Relation between Roots and Co-efficient of a Quadratic Equation (K.B + U.B)

Roots of standard quadratic equation $ax^2+bx+c=0$ **are**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

If $\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$

Sum of roots

$$S=\frac{-b+\sqrt{b^2-4ac}}{2a}+\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b+\sqrt{b^2-4ac}+(-b)-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2b}{2a}$$

$$\Rightarrow S=-\frac{b}{a}$$

Product of roots

$$P=\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$\because (a+b)(a-b)=a^2-b^2$$

$$=\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}$$

$$=\frac{b^2-(b^2-4ac)}{4a^2}$$

$$=\frac{b^2-b^2+4ac}{4a^2}$$

$$=\frac{4ac}{4a^2}$$

$$\Rightarrow P=\frac{c}{a}$$

Note **(K.B + U.B)**

(i) $S=-\frac{b}{a}=-\frac{\text{Coefficient of } x}{\text{Coefficent of } x^2}$

(ii) $P=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Quadratic Equation with Given Roots (K.B + U.B)

A quadratic equation whose roots are given can be obtained by using formula

$x^2-Sx+P=0$ **Or**

$x^2-(\text{Sum of roots})x+\text{Product of roots}=0$

Example 1: (Page # 26) (A.B)

Without solving, find the sum and product of roots of the equation $3x^2-5x+7=0$

Solution:

$3x^2-5x+7=0$

Here $a=3, b=-5, c=7$

Sum of roots $=S=-\frac{b}{a}$

$$=-\frac{-5}{3}=\frac{5}{3}$$

Product of roots $=P=\frac{c}{a}$

$$=\frac{7}{3}$$

To find unknown values involved in a given Quadratic Equation

Example 1: (Page # 27) (A.B)

Find the value of h, if the sum of roots is equal to 3-times the product of roots of the equation: $3x^2+(9-6h)x+5h=0$.

Solution:

$3x^2+(9-6h)x+5h=0$

Here $a = 3, b = 9 - 6h, c = 5h$

Let α, β be the roots of given equation

Then $\alpha + \beta = -\frac{b}{a} = -\frac{9-6h}{3} = \frac{6h-9}{3}$

And $\alpha\beta = \frac{c}{a} = \frac{5h}{3}$

According to given condition
Sum of roots = 3(Product of roots)

$\alpha + \beta = 3\alpha\beta$

$\frac{6h-9}{3} = 3\left(\frac{5h}{3}\right)$

$6h - 9 = 15h$

$6h - 15h = 9$

$-9h = 9$

$h = -1$

Exercise 2.3

Q.1 Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2-5x+3=0$ **(MTN 2017)** **(A.B)**

Here $a=1, b=-5, c=3$

Sum of roots $= -\frac{b}{a}$

$= -\frac{(-5)}{1}$

$= 5$

Product of roots $= \frac{c}{a}$

$= \frac{3}{1}$

$= 3$

(ii) $3x^2+7x-11=0$ **(A.B)**

(LHR 2017, SWL 2017, SGD 2016, D.G.K 2014)

Here

$a=3,\ b=7,\ c=-11$

Sum of roots $= -\frac{b}{a}$

$= -\frac{7}{3}$

Product of roots $= \frac{c}{a}$

$= \frac{-11}{3}$

$= -\frac{11}{3}$

(iii) $px^2-qx+r=0$ **(A.B)**

(LHR 2014, GRW 2014, SWL 2016, MTN 2017, SGD 2016)

Here $a=p, b=-q, c=r$

Sum of roots $= -\frac{b}{a}$

$= -\frac{(-q)}{p}$

$= \frac{q}{p}$

Product of roots $= \frac{c}{a}$

$= \frac{r}{p}$

(iv) $(a+b)x^2-ax+b=0$ **(A.B)**

(BWP 2014, 17)

Here

$A=a+b,\ B=-a,\ C=b$

Sum of roots $= -\frac{B}{A}$

$= -\frac{-a}{a+b}$

$= \frac{a}{a+b}$

Product of roots $= \frac{C}{A}$

$= \frac{b}{a+b}$

(v) $(l+m)x^2+(m+n)x+n-1=0$

Here

$a=l+m,\quad b=m+n,\quad c=n-l$

Sum of roots $= S = -\frac{b}{a}$

$= -\frac{m+n}{l+m}$

Product of roots $= P = \frac{c}{a}$

$= \frac{n-l}{l+m}$

(vi) $7x^2-5mx+9n=0$ **(A.B)**

Here

$a = 7, b = -5\text{m}, c = 9n$

Sum of roots $= -\frac{b}{a}$

$= -\frac{-5\text{m}}{7}$

$= \frac{5\text{m}}{7}$

Product of roots $= \frac{c}{a}$

$= \frac{9n}{7}$

Q.2 Find the value of *k*, if

(i) Sum of the roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots. (A.B)

Solution:

Let α, β be the roots of equation

$2kx^2 - 3x + 4k = 0$

Here

$a = 2k,\ b = -3,\ c = 4k$

$S = \alpha + \beta = -\dfrac{b}{a}$

$= \dfrac{-(-3)}{2k}$

$= \dfrac{3}{2k}$

$P = \alpha\beta = \dfrac{c}{a}$

$= \dfrac{4k}{2k}$

$= 2$

According to given condition

$S = 2P$

$\dfrac{3}{2k} = 2 \times 2$

$\dfrac{3}{2k} = \dfrac{4}{1}$

$3 \times 1 = 4 \times 2k$

$3 = 8k$

$\dfrac{3}{8} = k$

$k = \dfrac{3}{8}$

(ii) Sum of the roots of the equation $x^2 + (3k-7)x + 5k = 0$ is $\dfrac{3}{2}$ times the product of the roots. (A.B)

Solution:

$x^2 + (3k-7)x + 5k = 0$

Here, $a = 1,\ b = 3k-7,\ c = 5k$

Sum of roots = $S = \dfrac{-b}{a}$

$= -\dfrac{3k-7}{1}$

$= -3k + 7$

Product of roots = P = $\dfrac{c}{a}$

$= \dfrac{5k}{1}$

$= 5k$

According to given condition:

$S = \dfrac{3}{2}P$

$-3k + 7 = \dfrac{3}{2}(5k)$

$2(-3k+7) = 3(5k)$

$-6k + 14 = 15k$

$14 = 15k + 6k$

$14 = 21k$

$\dfrac{14}{21} = k$

$\dfrac{2}{3} = k$

Or $k = \dfrac{2}{3}$

Q.3 Find *k*,

(i) If sum of squares of the roots of the equation $4kx^2 + 3kx - 8 = 0$ is 2.

Solution **(FSD 2015)** **(A.B)**

$4kx^2 + 3kx - 8 = 0$

Here a = 4k, b = 3k, c = –8

Let α, β be the roots

Sum of roots = $\alpha + \beta = -\dfrac{b}{a}$

$= -\dfrac{3k}{4k}$

$\Rightarrow \alpha + \beta = -\dfrac{3}{4}$

Product of roots = $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{-8}{4k}$

$\Rightarrow \alpha\beta = \dfrac{-2}{k}$

According to given condition

$\alpha^2 + \beta^2 = 2$

$(\alpha+\beta)^2-2\alpha\beta=2$

Putting the values

$\left(-\frac{3}{4}\right)^2-2\left(-\frac{2}{k}\right)=2$

$\frac{9}{16}+\frac{4}{k}=2$

$\frac{4}{k}=2-\frac{9}{16}$

$\frac{4}{k}=\frac{32-9}{16}$

$\frac{4}{k}=\frac{23}{16}$

$\Rightarrow \quad \frac{k}{4}=\frac{16}{23}$

$k=\frac{64}{23}$

(ii) Sum of the squares of the roots of the equation $x^2-2kx+(2k+1)=0$ is 6. **(A.B)**

Solution:

Here

$a=1, b=-2k, c=2k+1$

Let, α,β be the roots of given equation,

Then

Sum of roots $=\alpha+\beta=-\frac{b}{a}$

$=-\left(\frac{-2k}{1}\right)$

$= 2k$

Product of roots $=\alpha\beta=\frac{c}{a}$

$=\frac{2k+1}{1}$

$=2k+1$

According to given condition

$\alpha^2+\beta^2=6$

Or $\quad (\alpha+\beta)^2-2\alpha\beta=6$

Putting the values

$(2k)^2-2(2k+1)=6$

$4k^2-4k-2-6=0$

$4k^2-4k-8=0$

$4(k^2-k-2)=0$

Or $\quad k^2-k-2=0$

$k^2-2k+k-2=0$

$k(k-2)+1(k-2)=0$

$(k-2)(k+1)=0$

Either

$k-2=0 \quad$ or $\quad k=2$

$k+1=0$

$k=-1$

Result

$k=2,-1$

Q.4 Find p, if

(i) The roots of the equation $x^2-x+p^2=0$ differ by unity. **(FSD 2015) (A.B)**

Let α,β be the roots of given equation.

Here $\quad a=1, \quad b=-1, \quad c=p^2$

Then $\quad \alpha+\beta=-\frac{b}{a}$

$=-\left(\frac{-1}{1}\right)$

$=1$

$\alpha\beta \quad =\frac{c}{a}$

$=\frac{p^2}{1}$

$=p^2$

According to given condition

$\alpha-\beta=1$

Taking square on both sides

$(\alpha-\beta)^2=1$

$\because (a+b)^2-(a-b)^2=4ab$

$(\alpha+\beta)^2-4\alpha\beta=1$

Putting the values

$(1)^2 - 4p^2 = 1$

$1 - 4p^2 = 1$

$1 - 1 = 4p^2$

$0 = 4p^2$

Or

$p^2 = 0$

By taking square root

$p = 0$

Result:

$p = 0$

(ii) The roots of the equation $x^2 + 3x + P - 2 = 0$ differ by 2.

Solution: **(A.B)**

$x^2 + 3x + P - 2 = 0$

Here $a = 1, b = 3, c = P - 2$

Let roots of given equation are α, β

Then sum of roots $= \alpha + \beta = -\frac{b}{a}$

$= -\frac{3}{1}$

$\Rightarrow \alpha + \beta = -3$

Product of roots $= \alpha\beta = \frac{c}{a}$

$= \frac{P-2}{1}$

According to given condition

$\alpha - \beta = 2$

Taking square of both sides

$(\alpha - \beta)^2 = 4$

$(\alpha + \beta)^2 - 4\alpha\beta = 4$

Putting the values

$(-3)^2 - 4(P - 2) = 4$

$9 - 4P + 8 = 4$

$9 + 8 - 4 = 4P$

$13 = 4P$

Or $P = \frac{13}{4}$

Result: $P = \frac{13}{4}$

Q.5 Find m, if

(i) The roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$ **(A.B)**

Solution:

$x^2 - 7x + 3m - 5 = 0$

Here $a = 1, b = -7, c = 3m - 5$

Let α, β be the roots of given equation

Then sum of roots $= \alpha + \beta = -\frac{b}{a}$

$= -\frac{-7}{1}$

$\Rightarrow \alpha + \beta = 7 \rightarrow (i)$

Product of roots $= \alpha\beta = \frac{c}{a}$

$= \frac{3m-5}{1}$

$\Rightarrow \alpha\beta = 3m - 5 \rightarrow (ii)$

According to given condition

$3\alpha + 2\beta = 4 \rightarrow (iii)$

Multiply equation (i) by '2'

$2\alpha + 2\beta = 14 \longrightarrow (iv)$

Sub. Equation (iii) & (iv)

$3\alpha + 2\beta = 4$

$\underline{-2\alpha \pm 2\beta = -14}$

$\alpha \quad = -10$

Put in equation (i)

$\alpha + \beta = 7$

$-10 + \beta = 7$

$\beta = 7 + 10$

$\beta = 17$

Putting the values of α and β in equation (ii)

$\alpha\beta = 3m - 5$

$-10(17) = 3m - 5$

$-170 + 5 = 3m$

$\frac{-165}{3} = m$

Or $m = -55$

Result:

$m = -55$

(ii) The roots of the equation $x^2+7x+3m-5=0$ satisfy the relation $3\alpha-2\beta=4$ **(A.B)**

Let α,β be the roots of given equation

Here

$a=1,\quad b=7,\quad c=3m-5$

Then

$\alpha+\beta=-\frac{b}{a}$

$=\frac{-7}{1}$

$\alpha+\beta=-7 \longrightarrow (i)$

$\alpha\beta=\frac{c}{a}$

$=\frac{3m-5}{1}$

$\alpha\beta=3m-5 \longrightarrow (ii)$

Also given

$3\alpha-2\beta=4 \longrightarrow (iii)$

Multiply equation (i) by 2

$2\alpha+2\beta=-14 \longrightarrow (iv)$

Adding equation (iii) and (iv)

$3\alpha-2\beta=4$

$\underline{2\alpha+2\beta=-14}$

$5\alpha\quad=-10$

$\alpha=-\frac{10}{5}$

$\Rightarrow\alpha=-2$

Put in equation (i)

$\alpha+\beta=-7$

$-2+\beta=-7$

$\beta=-7+2$

$\beta=-5$

Now putting the values in equation (ii)

$\alpha\beta=3m-5$

$-2(-5)=3m-5$

$10+5=3m$

$15=3m$

$\frac{15}{3}=m$

$5=m$

Result:

$m=5$

(iii) $3x^2-2x+7m+2=0$ **(A.B)**

Here $a=3, b=-2, c=7m+2$

Let α,β be the roots of given equation

Then sum of roots $=\alpha+\beta=-\frac{b}{a}$

$=-\frac{-2}{3}$

$\Rightarrow \alpha+\beta=\frac{2}{3} \longrightarrow (i)$

Product of roots $=\alpha\beta=\frac{c}{a}$

$=\frac{7m+2}{3} \longrightarrow (ii)$

Also given

$7\alpha-3\beta=18 \longrightarrow (iii)$

Multiply equation (i) by '3'

$3\alpha+3\beta=2 \longrightarrow (iv)$

Adding equation (iii) and (iv)

$7\alpha-3\beta=18$

$\underline{3\alpha+3\beta=2}$

$10\alpha\quad=20$

$\Rightarrow\alpha=2$

Put in equation (i)

$\alpha+\beta=\frac{2}{3}$

$2+\beta=\frac{2}{3}$

$\beta=\frac{2}{3}-2$

$=\frac{2-6}{3}$

$\beta=\frac{-4}{3}$

Now putting the values of α and β in equation (ii)

$2\left(\frac{-4}{3}\right)=\frac{7m+2}{3}$

$-8=7m+2$

$-8-2=7m$

$-10 = 7m$

$\frac{-10}{7} = m$

Result:

$m = \frac{-10}{7}$

Q.6 Find m, if sum and product of the roots of the following equations is equal to a given number λ.

(A.B)

(i) $(2m+3)x^2+(7m-5)x+(3m-10)=0$

Here

$a = 2m+3, b = 7m-5$ and $c = 3m-10$

Let α, β be the roots of given equation,

Then $\alpha+\beta = -\frac{b}{a}$

$= -\frac{7m-5}{2m+3}$

$= \frac{5-7m}{2m+3}$

And $\alpha\beta = \frac{c}{a}$

$= \frac{3m-10}{2m+3}$

According to given condition

$\alpha+\beta = \lambda = \alpha\beta$

(By using transitive property)

$\Rightarrow \alpha+\beta = \alpha\beta$

Putting the values

$\frac{5-7m}{2m+3} = \frac{3m-10}{2m+3}$

Or

$5-7m = 3m-10$

(By using cancellation property)

$-7m-3m = -10-5$

$-10m = -15$

$m = \frac{-15}{-10}$

$\Rightarrow m = \frac{3}{2}$

Result: $m = \frac{3}{2}$

(ii) $4x^2-(3+5m)x-(9m-17)=0$

(A.B)

Here

$a = 4, b = -(3+5m), c = -(9m-17)$

Let α, β be the roots

Then $\alpha + \beta = -\frac{b}{a}$

$= -\frac{-(3+5m)}{4}$

$= \frac{3+5m}{4}$

And $\alpha\beta = \frac{c}{a}$

$= \frac{-(9m-17)}{4}$

According to given condition:

$\alpha+\beta = \lambda$ and $\alpha\beta = \lambda$

$\Rightarrow \alpha+\beta = \alpha\beta$

$\because$ Transitive property of equality

Putting the values

$\frac{3+5m}{4} = \frac{-(9m-17)}{4}$

$\Rightarrow 3 + 5m = -9m + 17$

$5m + 9m = 17 - 3$

$14m = 14$

$\Rightarrow m = 1$

Result: $m = 1$