Mathematics-11

Exercise - 2.3

Q.1 Verify the commutative properties of union & intersection for the following pairs of sets.

(DGK 2021, LHR 2022, BWP 2023, MTN 2023, GRW 2023)

(i) $A=\{1,2,3,4,5\}, B=\{4,6,8,10\}$

Solution:

$A\cup B=\{1,2,3,4,5,6,8,10\}$ (i)

$B\cup A=\{1,2,3,4,5,6,8,10\}$ (ii)

From (i) and (ii) commutative property of union is satisfied.

Now $A\cap B=\{4\}$ (iii)

$B\cap A=\{4\}$ (iv)

From (iii) and (iv) commutative property of intersection is satisfied.

(i) [,]

Solution: ¥ ∪ ¢ = ¢ = ¢ ∪ ¥

Which satisfies commutative property of union. Also,

¥ ∩ ¢ = ¥ = ¢ ∩ ¥

Which satisfies commutative property of intersection.

(ii) $A=\{x \mid x\in$ ¡ $\wedge x\geq 0\}, B=$ ¡

Solution:

$A\cup B=\{x \mid x\in$ ¡ $\wedge x\geq 0\}\cup$ ¡ = ¡

and $B\cup A=A$ $\quad (\text{Q}\, A\subseteq B)$

So commutative property of union is satisfied.

Also $A\cap B=$

$\{x \mid x\in$ ¡ $\wedge x\geq 0\}\cap$ ¡ $=\{x \mid x\in$ ¡ $\wedge x\geq 0\}=A$

and $B\cap A=A$ $\quad (\text{Q}\, A\subseteq B)$

Which satisfies commutative property of intersection.

Q.2 Verify the properties for the sets A, B and C given below.

(a) $A=\{1,2,3,4\}, B=\{3,4,5,6,7,8\}$,

$C=\{5,6,7,9,10\}$

(i) Associativity of union.

$A\cup B=\{1,2,3,....,8\}$

$(A\cup B)\cup C=\{1,2,3,...,10\}$

$B\cup C=\{3,4,5,...,10\}$

$A\cup(B\cup C)=\{1,2,3,...,10\}$

Hence proved that

$A\cup(B\cup C)=(A\cup B)\cup C$

(ii) Associativity of intersection.

$A\cap B=\{3,4\}$

$(A\cap B)\cap C=\phi$

$B\cap C=\{5,6,7\}$

$A\cap(B\cap C)=\phi$

Hence proved that

$(A\cap B)\cap C=A\cap(B\cap C)$

(iii) Distributivity of union over intersection.

$B\cap C=\{5,6,7\}$

$A\cup(B\cap C)=\{1,2,3,4,5,6,7\}$ (i)

$A\cup B=\{1,2,3,...,8\}$

$A\cup C=\{1,2,3,4,5,6,7,9,10\}$

Now

$(A\cup B)\cap(A\cup C)=\{1,2,3,4,5,6,7\}$ (ii)

From (i) and (ii) we conclude that

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Which satisfies the property

(iv) Distributivity of intersection over union.

$B\cup C=\{3,4,5,...,10\}$

$A\cap(B\cup C)=\{3,4\}$ (i)

$A\cap C=\phi,\quad A\cap B=\{3,4\}$

$(A\cap B)\cup(A\cap C)=\{3,4\}$ (ii)

From (i) and (ii) we conclude that

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Which satisfies the property.

(b) $A=\phi, B=\{0\}, C=\{0,1,2\}$

(i) Associativity of union.

$A\cup B=\{0\}$

$(A\cup B)\cup C=\{0,1,2\}$ (i)

$B\cup C=\{0,1,2\}$

$A\cup(B\cup C)=\{0,1,2\}$ (ii)

From (i) and (ii) we get the proof.

(ii) Associativity of intersection.

$A\cap B=\phi$

$(A\cap B)\cap C=\phi$ (i)

$B\cap C=\{0\}$

$A\cap(B\cap C)=\phi$ (ii)

From (i) and (ii) we conclude the result.

(iii) Distributivity of union over intersection.

$B\cap C=\{0\}$

$A\cup(B\cap C)=\{0\}$ (i)

$A\cup B=\{0\}$

$A\cup C=\{0,1,2\}$

$(A\cup B)\cap(A\cup C)=\{0\}$ (ii)

From (i) and (ii) we get the proof.

(iv) Distributivity of intersection over union.

$B\cup C=\{0,1,2\}$

$A\cap(B\cup C)=\phi$ (i)

$A\cap B=\phi,\quad A\cap C=\phi$

$(A\cap B)\cup(A\cap C)=\phi$ (ii)

From (i) and (ii) we get the proof.

(c) □ ,□ ,□

(i) Associativity of union.

¥ ∪ ¢ = ¢ (∵□ ⊂□)

(¥ ∪ ¢)∪¤ = ¢ ∪¤ = ¤

(Q¢ ⊂ ¤) (i)

¢ ∪¤ = ¤ (∵□ ⊂□)

¥ ∪(¢ ∪¤)= ¥ ∪¤ = ¤

(Q¥ ⊂ ¤) (ii)

(i) and (ii) verify the property.

(ii) Associativity of intersection.

¥ ∩ ¢ = ¥ (∵□ ⊂□)

(¥ ∩ ¢)∩¤ = ¥ ∩¤ = ¥

(Q¥ ⊂ ¤) (i)

¢ ∩¤ = ¢ (∵□ ⊂□)

¥ ∩(¢ ∩¤)= ¥ ∩ ¢ = ¥

(Q¥ ⊂ ¢) (ii)

From (i) and (ii) we have proof.

(iii) Distributivity of union over intersection.

¢ ∩¤ = ¢ (Q ¢ ⊂¤)

¥ ∪(¢ ∩¤) = ¥ ∪ ¢ = ¢ (i)

(Q¥ ⊂ ¢)

(¥ ∪ ¢)∩(¥ ∪¤) = ¢ ∩¤

= ¢ (Q¢ ⊂¤) (ii)

From (i) & (ii) we get the result.

(iv) Distributivity of intersection over union.

¢ ∪¤ = ¤ (∵ ⊂)

¥ ∩(¢ ∪¤) = ¥ ∩¤ = ¥

(Q¥ ⊂¤) (i)

¥ ∩¢ = ¥ (Q¥ ⊂ ¢)

¥ ∩¤ = ¥ (Q¥ ⊂¤)

(¥ ∩¢)∪(¥ ∩¤) = ¥ ∪¥ = ¥

(ii)

From (i) and (ii) we get the proof.

Q.3 Verify de-Morgan's Laws for the following sets.

$U = \{1,2,3,...,20\}$

$A = \{2,4,6,...,20\}$

$B = \{1,3,5,7,...,19\}$

Solution: $A \cup B = \{1,2,3,...,20\}$

$(A \cup B)' = U - (A \cup B)$

$= \phi$

$A' = U - A$

$= \{1,3,5,...,19\}$

$B' = U - B$

$= \{2,4,6,...,20\}$

$A' \cap B' = \phi$

Which shows that $(A \cup B)' = A' \cap B'$

Now $A \cap B = \phi$

$(A \cap B)' = \{1,2,3,...,20\}$

$A' \cup B' = \{1,2,3,...,20\}$

Which shows that $(A \cap B)' = A' \cup B'$

Hence the proof.

Q.4 Let *U*= The set of English alphabets. (FSD 2023)

$A = \{x \mid x \text{ is a vowel}\}$

$B = \{y \mid y \text{ is a consonant}\}$

Verify de-Morgan's Laws.

Now, $U = \{a,b,c,d,e,...,z\}$

$A = \{a,e,i,o,u\}$

$B = \left\{\begin{matrix} b,c,d,f,g,h,j,k,l,m,n,p,q,r, \\ s,t,v,w,x,y,z \end{matrix}\right\}$

Now $A \cup B = \{a,b,c,...,z\}$

$(A \cup B)' = \phi$ (i)

$A' = U - A$

$= \left\{\begin{matrix} b,c,d,f,g,h,j,k,l,m,n,p,q,r, \\ s,t,v,w,x,y,z \end{matrix}\right\}$

$B' = U - B$

$= \{a,e,i,o,u\}$

$A' \cap B' = \phi$ (ii)

From (*i*) and (*ii*) $(A \cup B)' = A' \cap B'$

Also $A \cap B = \phi$

$(A \cap B)' = U - \phi = U$ (iii)

$A' \cup B' = \{a,b,c,...,z\} = U$ (iv)

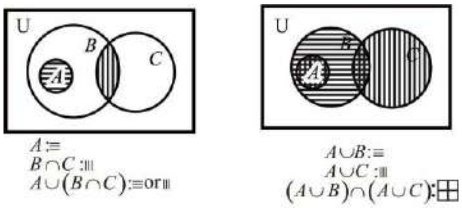
Form (iii) and (iv) $(A \cap B)' = A' \cup B'$

Q.5 With the help of Venn diagrams, verify the two distributive

properties in the following cases with respect to union and intersection.

(i) $A \subseteq B,\quad A \cap C = \phi$ **and B & C and over lapping.**

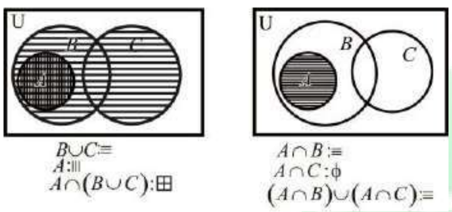
Distributivity of union over intersection.



From Venn diagrams

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Distributivity of intersection over union.

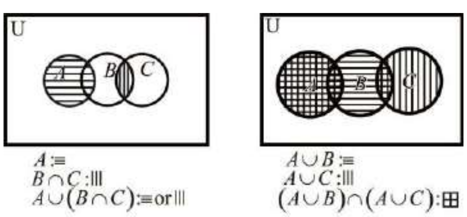


From Venn diagrams

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(ii) **A & B are overlapping, B & C are overlapping, but A & C are disjoint.**

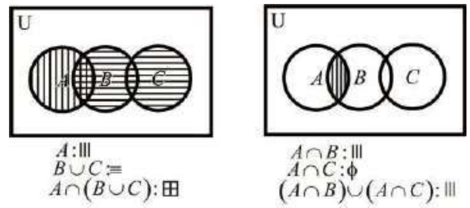
Distributivity of union over intersection.



From Venn diagrams

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Distributivity of intersection over union.



From Venn diagrams

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Q.6 **Taking any set, say** $A = \{1,2,3,4,5\}$ **verify the following.**

(i) $A \cup \phi = A$

$A \cup \phi = \{1,2,3,4,5\} \cup \phi = \{1,2,3,4,5\} = A$

(ii) $A \cup A = A$

$A \cup A = \{1,2,3,4,5\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5\}$

$= A$

(iii) $A \cap A = A$

$A \cap A = \{1,2,3,4,5\} \cap \{1,2,3,4,5\}$

$= \{1,2,3,4,5\}$

$= A$

Q.7 **If** $U = \{1,2,3,...,20\}, A = \{1,3,5,...,19\}$ **verify the following.**

(i) $A \cup A' = U$ **(GRW 21, FSD 2021)**

$A' = U - A$

$= \{2,4,6,...,20\}$

Now

$A \cup A' = \{1,3,5,...19\} \cup \{2,4,6,...,20\}$

$\{1,2,3,...,20\}$

$= U$

(ii) $A \cap U = A$

$A \cap U = \{1,3,5,...,19\} \cap \{1,2,3,...,20\}$

$= \{1,3,5,...,19\}$

$= A$

(iii) $A \cap A' = \phi$

$$A' = \{2, 4, 6, ..., 20\}$$

$$A \cap A' = \{1, 3, 5, ..., 19\} \cap \{2, 4, 6, ..., 20\}$$
$$= \phi$$

Q.8 From suitable properties of union and intersection deduce the following results.

(i) $A \cap (A \cup B) = A \cup (A \cap B)$ **(RWP 2022)**

Solution: $\text{L.H.S.} = A \cap (A \cup B)$

$$= (A \cap A) \cup (A \cap B)$$
(Distributive Law)
$$= A \cup (A \cap B) \quad (Q\ A \cap A = A)$$
= R.H.S.

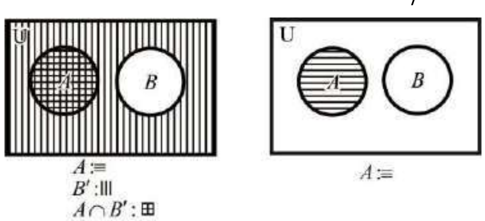
(ii) $A \cup (A \cap B) = A \cap (A \cup B)$

Solution: $\text{L.H.S.} = A \cup (A \cap B)$

$$= (A \cup A) \cap (A \cup B)$$
(Distributive Law)
$$= A \cap (A \cup B) \quad (Q\ A \cup A = A)$$
= R.H.S.

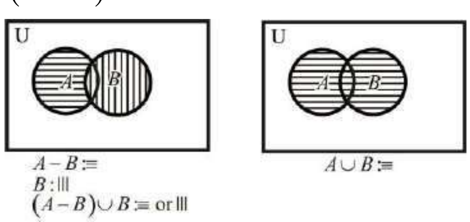
Using Venn diagrams, verify the following results.

(i) $A \cap B' = A$ iff $A \cap B = \phi$



From Venn diagrams $A \cap B' = A$

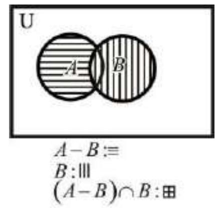
(ii) $(A - B) \cup B = A \cup B$



From Venn diagrams

$$(A - B) \cup B = A \cup B$$

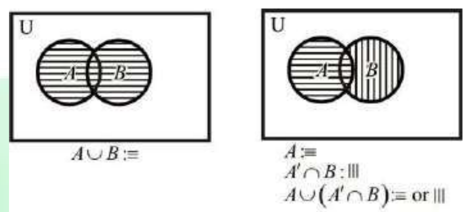
(iii) $(A - B) \cap B = \phi$



As such region does not exist so

$$(A - B) \cap B = \phi$$

(iv) $A \cup B = A \cup (A' \cap B)$



From Venn diagrams

$$A \cup B = A \cup (A' \cap B)$$