Exercise 2.7

Simultaneous Equation **(K.B)**

A system of equations having a common solution is called a system of simultaneous equations.

For example $x+2y=3,\ 2x-y=1$

having same solution (1,1).

Solution Set **(K.B)**

The set of all the ordered pairs (x,y), which satisfies the system of equations is called the solution set of the system.

Ordered Pair **(K.B)**

An ordered pair of real numbers x and y is a pair (x,y) in which elements are written in specific order.

For example: (x,y) is an ordered pair in which first elements (abscissa) is x and second element (ordinate) is y.

Note **(K.B)**

$(x,y)\neq(x,y)$. For example $(2,3)$ and $(3,2)$ are two different ordered pairs.

Example 1 (Page # 39) **(A.B)**

Solve the system of equations

$3x+y=4$ and $3x^2+y^2=52$.

Solution:

The given equations are

$3x+y=4\rightarrow(\text{i})$

$3x^2+y^2=52\rightarrow(\text{ii})$

From equation (i) $y=4-3x\rightarrow(\text{iii})$

Put value of y in equation (ii)

$3x^2+(4-3x)^2=52$

$3x^2+16-24x+9x^2-52=0$

$12x^2-24x-36=0$

$12(x^2-2x-3)=0$

$x^2-2x-3=0 \quad \because 12\neq 0$

By factorization

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

Either

$x-3=0$ or $x+1=0$

$x=3$ or $x=-1$

Put the values of x is equation (iii)

When $x=3$

$y=4-3x$

$y=4-3(3)$

$=4-9$

$y=-5$

when $x=-1$

$y=4-3x$

$y=4-3(-1)$

$=4+3$

$y=7$

$\therefore$ ordered pairs are $(3,-5)$ and $(-1,7)$

Thus, the solution set is $\{(3,-5),(-1,7)\}$

Example 2 (Page # 40) **(A.B)**

Solve the equations

$x^2+y^2+2x=8$ and $(x-1)^2+(y+1)^2=8$

Solution:

The given equation are

$x^2+y^2+2x=8\rightarrow(\text{i})$

$(x-1)^2+(y+1)^2=8\rightarrow(\text{ii})$

From equation (ii), we get

$x^2-2x+1+y^2+2y+1=8$

Or $x^2+y^2-2x+2y=6\rightarrow(iii)$

Subtracting equation (iiii) from equation (i) we have

$4x-2y=2$ or $2x-y=1$

$\Rightarrow \quad y=2x-1$

Put the value of y in equation (ii)

$(x-1)^2+(2x-1+1)^2=8$

$x^2-2x+1+4x^2-8=0$

$5x^2-2x-7=0$

$5x^2-7x+5x-7=0$

$x(5x-7)+1(5x-7)=0$

$(5x-7)(x+1)=0$

Either

$5x-7=0$ or $x+1=0$

$5x=7$ or $x=-1$

$\Rightarrow x=\dfrac{7}{5}$

Now putting the values of x in equation (iv), we have

When $x=\dfrac{7}{5}$ when $x=-1$

$y=2\left(\dfrac{7}{5}\right)-1$ $y=2(-1)-1$

$y=\dfrac{14}{5}-1=\dfrac{14-5}{5}=\dfrac{9}{5}$ $y=-3$

Thus, the solution set is

$\left\{(-1,-3),\left(\dfrac{7}{5},\dfrac{9}{5}\right)\right\}$

Example 3 (Page # 41) **(A.B)**

Solve the equations

$x^2+y^2=7$ and $2x^2+3y^2=18$

Solution:

Given equations are

$x^2+y^2=7$ (i)

$2x^2+3y^2=18$ (ii)

Multiply equation (i) with 3

$3x^2+3y^2=21$ (iii)

Subtracting equations (ii) from (iii)

$x^2=3 \Rightarrow x=\pm\sqrt{3}$

When $x=\sqrt{3}$, then from equation (i)

$x^2+y^2=7$ or

$3+y^2=7 \Rightarrow y^2=4 \Rightarrow y=\pm 2$

When $x=-\sqrt{3}$, then $y=\pm 2$

Thus, the required solution set is

$\{(\pm\sqrt{3},\pm 2)\}$.

Example 4 (Page # 41) **(A.B)**

Solve the equations

$x^2+y^2=20$ (i)

$6x^2+xy-y^2=0$ (ii)

The equation (ii) can be written as

$y^2-xy-6x^2=0$

$$\Rightarrow \quad y=\frac{-(-x)\pm\sqrt{(-x)^2-4\times 1\times(-6x^2)}}{2\times 1}$$

$$=\frac{x\pm\sqrt{x^2+24x^2}}{2}=\frac{x\pm\sqrt{25x^2}}{2}$$

$$=\frac{x\pm 5x}{2}$$

We have $y=\dfrac{x+5x}{2}=\dfrac{6x}{2}=3x$

Or $y=\dfrac{x-5x}{2}=\dfrac{-4x}{2}=-2x$

Substituting $y=3x$ in the equation (i), we get

$x^2+(3x)^2=20$

$x^2+9x^2=20$

$\Rightarrow 10x^2=20$

$\Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{2}$

When $x=\sqrt{2}, y=3(\sqrt{2})=3\sqrt{2}$ and

when $x=-\sqrt{2},\ y=3(-\sqrt{2})=-3\sqrt{2}$

Substituting $y=-2x$ in the equation (i), we have

$x^2+(-2x)^2=20$ or $x^2+4x^2=20$

$\Rightarrow 5x^2=20 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

When $x=2 \Rightarrow y=-2(2)=-4$

when $x=-2 \Rightarrow y=-2(-2)=4$

Thus, the solution is

$\{(\sqrt{2},3\sqrt{2}),(-\sqrt{2},-3\sqrt{2}),(2,-4),(-2,4)\}$.

Example 5 (Page # 42) **(A.B)**

Solve the equations

$x^2+y^2=40$ and $3x^2-2xy-y^2=80$

Solution:

Given equations are

$x^2+y^2=40 \rightarrow (\text{i})$

$3x^2-2xy-y^2=80\rightarrow(\text{ii})$

Multiplying equation (i) by 2, we have

$2x^2+2y^2=80\rightarrow(\text{iii})$

Subtracting the equation (iii) from equation (ii), we get

$x^2-2xy-3y^2=0\rightarrow(\text{iv})$

The equation (iv) can be written as

$x^2-3xy+xy-3y^2=0$

$x(x-3y)+y(x-3y)=0$

$(x-3y)(x+y)=0$

Either $x-3y$ or $x+y=0$

$\Rightarrow x=3y \rightarrow(v)$ or $x=-y\rightarrow(vi)$

Put in equation (i),

Equation (i)$\Rightarrow (3y)^2+y^2=40$

$10y^2=40$

$y^2=4$

$y=\pm2$

$y=2$ $\quad$ $y=-2$

eq.$(v)\Rightarrow$

$x=3(2)$ $\quad$ $x=3(-2)$

$x=6$ $\quad$ $x=-6$

Equation (i)$\Rightarrow (-y)^2+y^2=40$

$2y^2=40$

$y^2=20$

$y=\pm2\sqrt{5}$

$y=2\sqrt{5}$ $\quad$ $y=-2\sqrt{5}$

$eq.(vi)\Rightarrow$

$x=-(2\sqrt{5})$ $\quad$ $x=-(-2\sqrt{5})$

$x=-2\sqrt{5}$ $\quad$ $=2\sqrt{5}$

$\therefore$ The solution set is

$\{(6,2),(-6,-2),(2\sqrt{5},-2\sqrt{5}),(-2\sqrt{5},2\sqrt{5})\}$

Exercise 2.7

Solve the following simultaneous equations.

Q.1 $x+y=5$ **(A.B)**

$x^2-2y-14=0$

Solution:

$x+y=5\rightarrow(\text{i})$

$x^2-2y-14=0\rightarrow(\text{ii})$

From equation (i)

$y=5-x\rightarrow(\text{iii})$

Put in equation (ii)

$x^2-2y-14=0$

$x^2-2(5-x)-14=0$

$x^2-10+2x-14=0$

$x^2+2x-24=0$

$x^2+6x-4x-24=0$

$x(x+6)-4(x+6)=0$

$(x+6)(x-4)=0$

Either

$x+6=0$ or $x-4=0$

$\Rightarrow x=-6$ $\quad$ $x=4$

Put in equation (iii)

$y=5-x$ $\quad$ $y=5-x$

$=5-(-6)$ $\quad$ $=5-4$

$=5+6$ $\quad$ $=1$

$=11$

$\therefore$ **Solution Set** = $\{(-6,11),(4,1)\}$

Q.2 $3x-2y=1$ **(A.B)**

$x^2+xy-y^2=1$

Solution:

$3x-2y=1\rightarrow(\text{i})$

and $x^2+xy-y^2=1\rightarrow(\text{ii})$

From equation (i)

$3x-2y=1$

$3x=2y+1$

$x=\dfrac{2y+1}{3}\rightarrow(\text{iii})$

Now put in equation (ii)

$x^2+xy-y^2=1$

$$\left(\frac{1+2y}{3}\right)^2+\left(\frac{1+2y}{3}\right)y-y^2=1$$

$$\frac{1+4y+4y^2}{9}+\frac{y+2y^2}{3}-y^2=1$$

Multiplying both sides by LCM

We get

$$1+4y+4y^2+3\left(y+2y^2\right)-9y^2=9$$

$$1+4y+4y^2+3y+6y^2-9y^2-9=0$$

$$y^2+7y-8=0$$

$$y^2+8y-y-8=0$$

$$\Rightarrow y(y+8)-1(y+8)=0$$

$$(y+8)(y-1)=0$$

Either

$y+8=0$ or $y-1=0$

$y=-8$ $y=1$

Putting values in equation (iii)

When $y=-8$

$$x=\frac{1+2y}{3}$$

$$x=\frac{1+2(-8)}{3}$$

$$x=\frac{1-16}{3}$$

$$x=\frac{-15}{3}$$

$$x=-5$$

When $y=1$

$$x=\frac{1+2y}{3}$$

$$x=\frac{1+2(1)}{3}$$

$$x=\frac{3}{3}$$

$$x=1$$

$\therefore$ **Solution Set** = $\{(-5,-8),(1,1)\}$

Q.3 $x-y=7$ **(A.B)**

$$\frac{2}{x}-\frac{5}{y}=2$$

Solution:

$$x-y=7\rightarrow(\text{i})$$

$$\frac{2}{x}-\frac{5}{y}=2\rightarrow(\text{ii})$$

From equation (ii)

$$\frac{2}{x}-\frac{5}{y}=2$$

Multiple by xy

$$2y-5x=2xy$$

$$2y-5x-2xy=0$$

$$2y-2xy-5x=0\rightarrow(\text{iii})$$

From equation (i)

$$x=7+y\rightarrow(\text{iv})$$

Put in equation (iii)

$$2y-2(7+y)(y)-5(7+y)=0$$

$$2y-14y-2y^2-5y-35=0$$

$$-2y^2-17y-35=0$$

$$-\left(2y^2+17y+35\right)=0$$

$$\Rightarrow 2y^2+17y+35=0$$

$$2y^2+10y+7y+35=0$$

$$2y(y+5)+7(y+5)=0$$

$$(y+5)(2y+7)=0$$

Either

$y+5=0$ or $2y+7=0$

$y=-5$ $y=\frac{-7}{2}$

Now put in equation (i)

when $y=-5$

$$x-(-5)=7$$

$$x+5=7$$

$$x=7-5$$

$$x=2$$

when

$$y=\frac{-7}{2}$$

$$x-\left(\frac{-7}{2}\right)=7$$
$$x+\frac{7}{2}=7$$
$$x=7-\frac{7}{2}$$
$$x=\frac{14-7}{2}$$
$$x=\frac{7}{2}$$

$\therefore$ **Solution Set** $=\left\{(2,-5),\left(\frac{7}{2},\frac{-7}{2}\right)\right\}$

Q.4 $x+y=a-b$ **(A.B)**

$$\frac{a}{x}-\frac{b}{y}=2$$

Solution:

$$x+y=a-b\rightarrow\text{(i)}$$
$$\frac{a}{x}-\frac{b}{y}=2\rightarrow\text{(ii)}$$

From equation (ii)

$$\frac{a}{x}-\frac{b}{y}=2$$

Multiply by 'xy'

$$ay-bx=2xy\longrightarrow\text{(iii)}$$

From equation (i)

$$\Rightarrow x+y=a-b$$
$$y=a-b-x\rightarrow\text{(iv)}$$

Put in equation (iii)

$$a(a-b-x)-bx=2x(a-b-x)$$
$$a^2-ab-ax-bx=2ax-2bx-2x^2$$
$$2x^2-ax-2ax-bx+2bx+a^2-ab=0$$
$$2x^2-3ax+bx+a^2-ab=0$$
$$2x^2-(3a-b)x+(a^2-ab)=0$$

Here $A=2, B=-(3a-b), C=a^2-ab$

Using quadratic formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-(3a-b)]\pm\sqrt{[-(3a-b)]^2-4(2)(a^2-ab)}}{2(2)}$$
$$x=\frac{3a-b\pm\sqrt{(3a-b)^2-8(a^2-ab)}}{4}$$
$$=\frac{3a-b\pm\sqrt{9a^2-6ab+b^2-8a^2+8ab}}{4}$$
$$=\frac{3a-b\pm\sqrt{a^2+2ab+b^2}}{4}$$
$$=\frac{3a-b\pm\sqrt{(a+b)^2}}{4}$$
$$=\frac{3a-b\pm(a+b)}{4}$$

Either

$x=\frac{3a-b+a+b}{4}$ or $x=\frac{3a-b-a-b}{4}$

$=\frac{4a}{4}$ | $=\frac{2a-2b}{4}$

$x=a$ | $x=\frac{a-b}{2}$

Put in equation (iv)

$y=a-b-a$ | $y=a-b-\frac{a-b}{2}$

$y=-b$ | $y=\frac{2a-2b-(a-b)}{2}$

$y=\frac{a-b}{2}$

$\therefore$ **Solution Set** $=\left\{(a,-b),\left(\frac{a-b}{2},\frac{a-b}{2}\right)\right\}$

Q.5 $x^2+(y-1)^2=10$ **(A.B)**

$$x^2+y^2+4x=1$$

Solution:

$$x^2+(y-1)^2=10\rightarrow\text{(i)}$$
$$x^2+y^2+4x=1\rightarrow\text{(ii)}$$

Equation (i) $\Rightarrow$

$$x^2+y^2-2y+1=10$$
$$x^2+y^2-2y=9\rightarrow\text{(iii)}$$

Subtract equation (ii) and (iii)

$$x^2+y^2+4x \quad = 1$$
$$\underline{-x^2 \pm y^2 \quad \mp 2y = -9}$$
$$4x+2y=-8$$
$$2(2x+y)=-8$$
$$2x+y=-4$$
$$y=-4-2x \rightarrow \text{(iv)}$$

Put in equation (ii)

$$x^2+(-4-2x)^2+4x=1$$
$$x^2+[-(4+2x)]^2+4x=1$$
$$x^2+(4+2x)^2+4x=1$$
$$x^2+16+16x+4x^2+4x-1=0$$
$$5x^2+2x+15=0$$
$$5(x^2+4x+3)=0$$
$$\Rightarrow x^2+4x+3=0 \quad \because 5\neq 0$$
$$x^2+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+3)(x+1)=0$$

Either

$x+3=0$ or $x+1=0$

$x=-3$ $\quad x=-1$

Put in equation (iv)

y = – 4 – 2 (–3) $\quad$ y = – 4 – 2 (–1)

y = – 4 + 6 $\quad$ y = – 4 + 2

y = 2 $\quad$ y = –2

$\therefore$ **Solution Set**= $\{(-3,\ 2),(-1,\ -2)\}$

Q.6 $(x+1)^2+(y+1)^2=5$ **(A.B)**

$(x+2)^2+y^2-5$

Solution:

$$(x+1)^2+(y+1)^2=5 \rightarrow (i)$$
$$(x+2)^2+y^2-5 \rightarrow \text{(ii)}$$

From equation $\text{(i)} \Rightarrow$

$$x^2+2x+1+y^2+2y+1=5$$
$$x^2+y^2+2x+2y=3$$
$$x^2+y^2+2x+2y=3 \rightarrow \text{(iii)}$$
$$(x+2)^2+y^2=5$$
$$x^2+4x+4+y^2=5$$
$$x^2+y^2+4x=1 \rightarrow \text{(iv)}$$

Sub equation (iii) and (iv)

$$x^2+y^2+2x+2y=-3$$
$$\underline{-x^2 \pm y^2 \pm 4x \quad = -1}$$
$$-2x+2y=2$$
$$-x+y=1$$
$$y=x+1 \rightarrow \text{(v)}$$

Put in equation (iv)

$$x^2+(x+1)^2+4x=1$$
$$x^2+x^2+2x+1+4x-1$$
$$2x^2+6x=0$$
$$2x(x+3)=0$$

Either

$2x=0$ or $x+3=0$

$x=0$ $\quad x=-3$

Put in equation (v)

When $x=0$

y = 1 + 0

y = 1

When $x=-3$

$y=1-3$

$y=-2$

$\therefore$ **Solution Set** $=\{(0,1),(-3,-2)\}$

Q.7 $x^2+2y^2=22$ **(A.B)**

$5x^2+y^2=29$

Solution:

$$x^2+2y^2=22 \rightarrow \text{(i)}$$
$$5x^2+y^2=29 \rightarrow \text{(ii)}$$

Multiply equation (ii) by '2'

$$10x^2+2y^2=58 \longrightarrow \text{(iii)}$$

Subtract equation (i) and (iii)

$$10x^2+2y^2=58$$
$$\underline{-x^2 \pm 2y^2 = -22}$$
$$9x^2 \quad =36$$
$$x^2=4 \quad \text{(Div both sides by 9)}$$

Taking square root on both sides

$$x=\pm 2$$

Put $x^2 = 4$ in equation (ii)

$5(4) + y^2 = 29$

$20 + y^2 = 29$

$y^2 = 29 - 20$

$y^2 = 9$

Taking square root

$y = \pm 3$

$\therefore$ **Solution Set =** $\{(\pm 2, \pm 3)\}$

Q.8 $4x^2 - 5y^2 = 6$ **(A.B)**

$3x^2 + y^2 = 14$

Solution:

$4x^2 - 5y^2 = 6 \rightarrow (\text{i})$

$3x^2 + y^2 = 14 \rightarrow (\text{ii})$

Multiply equation (ii) by 5

$15x^2 + 5y^2 = 70 \rightarrow (\text{iii})$

Adding equation (i) and (iii)

$4x^2 - 5y^2 = 6$

$\underline{15x^2 + 5y^2 = 70}$

$19x^2 = 76$

$x^2 = \frac{76}{19}$

$x^2 = 4$

Taking square root

$x = \pm 2$

Put $x^2 = 4$ in equation (ii)

$3(4) + y^2 = 14$

$y^2 = 14 - 12$

$y^2 = 2$

Taking square root

$y = \pm\sqrt{2}$

$\therefore$ **Solution Set =** $\{(\pm 2, \pm\sqrt{2})\}$

Q.9 $7x^2 - 3y^2 = 4$ **(A.B)**

$2x^2 + 5y^2 = 7$

Solution:

$7x^2 - 3y^2 = 4 \rightarrow (\text{i})$

$2x^2 + 5y^2 = 7 \rightarrow (\text{ii})$

Multiply equation (i) by '5'

$35x^2 - 15y^2 = 20 \rightarrow (\text{iii})$

Multiply equation (ii) by '3'

$6x^2 + 15y^2 = 21 \rightarrow (\text{iv})$

Add equation (iii) and (iv)

$35x^2 - 15y^2 = 20$

$\underline{6x^2 + 15y^2 = 21}$

$41x^2 = 41$

$x^2 = 1$

On taking square root, we get

$x = \pm 1$

Put $x^2 = 1$ in equation (ii)

$2x^2 + 5y^2 = 7$

$2(1) + 5y^2 = 7$

$2 + 5y^2 = 7$

$5y^2 = 5$

$y^2 = \frac{5}{5}$

$y^2 = 1$

On taking the square root, we get

$y = \pm 1$

$\therefore$ **Solution Set** $= \{(\pm 1, \pm 1)\}$

Q.10 $3x^2 - y^2 = 3$ **(A.B)**

$x^2 + 4xy - 5y^2 = 0$

Solution:

$3x^2 - y^2 = 3 \rightarrow (\text{i})$

$x^2 + 4xy - 5y^2 = 0 \rightarrow (\text{ii})$

Equation (ii) $\Rightarrow$

$x^2 + 4xy - 5y^2 = 0$

$x^2 + 5xy - xy - 5y^2 = 0$

$x(x + 5y) - y(x + 5y) = 0$

$(x + 5y)(x - y) = 0$

Either

$x + 5y = 0$ or $x - y = 0$

$x = -5y \rightarrow (\text{iii})$ $\quad x = y \rightarrow (\text{iv})$

Put $x = -5y$ in equation (i)

$x^2 + 2y^2 = 3$

$(-5y)^2 + 2y^2 = 3$

$25y^2 + 2y^2 = 3$

$27y^2 = 3$

$y^2 = \frac{3}{27}$

$y^2 = \frac{1}{9}$

On taking square root, we get

$y = \pm\frac{1}{3}$

Putting in equation (iii)

When

$y = \frac{1}{3}$

$x = -5\left(\frac{1}{3}\right)$

$x = -\frac{5}{3}$

When $y = -\frac{1}{3}$

$x = -5\left(-\frac{1}{3}\right)$

$x = \frac{5}{3}$

Put $x = y$ in equation (i)

$x^2 + 2y^2 = 3$

$y^2 + 2y^2 = 3$

$3y^2 = 3$

$y^2 = 1$

On taking square root, we get

$y = \pm 1$

Put in equation (iv)

When y = 1

$x = 1$

When $y = -1$

$x = -1$

$\therefore$ **Solution Set**

$= \left\{(1,1),(-1,-1),\left(-\frac{5}{3},\frac{1}{3}\right),\left(\frac{5}{3},-\frac{1}{3}\right)\right\}$

Q.11 $3x^2 - y^2 = 26$ **(A.B)**

$3x^2 - 5xy - 12y^2 = 0$

Solution:

$3x^2 - y^2 = 26 \rightarrow \text{(i)}$

$3x^2 - 5xy - 12y^2 = 0 \rightarrow \text{(ii)}$

Equation (ii) $\Rightarrow 3x^2 - 5xy - 12y^2 = 0$

$3x^2 - 9xy + 4xy - 12y^2 = 0$

$3x(x-3y) + 4y(x-3y) = 0$

$(x-3y)(3x+4y) = 0$

Either

$x - 3y = 0 \rightarrow (a)$ or $3x + 4y = 0 \rightarrow (b)$

Equation $(a) \Rightarrow x = 3y \rightarrow \text{(iii)}$

Put in equation (i)

$3(3y)^2 - y^2 = 26$

$3(9y^2) - y^2 = 26$

$27y^2 - y^2 = 26$

$26y^2 = 26$

$y^2 = 1$

$\Rightarrow y = \pm 1$

Put in equation (iii)

When y = – 1

$x = 3(-1)$

$x = -3$

When y = 1

$x = 3(1)$

$x = 3$

Equation $(b) \Rightarrow 3x + 4y = 0$

$3x = -4y$

$x = \frac{-4}{3}y \rightarrow \text{(iv)}$

Put in equation (i)

$3\left(\frac{-4}{3}y\right)^2 - y^2 = 26$

$\left(\frac{16}{3}y^2\right) - y^2 = 26$

$\frac{16y^2 - 3y^2}{3} = 26$

$13y^2 = 26 \times 3$

$y^2 = 6$

$\Rightarrow y = \pm\sqrt{6}$

Put in equation (iv)

When $y = -\sqrt{6}$

$x = \frac{-4}{3}\left(-\sqrt{6}\right)$

$x = \frac{4\sqrt{6}}{3}$

When $y = \sqrt{6}$

$x = \frac{-4}{3}\left(\sqrt{6}\right)$

$x = \frac{-4\sqrt{6}}{3}$

$\therefore$ **Solution Set**

$= \left\{(-3,-1),(3,1),\left(\frac{-4\sqrt{6}}{3},\sqrt{6}\right),\left(\frac{4\sqrt{6}}{3},-\sqrt{6}\right)\right\}$

Q.12 $x^2 + xy = 5$ **(A.B)**

$y^2 + xy = 3$

Solution:

$x^2 + xy = 5 \rightarrow (\text{i})$

$y^2 + xy = 3 \rightarrow (\text{ii})$

Multiply equation (i) by '3' and equation (ii) '5'

$3x^2 + 3xy = 15 \rightarrow (\text{iii})$

$5y^2 + 5xy = 15 \rightarrow (\text{iv})$

Subtraction equation (iii) and (iv)

$3x^2 + 3xy = 15$

$5y^2 + 5xy = 15$

$3x^2 - 2xy - 5y^2 = 0 \rightarrow (\text{v})$

Equation (v)

$3x^2 - 5xy + 3xy - 5y^2 = 0$

$x(3x-5y) + y(3x-5y) = 0$

$(3x-5y)(x+y) = 0$

Either

$3x - 5y = 0$ or $x + y = 0$

$x = \frac{5y}{3} \rightarrow (\text{vi})$ $\quad x = -y \rightarrow (\text{vii})$

Put $x = \frac{5y}{3}$ in equation (i)

$\left(\frac{5y}{3}\right)^2 + \left(\frac{5y}{3}\right)y = 5$

$\frac{25}{9}y^2 + \frac{5y^2}{3} = 5$

$\frac{25y^2 + 15y^2}{9} = 5$

$25y^2 + 15y^2 = 45$

$40y^2 = 45$

$y^2 = \frac{9}{8}$

On taking square root, we get.

$y = \pm\frac{3}{2\sqrt{2}}$

Put $y = \frac{3}{2\sqrt{2}}$ in equation (vi)

$x = \frac{5}{3} \times \frac{3}{2\sqrt{2}}$

$x = \frac{5}{2\sqrt{2}}$

Now put $y = \frac{-3}{2\sqrt{2}}$ in equation (vi)

$x = \frac{5}{3}\left(\frac{-3}{2\sqrt{2}}\right)$

$x = \frac{-5}{2\sqrt{2}}$

Now put $x = -y$ in equation (i)

$(-y)^2 + (-y)(y) = 5$

$y^2 - y^2 = 5$

$0 = 5$

But $0 \neq 5$

$\therefore$ **Solution Set** $= \left\{\left(\frac{5}{2\sqrt{2}},\frac{3}{2\sqrt{2}}\right),\left(\frac{-5}{2\sqrt{2}},\frac{-3}{2\sqrt{2}}\right)\right\}$

Alternate Method

$x^2 + xy = 5 \rightarrow (\text{i})$

$y^2 + xy = 3 \rightarrow (\text{ii})$

$eq(i) \Rightarrow$

$x(x+y) = 5 \rightarrow (iii)$

$eq(ii) \Rightarrow$

$y(x+y) = 3 \rightarrow (iv)$

Dividing equation (*iii*) and (*iv*)

$$\frac{x(x+y)}{y(x+y)}=\frac{5}{3}$$

$$\frac{x}{y}=\frac{5}{3}$$

$$x=\frac{5y}{3}$$

Put $x=\frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3}\right)^2+\left(\frac{5y}{3}\right)y=5$$

$$\frac{25}{9}y^2+\frac{5y^2}{3}=5$$

$$\frac{25y^2+15y^2}{9}=5$$

$$25y^2+15y^2=45$$

$$40y^2=45$$

$$y^2=\frac{9}{8}$$

On taking square root, we get.

$$y=\pm\frac{3}{2\sqrt{2}}$$

Put $y=\frac{3}{2\sqrt{2}}$ in equation (vi)

$$x=\frac{5}{3}\times\frac{3}{2\sqrt{2}}$$

$$x=\frac{5}{2\sqrt{2}}$$

Now put $y=\frac{-3}{2\sqrt{2}}$ in equation (vi)

$$x=\frac{5}{3}\left(\frac{-3}{2\sqrt{2}}\right)$$

$$x=\frac{-5}{2\sqrt{2}}$$

$\therefore$ **Solution Set** $=\left\{\left(\frac{5}{2\sqrt{2}},\frac{3}{2\sqrt{2}}\right),\left(\frac{-5}{2\sqrt{2}},\frac{-3}{2\sqrt{2}}\right)\right\}$

Q.13 $x^2-2xy=7$ **(A.B)**

$xy+3y^2=2$

Solution:

$x^2-2xy=7\rightarrow$ (i)

$xy+3y^2=2\rightarrow$ (ii)

Multiply equation (i) by '2' and equation (ii) by '7'

$2x^2-4xy=14\rightarrow$ (iii)

$7xy+21y^2=14\rightarrow$ (iv)

Subtract equation (iii) and (iv)

$$\begin{array}{l}2x^2-4xy=14\\ \underline{-7xy\pm21y^2=-14}\\ 2x^2-11xy-21y^2=0\end{array}$$

$$2x^2-14xy+3xy-21y^2=0$$

$$2x(x-7y)+3y(x-7y)=0$$

$$(x-7y)(2x+3y)=0$$

Either $x-7y=0$ or $2x+3y=0$

$x=7y\longrightarrow$ (v)

Put in equation (ii)

$$(7y)y+3y^2=2$$

$$7y^2+3y^2=2$$

$$10y^2=2$$

$$5y^2=1$$

$$y^2=\frac{1}{5}$$

$$y=\pm\frac{1}{\sqrt{5}}$$

Put in equation (v)

when $y=-\frac{1}{\sqrt{5}}$

$$x=7\left(-\frac{1}{\sqrt{5}}\right)$$

$x=\frac{-3}{2}y\longrightarrow$ (vi)

Put in equation (ii)

$$\left(\frac{-3}{2}y\right)y+3y^2=2$$

$$\frac{-3y^2+6y^2}{2}=2$$

$$3y^2=4$$

$$y^2=\frac{4}{3}$$

$$y=\pm\frac{2}{\sqrt{3}}$$

Put in equation (vi)

when $y=\frac{-2}{\sqrt{3}}$

$$x=\frac{-3}{2}\left(-\frac{2}{\sqrt{3}}\right)$$

$$x=\frac{3}{\sqrt{3}}$$

$x=\frac{-7}{\sqrt{5}}$	$x=\sqrt{3}$
when	
$y=\frac{1}{\sqrt{5}}$	when $y=\frac{2}{\sqrt{3}}$
$x=7\left(\frac{1}{\sqrt{5}}\right)$	$x=\frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$
$x=\frac{7}{\sqrt{5}}$	$x=-\sqrt{3}$

$\therefore$ **Solution Set**

$$=\left\{\left(\frac{7}{\sqrt{5}},\frac{1}{\sqrt{5}}\right),\left(\frac{-7}{\sqrt{5}},\frac{1}{\sqrt{5}}\right),\left(-\sqrt{3},\frac{2}{\sqrt{3}}\right),\left(\sqrt{3},\frac{-2}{3}\right)\right\}$$