Convert the following theorems to logical form and prove them by constructing truth tables:

Q.1 $(A \cap B)' = A' \cup B'$ **(BWP 2021, FSD 2022)**

Solution: The corresponding logical form is : $(p \wedge q) =: \ p \vee : \ q$

p	q	$: p$	$: q$	$p \wedge q$	$: p \vee : q$	$: (p \wedge q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Last two columns show that : $(p \wedge q) =: \ p \vee : \ q$ and hence $(A \cap B)' = A' \cup B'$

Q.2 $(A \cup B) \cup C = A \cup (B \cup C)$

Solution: The corresponding logical form is $(p \vee q) \vee r = p \vee (q \vee r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Last two columns show that $(p \vee q) \vee r = p \vee (q \vee r)$ and hence

$(A \cup B) \cup C = A \cup (B \cup C)$

Q.3 $(A\cap B)\cap C = A\cap(B\cap C)$

Solution: The corresponding logical form is $(p\wedge q)\wedge r = p\wedge(q\wedge r)$

p	q	r	$p\wedge q$	$q\wedge r$	$(p\wedge q)\wedge r$	$p\wedge(q\wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Last two columns show that $(p\wedge q)\wedge r = p\wedge(q\wedge r)$ and hence

$(A\cap B)\cap C = A\cap(B\cap C)$

Q.4 $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ **(MTN 2022)**

Solution: The corresponding logical form is

$p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$

p	q	r	$p\vee q$	$p\vee r$	$q\wedge r$	$p\vee(q\wedge r)$	$(p\vee q)\wedge(p\vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F

Last two columns show that $p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$ and hence

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$