Mathematics-11
Exercise - 1.3

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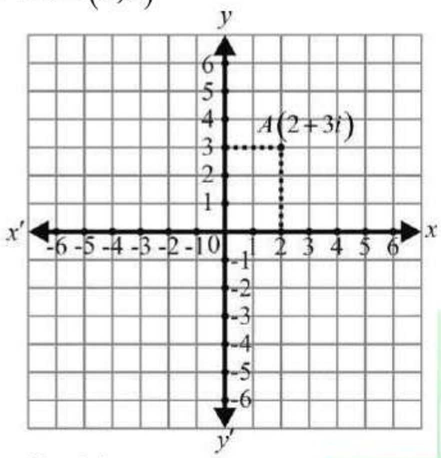
Q.1 Graph the following numbers in the complex plane.

(i) $2+3i$

Solution:

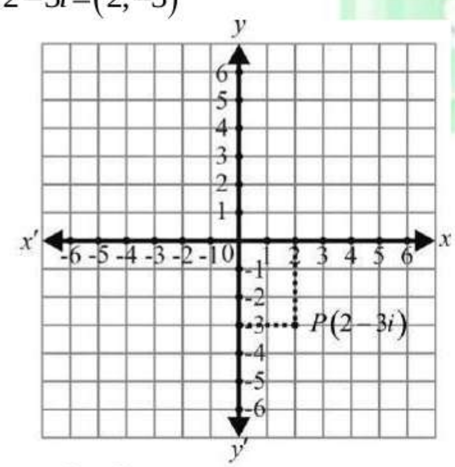
As $2+3i=(2,3)$



(ii) $2-3i$

Solution:

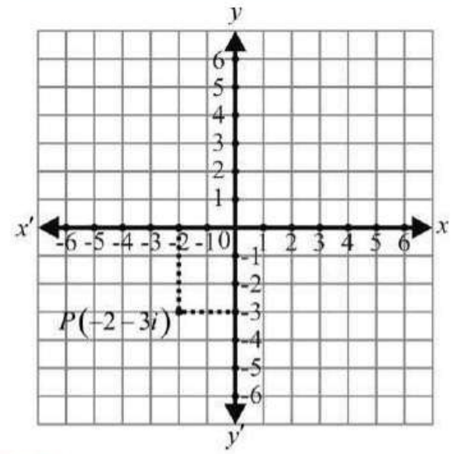
As $2-3i=(2,-3)$



(iii) $-2-3i$

Solution:

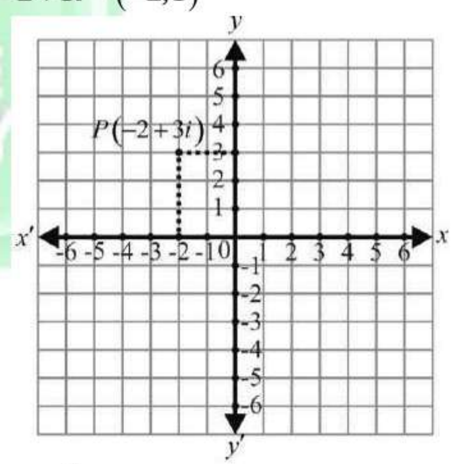
As $2-3i=(2,-3)$



(iv) $-2+3i$

Solution:

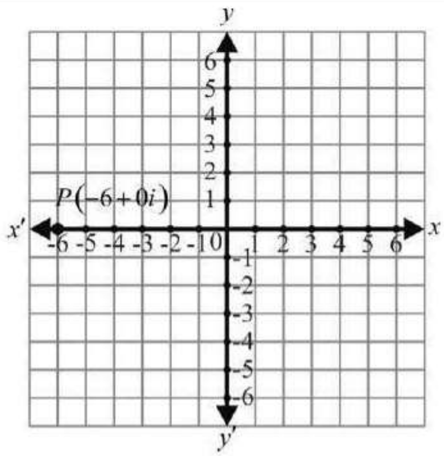
As $-2+3i=(-2,3)$



(v) -6

Solution:

$$-6=-6+0i=(-6,0)$$



(vi) i

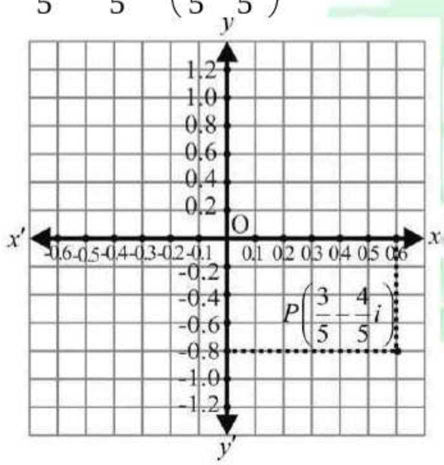
Solution:

$$i = 0 + i = (0,1)$$

(vii) $\frac{3}{5} - \frac{4}{5}i$

Solution:

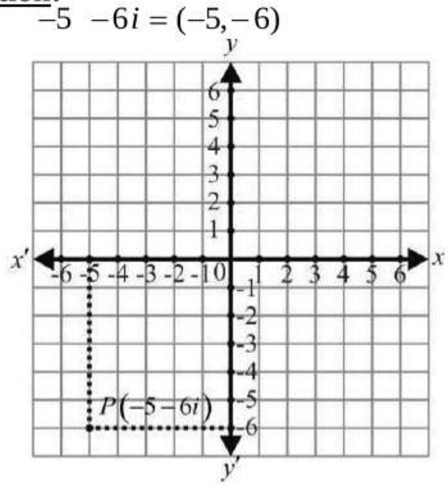
$$\frac{3}{5} - \frac{4}{5}i = \left(\frac{3}{5}, \frac{-4}{5}\right)$$

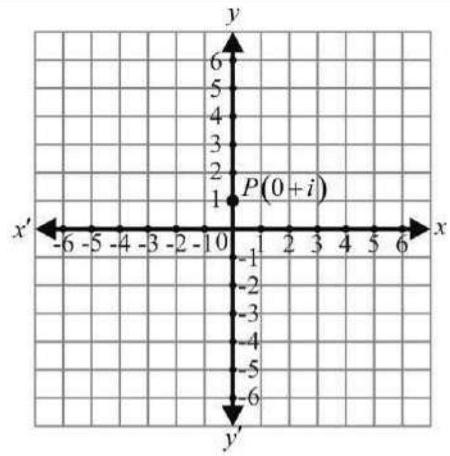


(viii) $-5-6i$

Solution:

$$-5 - 6i = (-5, -6)$$





Q.2 Find the multiplicative inverse of each of the following numbers:

(i) $-3i$

Solution:

Let $z = -3i$

$$\frac{1}{z} = \frac{1}{-3i}$$

$$= \frac{1}{-3i} \times \frac{3i}{3i}$$

$$= \frac{3i}{-9i^2}$$

$$= \frac{3i}{-9(-1)}$$

$$= \frac{3i}{9}$$

$$\frac{1}{z} = \frac{1}{3}i \text{ Answer}$$

(ii) $1 - 2i$

Solution:

Let $z = 1 - 2i$

$$\frac{1}{z} = \frac{1}{1-2i}$$

$$= \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i}{(1)^2 - (2i)^2}$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= \frac{1+2i}{1-4(-1)}$$

$$\frac{1}{z} = \frac{1+2i}{1+4}$$

$$\frac{1}{z} = \frac{1}{5} + \frac{2}{5}i \text{ Answer}$$

(iii) $-3-5i$

Solution:

Let $z = -3-5i$

$$\frac{1}{z} = \frac{1}{-3-5i}$$

$$\frac{1}{z} = \frac{1}{-3-5i} \times \frac{-3+5i}{-3+5i}$$

$$= \frac{-3+5i}{(-3)^2-(5i)^2}$$

$$\because a^2-b^2 = (a+b)(a-b)$$

$$= \frac{-3+5i}{9-25i^2}$$

$$= \frac{-3+5i}{9-25(-1)}$$

$$= \frac{-3+5i}{9+25}$$

$$= \frac{-3+5i}{34}$$

$$\frac{1}{z} = \frac{-3}{34} + \frac{5}{34}i \text{ Answer}$$

(iv) $(1, 2)$

Solution:

Let $z = (1, 2)$

$$z = 1+2i$$

$$\frac{1}{z} = \frac{1}{1+2i}$$

$$= \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$$

$$= \frac{1-2i}{(1)^2-(2i)^2}$$

$$\because a^2-b^2 = (a+b)(a-b)$$

$$= \frac{1-2i}{1-4(-1)}$$

$$= \frac{1-2i}{1+4}$$

$$= \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i$$

$$\frac{1}{z} = \left(\frac{1}{5}, \frac{-2}{5}\right) \text{ Answer}$$

Q.3 Simplify

(i) i^{101} ***GRW* 2019**

Solution:

$$= i^{101}$$

$$= i^{100}.i$$

$$= (i^2)^{50}.i$$

$$= (-1)^{50}.i$$

$$= 1.i$$

$$= i \text{ Answer}$$

(ii) $(-ai)^4$, $a \in \sqcap$

Solution:

$$= (-ai)^4$$

$$= (-a \times i)^4$$

$$= (-a)^4 i^4$$

$$= a^4(i^2)^2$$

$$= a^4(-1)^2$$

$$= a^4 \times 1$$

$$= a^4 \text{ Answer}$$

(iii) i^{-3}

Solution:

$$= i^{-3}$$

$$= \frac{1}{i^3}$$

$$= \frac{1}{i^2 .i}$$

$$= \frac{1}{-i}$$

$$= \frac{1}{-i} \times \frac{i}{i}$$

$= \frac{i}{-i^2}$

$= \frac{i}{-(-1)}$

$= i$ Answer

(iv) i^{-10}

Solution:

$= i^{-10}$

$= \frac{1}{i^{10}}$

$= \frac{1}{(i^2)^5}$

$= \frac{1}{(-1)^5}$

$= \frac{1}{-1}$

$= -1$ Answer

Q.4 Prove that $z = \bar{z}$ if z is real .

***FSD* 2019, *GRW* 2022, *MTN* 2022, *RWP* 2022-23, *SHW* 2023**

Proof:

Let $z = x + iy$ (i)

Suppose that $z = \bar{z}$

Now we have to prove z is real

As $z = \bar{z}$

$x + iy = x - iy$

$x + iy - x + iy = 0$

$2iy = 0$

As $2i \neq 0, y = 0$

Put in a equation (i)

$z = x + i(0)$

$z = x$

Hence z is a real number.

Conversely,

Suppose z is a real number

Now we have to prove that $z = \bar{z}$

Let $z = x + 0i = x$

$\bar{z} = x - 0i = x$

Hence

$z = \bar{z}$

Q.5 Simplify by expressing in the form $a + bi$

(i) $5 + 2\sqrt{-4}$

Solution:

$= 5 + 2\sqrt{-4}$

$= 5 + 2\sqrt{-1 \times 4}$

$= 5 + 2\sqrt{-1}\sqrt{4}$

$= 5 + 2(i)2$

$= 5 + 4i$ Answer

(ii) $(2 + \sqrt{-3})(3 + \sqrt{-3})$ ***GRW* 2021**

Solution:

$= (2 + \sqrt{-3})(3 + \sqrt{-3})$

$= (2 + \sqrt{-1 \times 3})(3 + \sqrt{-1 \times 3})$

$= (2 + \sqrt{-1}\sqrt{3})(3 + \sqrt{-1}\sqrt{3})$

$= (2 + i\sqrt{3})(3 + i\sqrt{3})$

$= 6 + i\,2\sqrt{3} + i3\sqrt{3} + i^2(\sqrt{3})^2$

$= 6 + i\,5\sqrt{3} + (-1)(3)$

$= 6 + i\,5\sqrt{3} - 3$

$= 3 + i5\sqrt{3}$ Answer

(iii) $\frac{2}{\sqrt{5} + \sqrt{-8}}$

Solution:

$= \frac{2}{\sqrt{5} + \sqrt{-8}}$

$= \frac{2}{\sqrt{5} + \sqrt{-1 \times 8}}$

$= \frac{2}{\sqrt{5} + \sqrt{-1}\sqrt{8}}$

$= \frac{2}{\sqrt{5} + i\sqrt{8}}$

$= \frac{2}{\sqrt{5} + i\sqrt{8}} \times \frac{\sqrt{5} - i\sqrt{8}}{\sqrt{5} - i\sqrt{8}}$

$= \frac{2\left[\sqrt{5} - i\sqrt{8}\right]}{(\sqrt{5})^2 - (i\sqrt{8})^2}$

$\because a^2 - b^2 = (a + b)(a - b)$

$= \frac{2\left[\sqrt{5} - i\sqrt{8}\right]}{5 - (-1)8}$

$$=\frac{2\left[\sqrt{5}-i\sqrt{8}\right]}{13}$$

$$=\frac{2\sqrt{5}}{13}-\frac{2\sqrt{8}}{13}i$$

$$=\frac{2\sqrt{5}}{13}-\frac{2\times2\sqrt{2}}{13}i$$

$$=\frac{2\sqrt{5}}{13}-\frac{4\sqrt{2}}{13}i \text{ Answer}$$

(iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

Solution:

$$=\frac{3}{\sqrt{6}-\sqrt{-12}}$$

$$=\frac{3}{\sqrt{6}-\sqrt{-1\times12}}$$

$$=\frac{3}{\sqrt{6}-i\sqrt{12}}$$

$$=\frac{3}{\sqrt{6}-i\sqrt{12}}\times\frac{\sqrt{6}+i\ \sqrt{12}}{\sqrt{6}+i\ \sqrt{12}}$$

$$=\frac{3\left[\sqrt{6}+i\sqrt{12}\right]}{\left(\sqrt{6}\right)^2-\left(i\sqrt{12}\right)^2}$$

$$\because a^2-b^2=(a+b)(a-b)$$

$$=\frac{3\left[\sqrt{6}+i\sqrt{12}\right]}{6-(-1)\,12}$$

$$=\frac{3\left[\sqrt{6}+i\sqrt{12}\right]}{18}$$

$$=\frac{\sqrt{6}+i\sqrt{12}}{6}$$

$$=\frac{\sqrt{6}}{6}+i\frac{\sqrt{6}\sqrt{2}}{6}$$

$$=\frac{1}{\sqrt{6}}+i\frac{\sqrt{2}}{\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}+i\frac{\sqrt{2}}{\sqrt{3}\sqrt{2}}$$

$$=\frac{1}{\sqrt{6}}+i\frac{1}{\sqrt{3}} \text{ Answer}$$

Q.6 Show that ∀ z ∈□

(i) $z^2+\bar{z}^2$ **is a real number.**

***GRW* 2019-21, *FSD* 2022, *BWP* 2023, *LHR* 2023**

Solution:

Let $z = x + iy$

$\bar{z} = x - iy$

$z^2+\bar{z}^2 = (x+i\,y)^2+(x-i\,y)^2$

$= x^2+(iy)^2+\cancel{2x(iy)}+x^2+(iy)^2-\cancel{2x(iy)}$

$= x^2+(-1)y^2+x^2+(-1)y^2$

$= 2x^2-2y^2$ is a real number

(ii) $(z-\bar{z})^2$

***GRW* 2018, *RWP* 2017, *LHR* 2022, *MTN* 2022**

Solution:

Let $z = x + iy$

$z = x - iy$

$(z-\bar{z}) = (x+iy)-(x-iy)$

$= x + i\,y - x + iy$

$z-\bar{z} = 2\,i\,y$

By taking square on both sides

$(z-\bar{z})^2 = (2iy)^2$

$= 4\,(-1)\ y^2$

$= -4\,y^2$ is a real number

Q.7 Simplify the following:

(i) $\left(\dfrac{-1}{2}+\dfrac{\sqrt{3}}{2}i\right)^3$

Solution:

$$=\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)^3$$

$$=\left(\frac{-1+\sqrt{3}i}{2}\right)^3$$

$$=\frac{(-1+\sqrt{3}i)^3}{2^3}$$

$$=\frac{1}{8}\left[-1+\sqrt{3}i\right]^3$$

$$=\frac{1}{8}\left[(-1)^3+3(-1)^2(\sqrt{3}i)+3(-1)(\sqrt{3}i)^2+(\sqrt{3}i)^3\right]$$
$$=\frac{1}{8}\left[-1+3\sqrt{3}i-3\times3(-1)+(\sqrt{3})^3i^3\right]$$
$$=\frac{1}{8}\left[-1+3\sqrt{3}i+9+(\sqrt{3})^2\sqrt{3}(-i)\right]$$
$$=\frac{1}{8}\left[-1+3\sqrt{3}i+9-3\sqrt{3}\,i\right]$$
$$=\frac{1}{8}[\,8\,]$$
$$=1 \text{ Answer}$$

(ii) $\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^3$ ***RWP* 2018, *FSD* 2022**

Solution:
$$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^3$$
$$=\left(\frac{-1-\sqrt{3}i}{2}\right)^3$$
$$=\frac{(-1-\sqrt{3}i)^3}{2^3}$$
$$=\frac{1}{8}\left[-1-\sqrt{3}i\right]^3$$
$$=\frac{1}{8}\left[(-1)^3-3(-1)^2(\sqrt{3}i)+3(-1)(\sqrt{3}i)^2-(\sqrt{3}i)^3\right]$$
$$=\frac{1}{8}\left[-1-3\sqrt{3}i-3\times3(-1)-(\sqrt{3})^3i^3\right]$$
$$=\frac{1}{8}\left[-1-3\sqrt{3}i+9-(\sqrt{3})^2\sqrt{3}(-i)\right]$$
$$=\frac{1}{8}\left[-1-3\sqrt{3}i+9+3\sqrt{3}\,i\right]$$
$$=\frac{1}{8}[8]$$
$$=1 \text{ Answer}$$

(iii) $\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

Solution:
$$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$$
$$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2+1}$$
$$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}$$
$$=\left(\frac{-1-\sqrt{3}i}{2}\right)^{-1}$$
$$=\frac{2}{-1-\sqrt{3}i}$$
$$=\frac{2}{-1-\sqrt{3}i}\times\frac{-1+\sqrt{3}i}{-1+\sqrt{3}i}$$
$$=\frac{2\left[-1+\sqrt{3}i\right]}{(-1)^2-(\sqrt{3}i)^2}$$
$$=\frac{2\left[-1+\sqrt{3}\,i\right]}{1-3(-1)}$$
$$=\frac{2\left[-1+\sqrt{3}\,i\right]}{4}$$
$$=\frac{-1+\sqrt{3}i}{2} \text{ Answer}$$

(iv) $(a+bi)^2$

Solution:
$$=(a+bi)^2$$
$$=a^2+(bi)^2+2(a)(bi)$$
$$=a^2+b^2(-1)+2abi$$
$$=a^2-b^2+2abi \text{ Answer}$$

(v) $(a+bi)^{-2}$

Solution:
$$=(a+bi)^{-2}$$
$$=\frac{1}{(a+bi)^2}$$
$$=\frac{1}{(a+bi)^2}\times\frac{(a-bi)^2}{(a-bi)^2}$$
$$=\frac{(a-bi)^2}{(a+bi)^2\,(a-bi)^2}$$
$$=\frac{a^2+(bi)^2-2(a)(bi)}{\left[a^2-(bi)^2\right]^2}$$

$$=\frac{a^2+b^2(-1)-2abi}{\left[a^2-b^2(-1)\right]^2}$$

$$=\frac{a^2-b^2-2abi}{(a^2+b^2)^2}$$

$$=\frac{a^2-b^2}{(a^2+b^2)^2}-\frac{2ab}{(a^2+b^2)^2}i \text{ Answer}$$

(vi) $(a+bi)^3$ ***BWP 2022***

Solution:

$$=(a+bi)^3$$

$$=a^3+3(a)^2(bi)+3a(bi)^2+(bi)^3$$

$$=a^3+3a^2bi+3ab^2(-1)+b^3i^3$$

$$=a^3+3a^2bi-3ab^2+b^3i^2.i$$

$$=a^3+3a^2bi-3ab^2+b^3(-1).i$$

$$=a^3-3ab^2+3a^2bi-b^3i$$

$$=(a^3-3ab^2)+(3a^2b-b^3)i \text{ Answer}$$

(vii) $(a-bi)^3$

Solution:

$$=(a-bi)^3$$

$$=a^3-3(a)^2(bi)+3a(bi)^2-(bi)^3$$

$$=a^3-3a^2bi+3ab^2(-1)-b^3i^3$$

$$=a^3-3a^2bi-3ab^2-b^3i^2.i$$

$$=a^3-3a^2bi-3ab^2-b^3(-1).i$$

$$=a^3-3a^2bi-3ab^2+b^3i$$

$$=(a^3-3ab^2)+(-3a^2b+b^3)i \text{ Answer}$$

(viii) $\left(3-\sqrt{-4}\right)^{-3}$

Solution:

$$=\left(3-\sqrt{-4}\right)^{-3}$$

$$=\left(3-\sqrt{-1}\sqrt{4}\right)^{-3}$$

$$=(3-i\,2)^{-3}$$

$$=\frac{1}{(3-2i)^3}$$

$$=\frac{1}{(3-2i)^3}\times\frac{(3+2i)^3}{(3+2i)^3}$$

$$=\frac{(3+2i)^3}{\left[(3-2i)(3+2i)\right]^3}$$

$$=\frac{3^3+3(3)^2(2i)+3(3)(2i)^2+(2i)^3}{\left[3^2-(2i)^2\right]^3}$$

$$=\frac{27+54i+9\times4(-1)+8i^3}{\left[9-4(-1)\right]^3}$$

$$=\frac{27+54i-36+8i^2i}{(13)^3}$$

$$=\frac{27+54i-36+8(-1)i}{13^3}$$

$$=\frac{-9+54i-8i}{2197}$$

$$=\frac{-9+46i}{2197}$$

$$=\frac{-9}{2197}+\frac{46}{2197}i \text{ Answer}$$