

EXERCISE 2.3

Differentiate w.r.t "x"

Q.1 $x^4 + 2x^3 + x^2$

Solution:

Let $y = x^4 + 2x^3 + x^2$

Differentiating both sides w.r.t "x",

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 + 2x^3 + x^2)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + 2\frac{d}{dx}(x^3) + \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = 4x^3 + 2(3x^2) + 2x$$

$$\boxed{\frac{dy}{dx} = 4x^3 + 6x^2 + 2x}$$

Q.2 $x^{-3} + 2x^{\frac{3}{2}} + 3$

Solution:

Let $y = x^{-3} + 2x^{\frac{3}{2}} + 3$

Differentiating both sides w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{-3} + 2x^{\frac{3}{2}} + 3\right)$$

$$\frac{dy}{dx} = -3x^{-4} + \cancel{2} \cdot \left(\frac{-3}{\cancel{2}}\right) x^{\frac{-5}{2}} + 0$$

$$\boxed{\frac{dy}{dx} = \frac{-3}{x^4} - \frac{3}{x^{\frac{5}{2}}}}$$

Q.3 $\frac{a+x}{a-x}$

Solution:

Let $y = \frac{a+x}{a-x}$

Differentiating both sides w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$\frac{dy}{dx} = \frac{(a-x)(1) - (a+x)(-1)}{(a-x)^2}$$

$$= \frac{a - \cancel{x} + a + \cancel{x}}{(a-x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2a}{(a-x)^2}}$$

Q.4 $\frac{2x-3}{2x+1}$

Solution:

Let $y = \frac{2x-3}{2x+1}$

Differentiating both sides w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-3}{2x+1}\right)$$

$$\frac{dy}{dx} = \frac{(2x+1)(2) - (2x-3)(2)}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{4x+2-4x+6}{(2x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{8}{(2x+1)^2}}$$

Q.5 $(x-5)(3-x)$

Solution:

Let $y = (x-5)(3-x)$

$$y = 3x - x^2 - 15 + 5x$$

$$y = 8x - x^2 - 15$$

Differentiating both sides w.r.t "x"

$$\frac{d}{dx}(y) = \frac{d}{dx}(8x - x^2 - 15)$$

$$\boxed{\frac{dy}{dx} = 8 - 2x}$$

Q.6 $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

Solution:

Let $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$$y = x + \frac{1}{x} - 2$$

Differentiating both sides w.r.t "x"

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Differentiation

$$\frac{dy}{dx} = \frac{d}{dx}(x + x^{-1} - 2)$$

$$\boxed{\frac{dy}{dx} = 1 - \frac{1}{x^2}}$$

Q.7

$$\frac{(1 + \sqrt{x})\left(x - x^{\frac{3}{2}}\right)}{\sqrt{x}}$$

Solution:

$$\text{Let } y = \frac{(1 + \sqrt{x})\left(x - x^{\frac{3}{2}}\right)}{\sqrt{x}}$$

$$y = \frac{(1 + \sqrt{x})(x)(1 - \sqrt{x})}{\sqrt{x}}$$

$$= \frac{x(1 - x)}{\sqrt{x}}$$

$$= \sqrt{x}(1 - x)$$

$$y = \sqrt{x} - x\sqrt{x}$$

$$y = x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

Differentiating w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x}}$$

Q.8

$$\frac{(x^2 + 1)^2}{x^2 - 1}$$

Solution:

$$\text{Let } y = \frac{(x^2 + 1)^2}{x^2 - 1}$$

Differentiating w.r.t "x",

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(x^2 + 1)^2}{x^2 - 1}\right)$$

$$\frac{dy}{dx} = \frac{(x^2 - 1) \cdot 2(x^2 + 1) \cdot 2x - (x^2 + 1)^2 \cdot (2x)}{(x^2 - 1)^2}$$

$$= \frac{4x(x^2 + 1)(x^2 - 1) - 2x(x^2 + 1)^2}{(x^2 - 1)^2}$$

$$= \frac{2x(x^2 + 1)\{2(x^2 - 1) - (x^2 + 1)\}}{(x^2 - 1)^2}$$

$$= \frac{2x(x^2 + 1)(x^2 - 3)}{(x^2 - 1)^2}$$

Q.9 $\frac{x^2 + 1}{x^2 - 3}$

Solution:

$$\text{Let } y = \frac{x^2 + 1}{x^2 - 3}$$

Differentiating w.r.t "x",

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2 + 1}{x^2 - 3}\right)$$

$$= \frac{(x^2 - 3) \frac{d}{dx}(x^2 + 1) - (x^2 + 1) \frac{d}{dx}(x^2 - 3)}{(x^2 - 3)^2}$$

$$\frac{dy}{dx} = \frac{(x^2 - 3)(2x) - (x^2 + 1)(2x)}{(x^2 - 3)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{2x^3} - 6x - \cancel{2x^3} - 2x}{(x^2 - 3)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-8x}{(x^2 - 3)^2}}$$

Q.10 $\frac{\sqrt{1+x}}{\sqrt{1-x}}$

Solution:

$$\text{Let } y = \sqrt{\frac{1+x}{1-x}}$$

$$\Rightarrow y = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

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Differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \left(\frac{1+x}{1-x} \right)^{-\frac{1}{2}} \frac{d}{dx} \left(\frac{1+x}{1-x} \right) \\ &= \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}} \left[\frac{(1-x) \frac{d}{dx}(1+x) - (1+x) \frac{d}{dx}(1-x)}{(1-x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}} \left[\frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}} \left[\frac{1-\cancel{x} + 1 + \cancel{x}}{(1-x)^2} \right] \\ &= \frac{1}{2} \frac{(1-x)^{\frac{1}{2}} (2)}{\sqrt{1+x} \cdot (1-x)^2} \\ \boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1+x} (1-x)^{\frac{3}{2}}}} \end{aligned}$$

Q.11 $\frac{2x-1}{\sqrt{x^2+1}}$

Solution:

Let $y = \frac{2x-1}{\sqrt{x^2+1}}$
 Differentiating w.r.t "x"
 $\frac{dy}{dx} = \frac{d}{dx} \left(\frac{2x-1}{\sqrt{x^2+1}} \right)$
 $= \frac{\sqrt{x^2+1} \frac{d}{dx}(2x-1) - (2x-1) \frac{d}{dx} \sqrt{x^2+1}}{(\sqrt{x^2+1})^2}$
 $= \frac{\sqrt{x^2+1}(2) - (2x-1) \left(\frac{1}{2\sqrt{x^2+1}} (2x) \right)}{x^2+1}$
 $\frac{dy}{dx} = \frac{2\sqrt{x^2+1} - \frac{x(2x-1)}{\sqrt{x^2+1}}}{x^2+1}$
 $\frac{dy}{dx} = \frac{2(x^2+1) - x(2x-1)}{\sqrt{x^2+1} (x^2+1)}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{2(x^2+1) - x(2x-1)}{(x^2+1)\sqrt{x^2+1}} \\ &= \frac{2x^2+2-2x^2+x}{(x^2+1)^{\frac{3}{2}}} \end{aligned}$$

$$\boxed{\frac{dy}{dx} = \frac{x+2}{(x^2+1)^{\frac{3}{2}}}}$$

Q.12 $\sqrt{\frac{a-x}{a+x}}$

Solution:

Let $y = \sqrt{\frac{a-x}{a+x}}$

Differentiating w.r.t "x"

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \sqrt{\frac{a-x}{a+x}} \\ \frac{dy}{dx} &= \frac{1}{2} \left(\frac{a-x}{a+x} \right)^{-\frac{1}{2}} \cdot \frac{d}{dx} \left(\frac{a-x}{a+x} \right) \\ &= \frac{1}{2} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}} \left[\frac{(a+x) \frac{d}{dx}(a-x) - (a-x) \frac{d}{dx}(a+x)}{(a+x)^2} \right] \\ &= \frac{1}{2} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}} \left[\frac{(a+x)(-1) - (a-x)(1)}{(a+x)^2} \right] \\ &= \frac{1}{2} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}} \left[\frac{-a-\cancel{x} - a+\cancel{x}}{(a+x)^2} \right] \\ &= \frac{1}{2} \frac{(a+x)^{\frac{1}{2}} (-2a)}{(a-x)^{\frac{1}{2}} (a+x)^2} \\ \boxed{\frac{dy}{dx} = \frac{-a}{\sqrt{a-x} \cdot (a+x)^{\frac{3}{2}}}} \end{aligned}$$

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Differentiation

Q.13 $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Solution:

$$\text{Let } y = \frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$$

Differentiating w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{x^2+1}{x^2-1} \right)^{-\frac{1}{2}} \cdot \frac{d}{dx} \left(\frac{x^2+1}{x^2-1} \right)$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{x^2-1}{x^2+1} \right)^{\frac{1}{2}} \frac{(x^2-1) \frac{d}{dx}(x^2+1) - (x^2+1) \frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{x^2-1}{x^2+1} \right)^{\frac{1}{2}} \cdot \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= \frac{1}{2} \left(\frac{x^2-1}{x^2+1} \right)^{\frac{1}{2}} \cdot \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$$

$$= \frac{1}{2} \frac{-4x}{\sqrt{x^2+1}(x^2-1)^{\frac{3}{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{\sqrt{x^2+1}(x^2-1)^{\frac{3}{2}}}}$$

Q.14 $\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

Solution:

$$\text{Let } y = \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$$

Rationalizing the denominator, we get

$$y = \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \cdot \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$$

$$y = \frac{(\sqrt{1+x} - \sqrt{1-x})^2}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2}$$

$$y = \frac{(\sqrt{1+x})^2 + (\sqrt{1-x})^2 - 2\sqrt{1-x^2}}{1+x - 1-x}$$

$$y = \frac{1+x + 1-x - 2\sqrt{1-x^2}}{2x}$$

$$y = \frac{1 - \sqrt{1-x^2}}{x}$$

Differentiating w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{1 - \sqrt{1-x^2}}{x} \right)$$

$$\frac{dy}{dx} = \frac{x \frac{d}{dx}(1 - \sqrt{1-x^2}) - (1 - \sqrt{1-x^2}) \frac{d}{dx}(x)}{x^2}$$

$$\frac{dy}{dx} = \frac{x \left[\frac{1(-\cancel{2}x)}{\cancel{2}\sqrt{1-x^2}} \right] - (1 - \sqrt{1-x^2})(1)}{x^2}$$

$$= \frac{x^2}{\sqrt{1-x^2}} - 1 + \sqrt{1-x^2}$$

$$= \frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{\sqrt{1-x^2}}$$

$$= \frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{x^2 \sqrt{1-x^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - \sqrt{1-x^2}}{x^2 \sqrt{1-x^2}}}$$

Q.15 $\frac{x\sqrt{a+x}}{\sqrt{a-x}}$

Solution:

$$\text{Let } y = x\sqrt{\frac{a+x}{a-x}}$$

Differentiating w.r.t "x"

Unit- 2

Differentiation

$$\frac{dy}{dx} = \frac{d}{dx} \left(x \sqrt{\frac{a+x}{a-x}} \right)$$

$$\frac{dy}{dx} = x \frac{d}{dx} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}} + \sqrt{\frac{a+x}{a-x}} \cdot \frac{d}{dx} (x)$$

$$\frac{dy}{dx} = x \frac{1}{2} \left(\frac{a+x}{a-x} \right)^{-\frac{1}{2}} \cdot \frac{d}{dx} \left(\frac{a+x}{a-x} \right) + \sqrt{\frac{a+x}{a-x}}$$

$$\frac{dy}{dx} = x \cdot \frac{1}{2} \sqrt{\frac{a-x}{a+x}} \cdot \frac{(a-x)(1) - (a+x)(-1)}{(a-x)^2} + \sqrt{\frac{a+x}{a-x}}$$

$$\frac{dy}{dx} = \frac{x}{2} \sqrt{\frac{a-x}{a+x}} \cdot \frac{a-x+a+x}{(a-x)^2} + \sqrt{\frac{a+x}{a-x}}$$

$$\frac{dy}{dx} = \frac{x}{2} \cdot \frac{2a}{\sqrt{a+x} \cdot (a-x)^{\frac{3}{2}}} + \sqrt{\frac{a+x}{a-x}}$$

$$\frac{dy}{dx} = \frac{ax}{\sqrt{a+x} \sqrt{a-x} (a-x)} + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{ax + (a+x)(a-x)}{\sqrt{a+x} (a-x)^{\frac{3}{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{a^2 - x^2 + ax}{\sqrt{a+x} (a-x)^{\frac{3}{2}}}}$$

Q.16 If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ show that

$$2x \frac{dy}{dx} + y = 2\sqrt{x}$$

Solution:

$$y = \sqrt{x} - \frac{1}{\sqrt{x}}$$

Differentiating w.r.t "x"

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2x^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{x+1}{2x\sqrt{x}}$$

$$2x \frac{dy}{dx} = \frac{x+1}{\sqrt{x}}$$

$$2x \frac{dy}{dx} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}$$

$$2x \frac{dy}{dx} + y = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$2x \frac{dy}{dx} = \sqrt{x} + \sqrt{x}$$

$$2x \frac{dy}{dx} + y = 2\sqrt{x}$$

Hence the proof.

Q.17 If $y = x^4 + 2x^2 + 2$, prove that

$$\frac{dy}{dx} = 4x\sqrt{y-1}$$

Solution:

$$y = x^4 + 2x^2 + 2$$

Differentiate w.r.t "x"

$$\frac{dy}{dx} = 4x^3 + 4x$$

$$\frac{dy}{dx} = 4x(x^2 + 1) \dots (i)$$

$$\text{Now } y = x^4 + 2x^2 + 2$$

$$y = x^4 + 2x^2 + 1 + 1$$

$$y - 1 = x^4 + 2x^2 + 1$$

$$y - 1 = (x^2 + 1)^2$$

$$\sqrt{y-1} = x^2 + 1 \dots (ii)$$

Now using equation (ii) and

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Differentiation

Putting the value of $x^2 + 1$ from equation (ii) into equation (i).

$$\frac{dy}{dx} = 4x\sqrt{y-1}$$

Hence the proof

The Chain Rule:

Theorem:

If g is differentiable at the point x and f is differentiable at the point $g(x)$, then the composition function $f \circ g$ is differentiable at the point x and

$$(f \circ g)'(x) = f'[g(x)] \cdot g'(x)$$

THE CHAIN RULE

(1) If $x = f(t)$ and $y = g(t)$ then $\frac{dx}{dt} = f'(t)$ and $\frac{dy}{dt} = g'(t)$ therefore $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

(2) If $y = [g(x)]^n$ and $u = g(x)$ then $y = u^n$ and $\frac{dy}{du} = nu^{n-1}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = nu^{n-1} \frac{du}{dx}$$

or

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} g'(x)$$

