Exercise 2.5

Q.1 Write the quadratic equations having following roots.

(a) 1, 5 **(K.B + A.B)**
(b) 4, 9 **(K.B + A.B)**
(c) –2, 3 **(K.B + A.B)**
(d) 0, –3 **(K.B + A.B)**
(e) 2, –6 **(K.B + A.B)**
(f) –1, –7 **(K.B + A.B)**
(g) $1+i, 1-i$ **(K.B + A.B)**
(h) $3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**

Solution:

(a) Roots of required equation are 1, 5
Then sum of roots = S = 1 + 5 = 6
And product of roots = P = 1 × 5 = 5
∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 5 = 0$$

(b) **(FSD 2016, 17, RWP 2017, RWP 2017)**
Roots of required equation are 4, 9
Then sum of roots = S = 4 + 9 = 13
And product of roots $= P = 4\times 9 = 36$
∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$
$$x^2 - 13x + 36 = 0$$

(c) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are –2, 3
Then sum of roots = S = –2 + 3 = 1
And product of roots = P = –2(3) = –6
∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$
$$x^2 - 1x + (-6) = 0$$ **(K.B + A.B)**
$$x^2 - x - 6 = 0$$

(d) **(SGD 2014, BWP 2017)**
Roots of required equation are 0, –3
Then sum of roots = S = 0 + (–3) = –3
And product of roots = P = 0(–3) = 0
∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$
$$x^2 - (-3)x + 0 = 0$$ **(K.B + A.B)**
$$x^2 + 3x = 0$$

(e) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are 2, –6
Sum of roots = S = 2 + (–6) = –4
Product of roots = P = 2(–6) = –12
∴ Required quadratic equation is:

$$x^2 - Sx + P = 2$$
$$x^2 - (-4)x + (-12) = 0$$
$$x^2 + 4x - 12 = 0$$ **(K.B + A.B)**

(f) **(LHR 2015, 17, RWP 2016)**
Roots of required equation are –1, –7
Sum of roots = S = –1 + (–7) = –8
Product of roots = P = –1(–7) = 7
∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$
$$x^2 - (-8)x + 7 = 0$$ **(K.B + A.B)**
$$x^2 + 8x + 7 = 0$$

(g) **(D.G.K 2014, SGD 2017)**
$1+i, 1-i$ **(K.B + A.B)**
Roots of the required equation are
$1+i,\ 1-i$

Sum of roots $= S = (1+i)+(1-i)$
$= 1+i+1-i$
$= 2$

Product of roots $= P = (1+i)(1-i)$
$= (1)^2 - (i)^2$
$= 1 - i^2$
$= 1-(-1)$
$= 1+1$
$= 2$

Required quadratic equation is

$$x^2 - Sx + P = 0$$

$x^2 - 2x + 2 = 0$

(h) Roots of required equation are $3+\sqrt{2},\ 3-\sqrt{2}$ **(K.B + A.B)**

Sum of roots $= \left(3+\sqrt{2}\right)+\left(3-\sqrt{2}\right)$

$= 3+\sqrt{2}+3-\sqrt{2}$

$= 6$

Product of roots $= \left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$

$= (3)^2 - \left(\sqrt{2}\right)^2$

$= 9 - 2$

$= 7$

$\therefore$ Required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 7 = 0$

Q.2 If α, β are the roots of the equation $x^2 - 3x + 6 = 0$. Form equations whose roots are

(a) $2\alpha+1,\ 2\beta+1$

(b) α^2, β^2

(c) $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

(d) $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

(e) $\alpha+\beta,\ \dfrac{1}{\alpha}+\dfrac{1}{\beta}$

Solution: **(K.B + A.B)**

$x^2 - 3x + 6 = 0$

Here $a = 1, b = -3, c = 6$

Roots of given equations are α, β

Then $\alpha + \beta = -\dfrac{b}{a}$

$= -\dfrac{-3}{1}$

$= 3$

$\alpha\beta = \dfrac{c}{a}$

$= \dfrac{6}{1}$

$= 6$

(a) Roots of required equation are $2\alpha+1,\ 2\beta+1$

(FSD 2015) **(K.B + A.B)**

Sum of roots = S = $(2\alpha+1)(2\beta+1)$

$= 2\alpha+1+2\beta+1$

$= 2\alpha+2\beta+2$

$= 2(\alpha+\beta)+2$

$= 2(3) + 2$

$= 6 + 2$

$= 8$

Product of roots = P $= (2\alpha+1)(2\beta+1)$

$= 4\alpha\beta+2\alpha+2\beta+1$

$= 4(6) + 6 + 1$

$= 24 + 7 = 31$

$\therefore$ Required quadratic equation is:

$x^2 - Sx + P = 0$

$x^2 - 8x + 31 = 0$

(b) Roots of required equation are α^2, β^2

Sum of roots = S = $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(6)$

$= 9 - 12$

$= -3$ **(K.B + A.B)**

Product of roots = P = $\alpha^2\beta^2$

$= (\alpha\beta)^2$

$= (6)^2$

$= 36$

$\therefore$ Required quadratic equation is:

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 36 = 0$

$x^2 + 3x + 36 = 0$

(c) Roots of required equation are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

(K.B + A.B)

Sum of roots = S = $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$

$= \dfrac{\beta+\alpha}{\alpha\beta}$

$= \dfrac{\alpha+\beta}{\alpha\beta}$

$= \dfrac{3}{6}$

$$= \frac{1}{2}$$

Product of roots = P = $\frac{1}{\alpha}.\frac{1}{\beta}$

$$= \frac{1}{\alpha\beta}$$

$$= \frac{1}{6}$$

∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiply by '6'

$$6x^2 - 3x + 1 = 0$$

(d) Roots of required equation are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(K.B + A.B)

$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(3)^2 - 2(6)}{6}$$

$$= \frac{9-12}{6} = \frac{-3}{6}$$

$$\Rightarrow S = -\frac{1}{2}$$

$$P = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$$

$$\Rightarrow P = 1$$

Required quadratic equation is

$$x^2 - Sx + p = 0$$

Or $x^2 - \left(-\frac{1}{2}\right)x + 1 = 0$

$x^2 + \frac{1}{2}x + 1 = 0$ (Multiplying by 2)

$$2x^2 + x + 2 = 0$$

(e) **Roots of required equation,**

$\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$ **(K.B + A.B)**

$$S = (\alpha+\beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha+\beta) + \left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$$= (\alpha+\beta) + \left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= 3 + \frac{3}{6} = \frac{18+3}{6}$$

$$= \frac{21}{6} = \frac{7}{2}$$

$$P = (\alpha+\beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha+\beta)\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$$= 3\left(\frac{3}{6}\right)$$

$$= (3)\left(\frac{1}{2}\right)$$

$$P = \frac{3}{2}$$

Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying by (2)

$$2x^2 - 7x + 3 = 0$$

Q.3 **If α, β are the roots of the equation $x^2 + px + q = 0$ form equations whose roots are:**

(a) α^2, β^2 **(FSD 2015)**

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

$$x^2 + px + q = 0$$

Here

$$a = 1, b = p, c = q$$

Roots of given equation are α, β

Then $\alpha+\beta=-\dfrac{b}{a}=-p$

$\alpha\beta=\dfrac{c}{a}=q$

(a) Roots of required equation are α^2,β^2

(K.B + A.B)

Sum of roots = S = $\alpha^2+\beta^2$

$= (\alpha+\beta)^2-2\alpha\beta$

$= (-P)^2-2q$

$= p^2-2q$

Product of roots = P = $\alpha^2\beta^2$

$= (\alpha\beta)^2$

$= (q)^2$

$= q^2$

$\therefore$ Required quadratic equation is

$x^2-Sx+P=0$

$x^2-(p^2-2q)x+q^2=0$

(b) Roots of required equation are $\dfrac{\alpha}{\beta},\dfrac{\beta}{\alpha}$

(K.B + A.B)

Sum of roots = S = $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

$= \dfrac{\alpha^2+\beta^2}{\alpha\beta}$

$= \dfrac{p^2-2q}{q}$

Product of roots = P = $\dfrac{\alpha}{\beta}.\dfrac{\beta}{\alpha}$

$= 1$

$\therefore$ Required quadratic equation is:

$x^2-Sx+P=0$

$\Rightarrow x^2-\dfrac{p^2-2q}{q}x+1=0$

Multiply by 'q'

$qx^2-(p^2-2q)x+q=0$