Mathematics-11
Exercise - 2.7

Q.1 Complete the table, indicating by a tick mark those properties which are satisfied by the specified set of numbers.

Set of Numbers → / Property ↓		Natural	Whole	Integers	Rational	Real
Closure	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Associative	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Identity	$\oplus$		✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Inverse	$\oplus$			✓	✓	✓
	$\otimes$					
Commutative	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓

Q.2 What are field axioms? In what respect does the field of real numbers differ from that of complex numbers?

Solution: **Field:** A non empty set F is said to be a field if for all $x, y, z \in F$, the following axioms are satisfied.

1) $x + y \in F$

2) $x + (y + z) = (x + y) + z$

3) There exists $0 \in F$ such that $x + 0 = 0 + x = x$

4) There exists $-x \in F$ such that $x + (-x) = 0 = -x + x$

5) $x + y = y + x$

6) $xy \in F$

7) $x(yz) = (xy)z$

8) There exists $1 \in F$ such that $x.1 = 1.x = x$

9) There exists $\frac{1}{x}\in F$ such that $\frac{1}{x}.x = x.\frac{1}{x} = 1$ $(x \neq 0)$

10) $xy = yx$

11) $x(y+z) = xy + xz$ and $(x+y)z = xz + yz$

The field of real numbers differ from the field of complex numbers in a way that real field holds order axioms where as field of complex numbers does not hold order axioms.

Q.3 Show that the adjoining table is that of multiplication of the elements of the set of residue classes of modulo 5.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

The zeroes in the second row are produced by the 0 in the first column which shows that this is a product table. It is also noted that whenever a number equal to or greater than 5 is obtained, we divide it by 5 and write the reminder. Hence given table is that of multiplication of the elements of the set of residue classes modulo 5.

Q.4 Prepare a table of addition of the elements of the set of residue classes modulo 4.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Q.5 Which of the following binary operations shown in tables (a) and (b) is commutative?

(a)

$*$	a	b	c	d
a	a	c	b	d
b	b	c	b	a
c	c	d	b	c
d	a	a	b	b

(b)

$*$	a	b	c	d
a	a	c	b	d
b	c	d	b	a
c	b	b	a	c
d	d	a	c	d

In table (a) we have $a*c = b, c*a = c \Rightarrow$ $a*c \neq c*a$ so operation $*$ is not commutative.

In table (b) elements across the diagonal are same, so operation $*$ is commutative. e.g. $a*b = b*a$

$c = c$

Q.6 Supply the missing elements of the third row of the given table so that the operation $*$ may be associative.

$*$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c				
d	d	c	c	d

Solution:

$*$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	p	q	r	s
d	d	c	c	d

Let missing elements be p, q, r and s

$p = c*a$ $= (d*b)*a$ $= d*(b*a)$ $= d*b$ $= c$	$q = c*b$ $= (d*b)*b$ $= d*(b*b)$ $= d*a$ $= d$	$r = c*c$ $= (d*b)*c$ $= d*(b*c)$ $= d*c$ $= c$	$s = c*d$ $= (d*b)*d$ $= d*(b*d)$ $= d*d$ $= d$

Q.7 Which operation is represented by the adjoining table? Name the identity element of the relevant set, if it exists. Is the operation associative? Find the inverses of 0,1,2,3, if they exist.

※	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

The second row of the given table is obtained by adding 0 in 0,1,2,3. This shows that the operation is addition. It is also noted that whenever a number equal to or greater than 4 is obtained, we divide it by 4 and write the remainder. So the binary operation is addition modulo 4.

0 is identity.

Clearly the binary operation is associative. e.g. $(1*2)*3 = 1*(2*3)$

$$3*3 = 1*1$$
$$2 = 2$$

The inverse of 0 is 0.

The inverse of 1 is 3

The inverse of 2 is 2
The inverse of 3 is 1