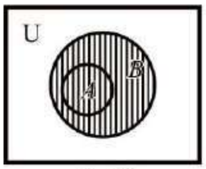
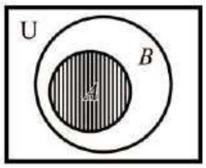
Mathematics-11
Exercise - 2.2

Q.1 Exhibit $A\cup B$ and $A\cap B$ by Venn diagrams in the following cases.

(i) $A\subseteq B$

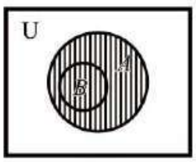


$A\cup B$

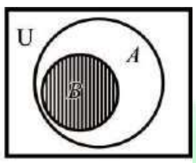


$A\cap B$

(ii) $B\subseteq A$

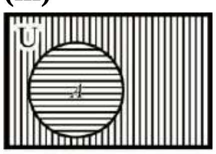


$A\cup B$



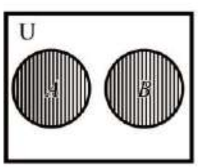
$A\cap B$

(iii) $A\cup A'$

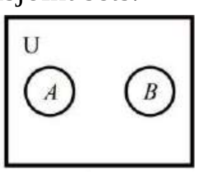


A:≡
A':|||
$A\cup A'$:≡ or |||

(iv) A and B are disjoint sets.

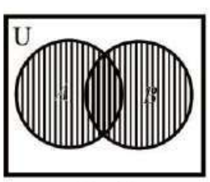


$A\cup B$

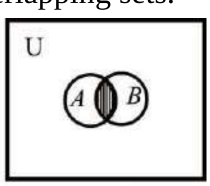


$A\cap B$

(v) A and B are overlapping sets.



$A\cup B$

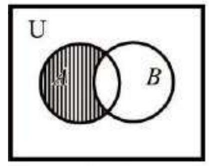


$A\cap B$

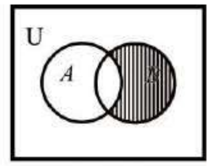
Q.2 Show $A-B$ and $B-A$ by Venn diagrams when:

(i) A and B are overlapping sets.

(SHW 2021, BWP 2023)

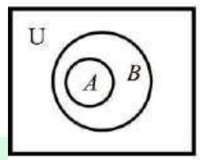


$A-B$

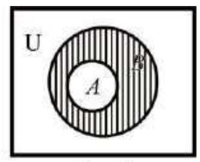


$B-A$

(ii) $A\subseteq B$ **(SHW 2022)**

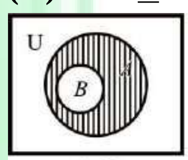


$A-B$

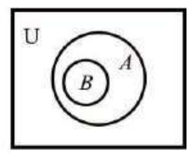


$B-A$

(iii) $B\subseteq A$ **(DGK 2022, MTN 2022)**



$A-B$



$B-A$

Q.3 Under what conditions on A and B are the following statements true?

(i) $A\cup B=A$

Solution: $B\subseteq A$

(ii) $A\cup B=B$

Solution: $A\subseteq B$

(iii) $A-B=A$

Solution: $A\cap B=\phi$

(iv) $A\cap B=B$

Solution: $B\subseteq A$

(v) $n(A\cup B)=n(A)+n(B)$

Solution: $A\cap B=\phi$

(vi) $n(A\cap B)=n(A)$

Solution: $A\subseteq B$

(vii) $A-B=A$

Solution: $A \cap B = \phi$

(viii) $n(A \cap B) = 0$

Solution: $A \cap B = \phi$

(ix) $A \cup B = U$

Solution: $B = A'$

(x) $A \cup B = B \cup A$

Solution: It holds always

(xi) $n(A \cap B) = n(B)$

Solution: $B \subseteq A$

(xii) $U - A = \phi$

Solution: $U = A$

Q.4 Let $U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{2,4,6,8,10\}$

$B = \{1,2,3,4,5\}$ and $C = \{1,3,5,7,9\}$

List the members of each of the following sets.

(i) A^c , $A^c = U - A = \{1,3,5,7,9\}$

(ii) B^c, $B^c = U - B = \{6,7,8,9,10\}$

(iii) $A \cup B$,

$A \cup B = \{1,2,3,4,5,6,8,10\}$

(iv) $A - B$, $A - B = \{6,8,10\}$

(v) $A \cap C$, $A \cap C = \phi$

(vi) $A^c \cup C^c$, $A^c = \{1,3,5,7,9\}$,

$C^c = \{2,4,6,8,10\}$ and

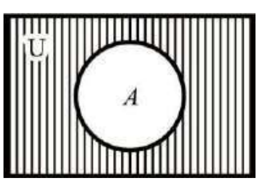
$A^c \cup C^c = \{1,2,3,...,10\} = U$

(vii) $A^c \cup C$, $A^c \cup C = \{1,3,5,7,9\}$

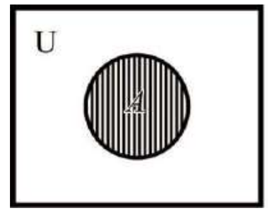
(viii) U^c , $U^c = \phi$

Q.5 **Using Venn diagrams, if necessary, find the single sets equal to the following.**

(i) $A^c = U - A$

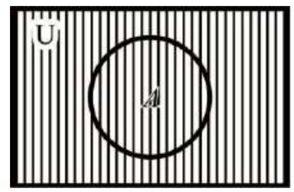


(ii) $A \cap U$



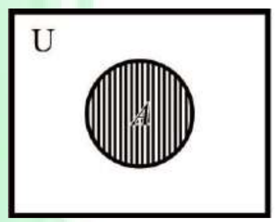
$A \cap U = A$

(iii) $A \cup U$



$A \cup U = U$

(iv) $A \cup \phi$

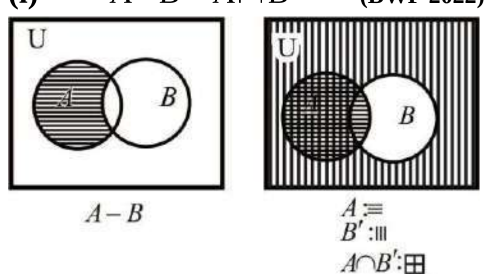


$A \cup \phi = A$

(v) $\phi \cap \phi = \phi$

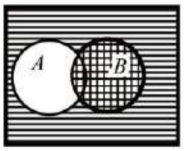
Q.6 **Use Venn diagrams to verify the following**

(i) $A - B = A \cap B^c$ **(BWP 2022)**

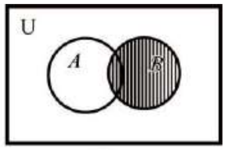


From Venn diagrams $A - B = A \cap B^c$

(ii) $(A - B)^c \cap B = B$



$(A-B)'$:≡
B: |||
$(A-B)'\cap B$: ⊞



B: |||

From Venn diagrams

$(A-B)^c \cap B = B$

De Morgan's laws:

(i) $(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$

Proof:-

(i) $(A\cup B)' = A'\cap B'$

Let $x\in (A\cup B)'$

$\Rightarrow x\notin A\cup B$

$\Rightarrow x\notin A$ and $x\notin B$

$\Rightarrow x\in A'$ and $x\in B'$

$\Rightarrow x\in A'\cap B'$

But x is an arbitrary member of $(A\cup B)'$.

Therefore $(A\cup B)'\subseteq A'\cap B'$ (i)

Conversely, suppose that

$y\in A'\cap B'$

$\Rightarrow y\in A'$ and $y\in B'$

$\Rightarrow y\notin A$ and $y\notin B$

$\Rightarrow y\notin A\cup B$

$\Rightarrow y\in (A\cup B)'$

Therefore $A'\cap B'\subseteq (A\cup B)'$ (ii)

From (i) and (ii) we conclude that

$(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$

Let $x\in (A\cap B)'$

$\Rightarrow x\notin A\cap B$

$\Rightarrow x\notin A$ or $x\notin B$

$\Rightarrow x\in A'$ or $x\in B'$

$\Rightarrow x\in A'\cup B'$

Therefore $(A\cap B)'\subseteq A'\cup B'$ (i)

Conversely, suppose that

$y\in A'\cup B'$

$\Rightarrow y\in A'$ or $y\in B'$

$\Rightarrow y\notin A$ or $y\notin B$

$\Rightarrow y\notin (A\cap B)$

$\Rightarrow y\in (A\cap B)'$

Therefore $A'\cup B'\subseteq (A\cap B)'$ (ii)

From (i) and (ii) we conclude that

$(A\cap B)' = A'\cup B'$

Distributive laws

(i) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

(ii) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

Proof:

(i) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Let $x\in A\cup(B\cap C)$

$\Rightarrow x\in A$ or $x\in B\cap C$

$\Rightarrow x\in A$ or $x\in B$ and $x\in C$

$\Rightarrow x\in A$ or $x\in B$ and $x\in A$ or $x\in C$

$\Rightarrow x\in (A\cup B)$ and $x\in (A\cup C)$

$\Rightarrow x\in (A\cup B)\cap(A\cup C)$

Thus

$A\cup(B\cap C)\subseteq (A\cup B)\cap(A\cup C)$ (i)

Conversely suppose that

$y\in (A\cup B)\cap(A\cup C)$

$\Rightarrow y\in (A\cup B)$ and $y\in (A\cup C)$

$\Rightarrow y\in A$ or $y\in B$ and $y\in A$ or $y\in C$

$\Rightarrow y\in A$ or $y\in B$ and $y\in C$

$\Rightarrow y\in A$ or $y\in (B\cap C)$

$\Rightarrow y\in A\cup(B\cap C)$

Thus

$(A\cup B)\cap(A\cup C)\subseteq A\cup(B\cap C)$ (ii)

From (i) and (ii)

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(ii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Let $x\in A\cap(B\cup C)$

$\Rightarrow x\in A$ and $x\in(B\cup C)$

$\Rightarrow x\in A$ and $x\in B$ or $x\in C$

$\Rightarrow x\in A$ and $x\in B$ or $x\in A$ and $x\in C$

$\Rightarrow x\in(A\cap B)$ or $x\in(A\cap C)$

$\Rightarrow x\in(A\cap B)\cup(A\cap C)$

Therefore

$A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)$ (i)

Conversely let

$y\in(A\cap B)\cup(A\cap C)$

$\Rightarrow y\in(A\cap B)$ or $y\in(A\cap C)$

$\Rightarrow y\in A$ and $y\in B$ or $y\in A$ and $y\in C$

$\Rightarrow y\in A$ and $y\in B$ or $y\in C$

$\Rightarrow y\in A$ and $y\in(B\cup C)$

$\Rightarrow y\in A\cap(B\cup C)$

Therefore

$(A\cap B)\cup(A\cap C)\subseteq A\cap(B\cup C)$ (ii)

From (i) and (ii)

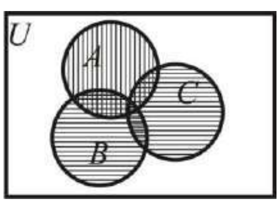
$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Verification of the properties with the help of Venn diagrams

(i) Associative property of union

$A\cup(B\cup C)=(A\cup B)\cup C$

L.H.S = $A\cup(B\cup C)$

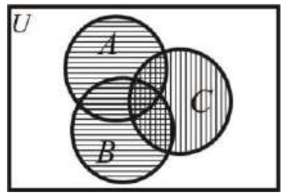


A: |||

$B\cup C$: ≡

$A\cup(B\cup C)$: |||, ≡ or ⊞

R.H.S= $(A\cup B)\cup C$



$A\cup B$: ≡

C: |||

$(A\cup B)\cup C$: |||, ≡ or ⊞

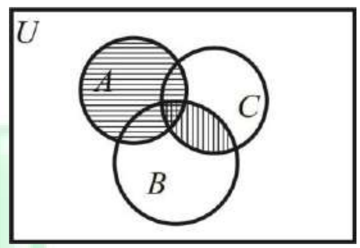
From two diagrams, we can see that

$A\cup(B\cup C)=(A\cup B)\cup C$

(ii) Associative property of intersection

$A\cap(B\cap C)=(A\cap B)\cap C$

L.H.S $=A\cap(B\cap C)$

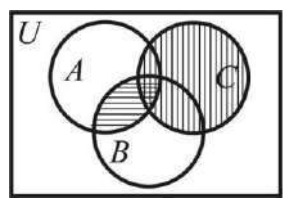


A: ≡

$B\cap C$: |||

$A\cap(B\cap C)$: ⊞

R.H.S = $(A\cap B)\cap C$



$A\cap B$: ≡

C: |||

$(A\cap B)\cap C$: ⊞

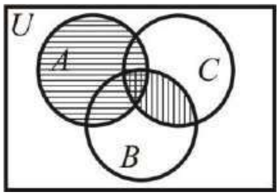
From two diagrams we can see that

$A\cap(B\cap C)=(A\cap B)\cap C$

(iii) Distributive laws

Proof: (a) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

L.H.S $= A\cup(B\cap C)$

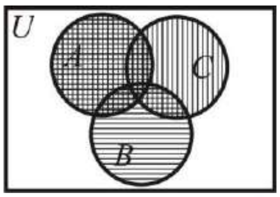


A:

$B\cap C$:

$A\cup(B\cap C)$: , or

R.H.S $= (A\cup B)\cap(A\cup C)$



$A\cup B$:

$A\cup C$:

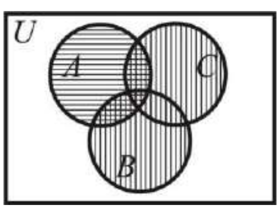
$(A\cup B)\cap(A\cup C)$:

From two diagrams, we can see that

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$

(b) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

L.H.S = $A\cap(B\cup C)$

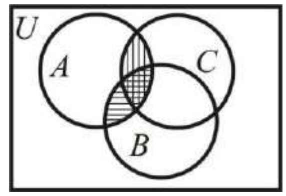


A:

$B\cup C$:

$A\cap(B\cup C)$:

R.H.S = $(A\cap B)\cup(A\cap C)$



$A\cap B$:

$A\cap C$:

$(A\cap B)\cup(A\cap C)$: , or

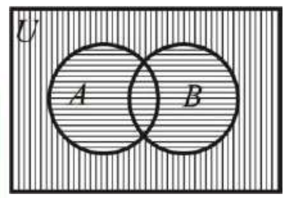
From two diagrams

$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$$

(iv) De Morgan's Laws

(a) $(A\cup B)'=A'\cap B'$

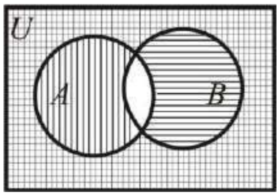
L.H.S $=(A\cup B)'$



$A \cup B$:

$(A \cup B)'$:

R.H.S $= A' \cap B'$



A' :

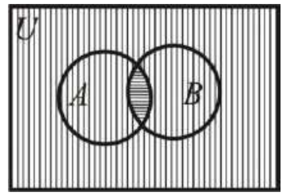
B' :

$A' \cap B'$:

From two diagrams $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

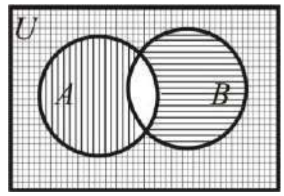
L.H.S $= (A \cap B)'$



$A \cap B$:

$(A \cap B)'$:

R.H.S $= A' \cup B'$



A' :

B' :

$A' \cup B'$: , or

From two diagrams $(A \cap B)' = A' \cup B'$