



Mathematics-10
Exercise 2.4

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Symmetric Function of the Roots of Quadratic Equation (K.B + U.B)

(MTN 2014, FSD 2014)

A function in which the roots involved are such that the value of the expression remains same, when roots are interchanged is called symmetric function. i.e. $f(\alpha, \beta) = f(\beta, \alpha)$

Some symmetric functions are:

$$\alpha^2 + \beta^2, \quad \alpha^3 + \beta^3, \quad \frac{1}{\alpha} + \frac{1}{\beta}$$

Example: (Page # 30) (K.B + U.B)

Verify that

$\alpha^2 + \beta^2 + 2\alpha\beta$ is Symmetric

Verification:

$$\text{Let } f(\alpha, \beta) = \alpha^2 + \beta^2 + 2\alpha\beta \rightarrow (i)$$

$$\begin{aligned} f(\beta, \alpha) &= (\beta)^2 + (\alpha)^2 + 2\beta\alpha \\ &= \beta^2 + \alpha^2 + 2\alpha\beta \\ &= \alpha^2 + \beta^2 + 2\alpha\beta \rightarrow (ii) \end{aligned}$$

From equation (i) and (ii) we get

$$f(\alpha, \beta) = f(\beta, \alpha)$$

Hence $\alpha^2 + \beta^2 + 2\alpha\beta$ is symmetric

Example: (Page # 30) (A.B + U.B)

Find the value of $\alpha^3 + \beta^3 + 3\alpha\beta$, if $\alpha = 2, \beta = 1$. Also find the value of $\alpha^3 + \beta^3 + 3\alpha\beta$ if $\alpha = 1, \beta = 2$.

Solution:

When $\alpha = 2, \beta = 1$

$$\begin{aligned} \alpha^3 + \beta^3 + 3\alpha\beta &= (2)^3 + (1)^3 + 3(2)(1) \\ &= 8 + 1 + 6 = 15 \end{aligned}$$

When $\alpha = 1, \beta = 2$

$$\begin{aligned} \alpha^3 + \beta^3 + 3\alpha\beta &= (1)^3 + (2)^3 + 3(1)(2) \\ &= 1 + 8 + 6 = 15 \end{aligned}$$

Note

Expression $\alpha^3 + \beta^3 + 3\alpha\beta$ represents a symmetric function.

Exercise 2.4

Q.1 If α, β are roots of the equation

$$x^2 + px + q = 0, \text{ then evaluate}$$

- (i) $\alpha^2 + \beta^2$
- (ii) $\alpha^3\beta + \alpha\beta^3$
- (iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(i) Solution: (A.B + U.B)

$$x^2 + px + q = 0$$

Here $a = 1, b = p, c = q$

Roots of given equation are α, β

Then

$$\begin{aligned} \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{p}{1} \\ &= -p \\ \alpha\beta &= \frac{c}{a} \\ &= \frac{q}{1} \\ &= q \end{aligned}$$

(i) Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\begin{aligned} \alpha^2 + \beta^2 &= (-p)^2 - 2(q) \\ &= p^2 - 2q \end{aligned}$$

(ii) $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2) \quad (A.B)$

$$\begin{aligned} &= \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta] \\ &= q[(-p)^2 - 2(q)] \end{aligned}$$

$$\Rightarrow \alpha^3\beta + \alpha\beta^3 = q(p^2 - 2q)$$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

(LHR 2015) (K.B + U.B)

Unit-2

Theory of Quadratic Equations

$$\begin{aligned} &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \\ &= \frac{(-p)^2 - 2(q)}{q} \\ \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= \frac{p^2 - 2q}{q} \end{aligned}$$

Q.2 If α, β are the roots of the equation $4x^2 - 5x + 6 = 0$, then find the value of

- (i) $\frac{1}{\alpha} + \frac{1}{\beta}$ **(K.B + A.B)**
- (ii) $\alpha^2\beta^2$ **(A.B + U.B)**
- (iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ **(U.B + A.B)**
- (iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ **(K.B + U.B)**

(LHR 2016, GRW 2014, SWL 2016, MTN 216, SGD 2015, D.G.K 2014)

Solution:

$$4x^2 - 5x + 6 = 0$$

Here $a = 4, b = -5, c = 6$

Since α, β be the roots of the given equation

Then

$$\begin{aligned} \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{(-5)}{4} \end{aligned}$$

$$= \frac{5}{4}$$

$$\begin{aligned} \alpha\beta &= \frac{c}{a} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(i)} \quad \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\beta + \alpha}{\alpha\beta} \\ &= \frac{\alpha + \beta}{\alpha\beta} \end{aligned}$$

$$\begin{aligned} &= \frac{5}{4} \\ &= \frac{4}{3} \\ &= \frac{2}{2} \\ &= \frac{5}{4} \times \frac{2}{3} \\ \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{5}{6} \\ \text{(ii)} \quad \alpha^2\beta^2 &= (\alpha\beta)^2 \\ &= \left(\frac{3}{2}\right)^2 \\ \Rightarrow \alpha^2\beta^2 &= \frac{9}{4} \\ \text{(iii)} \quad \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} &= \frac{\beta + \alpha}{\alpha^2\beta^2} \\ &= \frac{\alpha + \beta}{(\alpha\beta)^2} \\ &= \frac{5}{4} \\ &= \left(\frac{3}{2}\right)^2 \\ &= \frac{5}{4} \\ &= \frac{4}{9} \\ &= \frac{1}{4} \\ &= \frac{5}{4} \times \frac{4}{9} \\ \Rightarrow \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} &= \frac{5}{9} \\ \text{(iv)} \quad \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} \\ &= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} \\ \text{Putting the values} \\ &= \frac{\left(\frac{5}{4}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{4}\right)}{\frac{3}{2}} \end{aligned}$$

Unit-2

Theory of Quadratic Equations

$$\begin{aligned}
 &= \frac{125}{64} - \frac{45}{8} \\
 &= \frac{3}{\frac{2}{2}} \\
 &= \frac{125-360}{64} \times \frac{2}{3} \\
 &= \frac{-235}{64} \times \frac{2}{3} \\
 \Rightarrow &\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = -\frac{235}{96}
 \end{aligned}$$

Q.3 If α, β are the roots of the equation $lx^2 + mx + n = 0$ ($l \neq 0$), then find the values of

- (i) $\alpha^3\beta^2 + \alpha^2\beta^3$ **(A.B + U.B)**
 (ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ **(K.B + U.B)**

Solution:

$$lx^2 + mx + n = 0$$

Roots of given equation are α, β

$$\text{Then } \alpha + \beta = -\frac{b}{a} = -\frac{m}{l}$$

$$\begin{aligned}
 \alpha\beta &= \frac{c}{a} \\
 &= \frac{n}{l}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \text{Now } \alpha^3\beta^2 + \alpha^2\beta^3 &= \alpha^2\beta^2(\alpha + \beta) \\
 &= (\alpha\beta)^2(\alpha + \beta) \\
 &= \left(\frac{n}{l}\right)^2 \times \left(-\frac{m}{l}\right) \\
 &= \frac{n^2}{l^2} \times \left(-\frac{m}{l}\right) \\
 \Rightarrow \alpha^3\beta^2 + \alpha^2\beta^3 &= -\frac{mn^2}{l^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} \\
 &= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}
 \end{aligned}$$

(FSD 2017, SWL 2017, BWP 2014, D.G.K 2017)

$$\begin{aligned}
 &= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} \\
 &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\left(-\frac{m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2} \\
 &= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}} \\
 &= \frac{m^2 - 2ln}{l^2} \times \frac{l^2}{n^2} \\
 \Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{m^2 - 2ln}{n^2}
 \end{aligned}$$