Synthetic Division **(K.B)**

It is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial.
It is a short cut of long division.

Example 3: (Page # 35) **(A.B)**

Use synthetic division, divide the polynomial $P(x)=5x^4+x^3-3x$ by $x-2$.

Solution:

$P(x)=5x^4+x^3+0x^2-3x+0$

Here $x-a=x-2 \Rightarrow x=2$

	5	1	0	-3	0
2	↓	10	22	44	82
	5	11	22	41	82

$\therefore Q(x)=5x^3+11x^2+22x+41$

$R=82$

Example 3: (Page # 36) **(A.B)**

Using synthetic division, find the value of h. If the zero of polynomial $p(x)=3x^2+4x-7h$ is 1.

Solution:

$p(x)=3x^2+4x-7h$ and its zero is 1.

Then by the synthetic division.

1	3	4	$-7h$
		3	7
	3	7	$7-7h$

Remainder $=7-7h$

Since 1 is the zero of the polynomial, therefore,

Remainder $=0$, that is

$7-7h=0$

$7=7h$

$\Rightarrow h=1$

Example 4: (Page # 36) **(A.B)**

Using synthetic division, find the values of l and m, if $x-1$ and $x+1$ are the factors of the polynomial

$P(x)=x^3+3lx^2+mx-1$

Solution:

Since $x-1$ and $x+1$ are the factors of $P(x)=x^3+3lx^2+mx-1$

Therefore, 1 and -1 are zeros of polynomial $P(x)$.

Now by synthetic division

1	1	$3l$	m	-1
	↓	-1	$-3l+1$	$3l+m+1$
	1	$3l-1$	$3l+m+1$	$3l+m$

Since 1 is the zero of polynomial, therefore, remainder is zero, that is,

$3l+m=0 \rightarrow (i)$

And

-1	1	$3l$	m	-1
	↓	-1	$-3l+1$	$3l-m-1$
	1	$3l-1$	$3l+m+1$	$3l-m-2$

Since 1 is the zero of polynomial, therefore, remainder is zero, that is,

$3l-m-2=0 \rightarrow (ii)$

Adding equations (i) and (ii), we get

$6l-2=0$

$6l=2 \Rightarrow l=\frac{2}{6}=\frac{1}{3}$

Put the value of l in equation (i)

$3\left(\frac{1}{3}\right)+m=0$ or

$1+m=0 \Rightarrow m=-1$

Thus $l=\frac{1}{3}$ and $m=-1$

Example 6: (Page # 38) **(A.B)**

By synthetic division, solve the equation $x^4-49x^2+36x+252=0$ having roots -2 and 6.

Solution:

Since –2 and 6 are the roots of the given equation $x^4-49x^2+36x+252=0$.
Then by synthetic division, we get

	1	0	–49	36	252
–2	↓	–2	4	90	–252
	1	–2	–45	126	0
6		6	24	–126	
	1	4	–21	0	

∵ The depressed equation is

$x^2+4x-21=0$

$x^2+7x-3x-21=0$

$x(x+7)-3(x+7)=0$

$(x+7)(x-3)=0$

Either $x+7=0$ or $x-3=0$

$x=-7$ or $x=3$

Thus $-2, 6-7$ and 3 are the roots of the given equation.

Exercise 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2+7x-1)\div(x+1)$

(ii) $(4x^3-5x+15)\div(x+3)$

(iii) $(x^3+x^2-3x+2)\div(x-2)$

Solution:

(i) $P(x)=x^2+7x-1$

(FSD 2016, 17) (A.B)

	1	7	–1
–1	↓	–1	–6
	1	6	–7

$\therefore Q(x) = x + 6$

R= –7

(ii) $P(x)=4x^3-5x+15$ **(A.B)**

(SWL 2016, SGD 2017, MTN 2016)

$= 4x^3+0x^2-5x+15$

	4	0	–5	15
–3	↓	–12	36	–93
	4	–12	31	–78

$\therefore\ Q(x)=4x^2-12x+31$

R = –78

(iii) $P(x)=x^3+x^2-3x+2$ **(A.B)**

(GRW 2017, FSD 2015, MTN 2017, D.G.K 2015)

	1	1	–3	2
2	↓	2	6	6
	1	3	3	8

$\therefore\ Q(x)=x^2+3x+3$

R = 8

Q.2 Find the value of h using synthetic division, if **(A.B)**

(i) 3 is the zero of the polynomial $2x^3-3hx^2+9$

(ii) 1 is the zero of the polynomial x^3-2hx^2+11

(iii) –1 is the zero of the polynomial $2x^3+5hx-23$

Solution:

(i) $P(x)=2x^3-3hx^2+9$

(SWL 2014) (A.B)

$= 2x^3-3hx^2+0x+9$

	2	$-3h$	0	9
3	↓	6	$-9h+18$	$-27+54$
	2	$-3h+6$	$-9h+18$	$-27h+63$

Since 3 is the zero of given polynomial, $R=0$

$\Rightarrow$ –27h + 63 = 0

–27h = –63

$h = \frac{-63}{-27}$

$\Rightarrow h = \frac{7}{3}$

(ii) $P(x)=x^3-2hx^2+11$ **(A.B)**

$= x^3-2hx^2+0x+11$

	1	–2h	0	11
1	↓	1	–2h+1	–2h+1
	1	–2h+1	–2h+1	–2h+12

Since 1 is zero of given polynomial, R = 0

$\Rightarrow$ $-2h + 12 = 0$

$-2h = -12$

$h = \frac{-12}{-2}$

$h = 6$

Result:

$h = 6$

(iii) $P(x) = 2x^3 + 5hx - 23$ **(A.B)**

$= 2x^3 + 0x^2 + 5hx - 23$

	2	0	5h	–23
–1	↓	–2	2	–5h–2
	1	–2h+1	–2h+1	$\lfloor -5h-25$

Since 1 is zero of given polynomial, R = 0

$\Rightarrow -5h - 25 = 0$

$-5h = 25$

$h = -\frac{25}{5} = -5$

Result:

$h = -5$

Q.3 Use synthetic division to find the values of *l* and m, if (A.B)

(i) $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 + 4x^2 + 2l + m$

(ii) $(x - 1)$ and $(x + 1)$ are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

Solution:

(i) $P(x) = x^3 + 4x^2 + 2lx + m$ **(A.B)**

	1	4	2*l*	m
–3	↓	–3	–3	–6*l*–9
	1	1	2*l*–3	$\lfloor -6l + m + 9$
2	↓	2	6	
	1	3	$\lfloor 2l + 3$	

Since $x - 2$ is a factor, R = 0

$\Rightarrow$ $2l + 3 = 0$

$2l = -3$

$\Rightarrow$ $l = \frac{-3}{2}$

Also $x + 3$ is a factor, R = 0

$\Rightarrow$ $-6l + m + 9 = 0$

$-6\left(\frac{-3}{2}\right) + m + 9 = 0 \quad \because l = \frac{-3}{2}$

$9 + m + 9 = 0$

$m + 18 = 0$

$\Rightarrow$ $m = -18$

Result

$l = \frac{-3}{2}$, $m = -18$

(ii) $P(x) = x^3 - 3lx^2 + 2mx + 6$ **(A.B)**

	1	–3*l*	2m	6
1	↓	1	–3*l*+1	–3*l*+2m+1
	1	–3*l* +1	–3*l*+2m+1	$\lfloor -3l + 2m + 7$
-1	↓	–1	3*l*	
	1	–3*l*	$\lfloor 2m + 1$	

Since $x + 1$ is a factor, R = 0

$\Rightarrow$ $2m + 1 = 0$

$2m = -1$

$\Rightarrow$ $m = \frac{-1}{2}$

Also $x - 1$ is a factor, R = 0

$-3l + 2m + 7 = 0$

$-3l + 2\left(-\frac{1}{2}\right) + 7 = 0 \because m = -\frac{1}{2}$

$-3l - 1 + 7 = 0$

$-3l + 6 = 0$

$-3l = -6$

$\Rightarrow$ $l = 2$

Result

$l = 2$, $m = -\frac{1}{2}$

Q.4 Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

(iii) –1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

Solution:

(i) $P(x) = x^3 - 28x + 48$ **(A.B)**

$= x^3 + 0x^2 - 28x + 48$

	1	0	–28	48
2	↓	2	4	–48
	1	2	–24	$\lfloor 0$

$\therefore$ Depressed equation is:

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x + 6) - 4(x + 6) = 0$

$(x+6)(x-4)=0$

Either

$x+6=0$ or $x-4=0$

$x=-6$ $x=4$

Thus 2, 4 and –6 are the roots of the given equation.

$\therefore$ Solution Set $=\{2,4,-6\}$

(ii) $P(x) = 2x^3 - 3x^2 - 11x + 6$ **(A.B)**

	2	–3	–11	6
3	↓	4	9	–6
	2	3	–2	0

$\therefore$ Depressed equation is:

$2x^2+3x-2=0$

$2x^2+4x-x-2=0$

$2x(x+2)-1(x+2)=0$

$(x+2)(2x-1)=0$

Either

$x+2=0$ Or $2x-1=0$

$x=-2$ $2x=1$

$x=\frac{1}{2}$

$\therefore$ **Solution Set** $=\left\{3,-2,\frac{1}{2}\right\}$

(iii) $P(x)=4x^3-x^2-11x-6$ **(A.B)**

	4	–1	–11	–6
–1	↓	–4	5	6
	4	–5	–6	0

$\therefore$ Depressed equation is

$4x^2-5x-6=0$

$4x^2-8x+3x-6=0$

$4x(x-2)+3(x-2)=0$

$(x-2)(4x+3)=0$

Either

$x-2=0$ or $4x+3=0$

$x=2$ $4x=-3$

$x=-\frac{3}{4}$

Thus $-1,2,-\frac{3}{4}$ are the roots of the given equation.

$\therefore$ **Solution Set** $=\left\{-1,2,-\frac{3}{4}\right\}$

Q.5 Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation $x^4-10x^2+9=0$

(ii) 3 and –4 are the roots of the equation $x^4+2x^3-13x^2-14x+24=0$

Solution: **(A.B)**

(i) $P(x)=x^4+0x^3-10x^2+0x+9$

1	1	0	–10	0	9
		1	1	–9	–9
3	1	1	–9	–9	0
		3	12	9	
	1	4	3	0	

$\therefore$ Depressed equation is

$x^2+4x+3=0$

$x^2+3x+x+3=0$

$x(x+3)+1(x+3)=0$

$(x+3)(x+1)=0$

Either

$x+3=0$ or $x+1=0$

$x=-3$ $x=-1$

$\therefore$ **Solution Set** $=\{\pm3,\pm1\}$

(ii) $P(x)=x^4+2x^3-13x^2-14x+24$

	1	2	–13	–14	24
3	↓	3	15	6	–24
	1	5	2	–8	0
–4	↓	–4	–4	8	
	1	1	–2	0	

$\therefore$ Depressed equation is:

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$x(x+2)-1(x+2)=0$

$(x+2)(x-1)=0$

Either

$x+2=0$ or $x-1=0$

$x=-2$ $x=1$

Thus –2, –4, 1 and 3 are the roots of the given equation.

$\therefore$ **Solution Set** $= \{-2, -4, 1, 3\}$