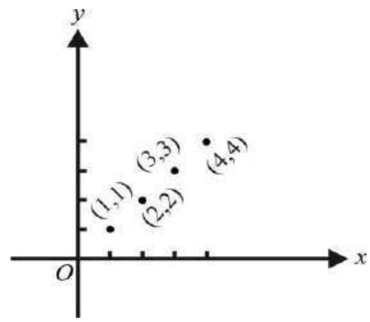
Mathematics-11

Exercise - 2.6

Q.1 For $A=\{1,2,3,4\}$, find the following relations in A. State the domain and range of each relation. Also draw the graph of each.

(i) $\{(x,y) \mid y=x\}$



Solution:

Let $R_1=\{(x,y) \mid y=x\}=\{(1,1),(2,2),(3,3),(4,4)\}$

The domain of R_1 is $\{1,2,3,4\}$

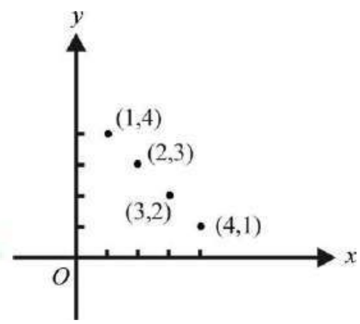
Range of R_1 is $\{1,2,3,4\}$

(ii) $\{(x,y) \mid y+x=5\}$ **(LHR 2021, DGK 2022)**

Solution:

Let $R_2=\{(x,y) \mid y+x=5\}$

$=\{(1,4),(2,3),(3,2),(4,1)\}$



Domain of $R_2=\{1,2,3,4\}$

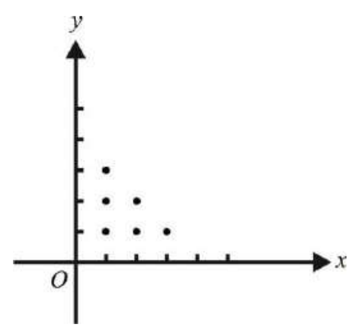
Range of $R_2=\{1,2,3,4\}$

(iii) $\{(x,y) \mid x+y<5\}$ **(FSD 2022)**

Solution:

Let $R_3=\{(x,y) \mid x+y<5\}$

$=\{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$



Domain of $R_3=\{1,2,3\}$

Range of $R_3=\{1,2,3\}$

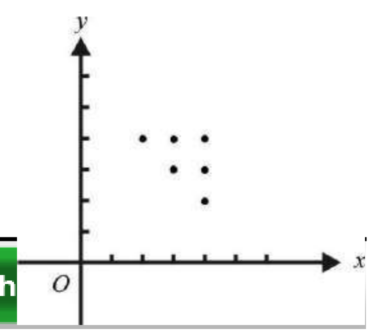
(iv) $\{(x,y) \mid x+y>5\}$ **(MTN 2021, GRW 2021)**

Solution:

Let $R_4=\{(x,y) \mid x+y>5\}$

$=\{(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)\}$

Domain of $R_4=\{2,3,4\}$

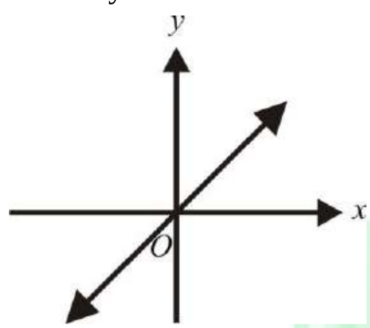


Range of $R_4 = \{2, 3, 4\}$

Q.2 Repeat Q:1 when $A = ¡$, the set of real numbers. Which of the real lines are functions?

(i) $\{(x,y) \mid y = x\}$

The domain of above relation is ¡ and range is also ¡ . The graph gives straight line passing through origin. Given relation is a function since each value of x gives unique value of y.



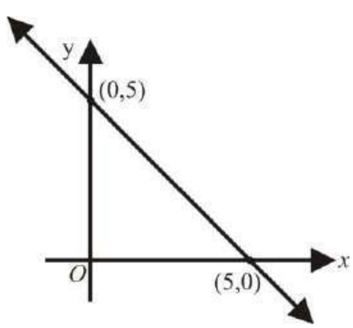
(ii) $\{(x,y) \mid y + x = 5\}$

Using

$y + x = 5,$ When $y = 0,$ $x = 5$

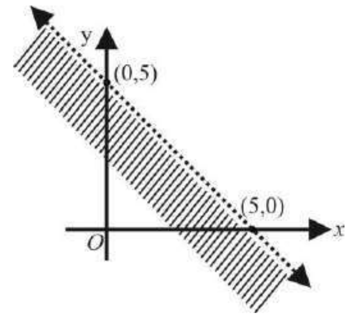
And when $x = 0,$ $y = 5$, so (5,0) and (0,5) lie on the graph. The domain and range is ¡ .

Given relation is a function since each value of x gives unique value of y



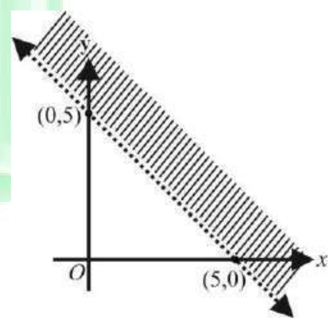
(iii) $\{(x,y) \mid x + y < 5\}$

Using $x + y = 5$, when $x = 0, y = 5$ and when $y = 0, x = 5$. The graph is shown in figure. The domain and range is ¡ . Clearly given relation is not a function.



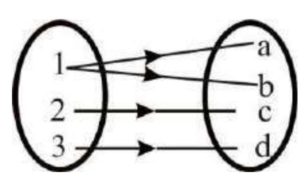
(iv) $\{(x,y) \mid x + y > 5\}$

Using $x + y = 5$, we get (5,0) and (0,5) on graph as shown in fig. The domain & range is ¡ . Clearly given relation is not a function.



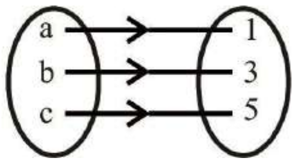
Q.3 Which of the following diagrams represent functions and of which type?

(i)



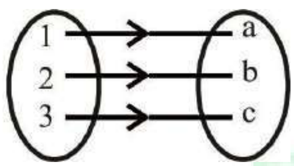
The above figure does not represent a function since element 1 has two images a and b, while for function each element in domain must have a unique image.

(ii)



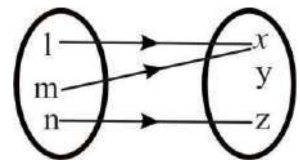
The figure represents a function since every element in domain has a unique image. Also distinct elements have distinct images, therefore it is a one-to-one function. It is also an on to function. Hence given figure represents a bijective function.

(iii)



The figure represents a function since every element in domain has a unique image. Also distinct elements have distinct images, therefore it is a one-to-one function. It is also an on to function. Hence given figure represents a bijective function

(iv)



Each element in domain has unique image, so this represents a function. But distinct elements do not have distinct images, so this is not a 1-1 function. As range $\neq \{x, y, z\}$, so given figure represents an into function.

Q.4 Find inverse of each of the following relations. Tell whether each relation and its inverse is a function or not.

(i) $\{(2,1),(3,2),(4,3),(5,4),(6,5)\}$

Solution:

Let $R=\{(2,1),(3,2),(4,3),(5,4),(6,5)\}$ then its inverse is

$R^{-1}=\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Both R and R^{-1} are functions.

(ii) $\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$ **(DGK 2021, 22)**

Solution:

Let $R=\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$

Then $R^{-1}=\{(3,1),(5,2),(7,3),(9,4),(11,5)\}$

Both R and R^{-1} are functions.

(iii) $\{(x,y) \mid y = 2x+3, x \in \mathbb{R}\}$

Solution:

Let $R = \{(x,y) \mid y = 2x+3, x \in \mathbb{R}\}$

$y = 2x+3 \Rightarrow 2x = y-3 \Rightarrow x = \dfrac{y-3}{2}$

replace x by y

$y = \dfrac{x-3}{2}$

Then $R^{-1} = \left\{(x,y) \mid y = \dfrac{x-3}{2}, x \in \mathbb{R}\right\}$

Both R and R^{-1} are functions.

(iv) $\{(x,y) \mid y^2 = 4ax, x \geq 0\}$ **(SGD 2021)**

Solution:

Let $R = \{(x,y) \mid y^2 = 4ax, x \geq 0\}$

$y^2 = 4ax \Rightarrow \quad y = \pm 2\sqrt{ax}$

Which shows that we get two values of y for one value of x so the above relation is not a function.

Now $y^2 = 4ax \Rightarrow \quad x = \dfrac{y^2}{4a}$

Interchanging x and y, we get $y = \dfrac{x^2}{4a}$

Hence $R^{-1} = \left\{(x,y) \mid y = \dfrac{x^2}{4a}, y \geq 0\right\}$. Clearly R^{-1} is a function.

(v) $\{(x,y) \mid x^2 + y^2 = 9, |x| \leq 3, |y| \leq 3\}$

Solution:

Let $R = \{(x,y) \mid x^2 + y^2 = 9, |x| \leq 3, |y| \leq 3\}$

Using $x^2 + y^2 = 9$ we get $y = \pm\sqrt{9-x^2}$

This shows that there are two values of y for one value of x. Hence R is not a function.

Interchanging x and y we get $y^2 + x^2 = 9$. Hence

$R^{-1} = \{(x,y) \mid y^2 + x^2 = 9, |x| \leq 3, |y| \leq 3\}$

Clearly R^{-1} is not a function.