frac{bc}{bd}}$$ [Golden rule of fraction]

$$= \frac{ad.\frac{1}{bd} + bc.\frac{1}{bd}}{ad.\frac{1}{bd} - bc.\frac{1}{bd}}$$ [Rule for product of fractions]

$$= \frac{(ad+bc)\ .\frac{1}{bd}}{(ad-bc)\ .\frac{1}{bd}}$$ [Distributive Law]

$$= \frac{(ad+bc)\ .\frac{1}{bd}\cdot \mathrm{bd}}{(ad-bc)\cdot\frac{1}{bd}\cdot bd}$$ [Golden rule of fraction]

$$= \frac{(ad+bc)\cdot 1}{(ad-bc)\cdot 1}$$ [Multiplicative Inverse]

$$= \frac{ad+bc}{ad-bc}$$ [Multiplicative Identity]

(iv) $\dfrac{\frac{1}{a}-\frac{1}{b}}{1-\frac{1}{a}.\frac{1}{b}}$ **(RWP 2017)**

Solution:

$$\frac{\frac{1}{a}-\frac{1}{b}}{1-\frac{1}{a}.\frac{1}{b}}$$

$$= \frac{\frac{1\times b}{a\times b}-\frac{a\times 1}{a\times b}}{\frac{1\times ab}{1\times ab}-\frac{1}{ab}}$$ [Golden rule of fraction]

$$= \frac{\frac{b}{ab}-\frac{a}{ab}}{\frac{ab}{ab}-\frac{1}{ab}}$$ [Closure Law]

$$= \frac{b\times\frac{1}{ab}-a\times\frac{1}{ab}}{ab\times\frac{1}{ab}-1\times\frac{1}{ab}}$$ [Rule for product of fractions]

$$= \frac{(b-a)\ \frac{1}{ab}}{(ab-1)\ \frac{1}{ab}}$$ [By Distributive law]

$$= \frac{(b-a)\cdot\frac{1}{ab}\cdot ab}{(ab-1)\cdot\frac{1}{ab}\cdot ab}$$ [Golden rule of fraction]

$$= \frac{(b-a)\cdot 1}{(ab-1)\cdot 1}$$ [Multiplicative inverse]

$$= \frac{b-a}{ab-1}$$ [Multiplicative identity]