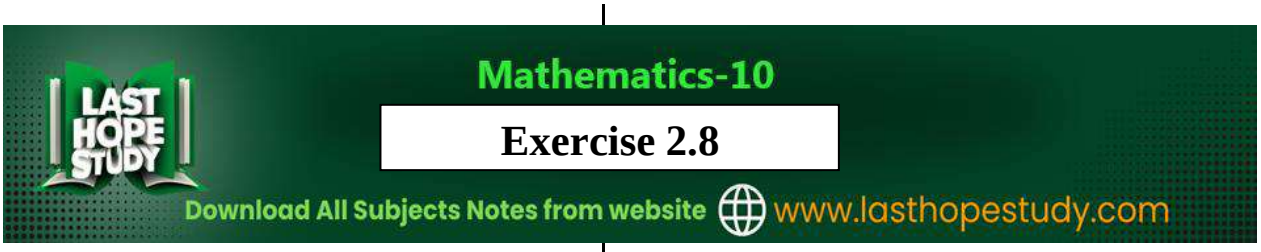

Problems Leading to Quadratic Equations (A.B + K.B + U.B)

Example 1: (Page # 43)

Three less than a certain number multiplied by 9 less than twice the number is 104. Find the number.

Solution:

Let the required number $= x$

Then, three less than the number $= x-3$

And, 9 less than twice the number $= 2x-9$

According to given condition

$(x-3)(2x-9)=104$

$2x^2-9x-6x+27-104=0$

$2x^2-15x-77=0$

$2x^2-22x+7x-77=0$

$2x(x-11)+7(x-11)=0$

$(x-11)(2x+7)=0$

Either

$x-11=0$ or $2x+7=0$

$\Rightarrow x=11$ or $x=-\frac{7}{2}$

Result:

Thus, required number is either 11 or $-\frac{7}{2}$

Example 2: (Page # 44) **(A.B)**

The length of rectangle is 4cm more than its breadth. If the area of rectangle is $45cm^2$. Find its sides.

Solution:

Let breadth of rectangle $= x$

Then, length of rectangle $= x+4$

Area of rectangle $= 45cm^2$

According to given condition

$x(x+4)=45$

$x^2+4x-45=0$

$x^2+9x-5x-45=0$

$x(x+9)-5(x+9)=0$

$(x+9)(x-5)=0$

Either

$x+9=0$ or $x-5=0$

$\Rightarrow x=-9$ or $x=5$

(Neglecting –ve value)

$\therefore\ x+4=5+4=9$

Result:

Thus, the breadth is 5cm and length is 9cm

Exercise 2.8

Q.1 The product of two positive consecutive numbers is 182. Find the numbers. **(A.B)**

Solution:

Let two positive consecutive numbers are $x, x+1$

According to given condition:

$x(x+1)=182$

$x^2+x-182=0$

$x^2+14x-13x-182=0$

$x(x+14)-13(x+14)=0$

$(x+14)(x-13)=0$

Either

$x+14=0$ or $x-13=0$

$x=-14$ $x=13$

(Ignore negative value)

Therefore,

$x=13$

$\Rightarrow$ $x+1=13+1=14$

Result:

Thus, required Numbers are 13 and 14.

Q.2 The sum of squares of three positive consecutive numbers is 77. Find the numbers. (A.B + K.B)
(SWL 2015)

Solution:

Let three consecutive numbers are
$x, x+1, x+2$
According to given condition:
$x^2+(x+1)^2+(x+2)^2=77$
$x^2+x^2+2x+1+x^2+4x+4-77=0$
$3x^2+6x-72=0$
$3(x^2+2x-24)=0$
$x^2+2x-24=0$
$x^2+6x-4x-24=0$
$x(x+6)-4(x-6)=0$
$(x+6)(x-4)=0$
Either
$x+6=0$ or $x-4=0$
$x=-6$ $x=4$
(Ignore negative value)
Therefore, $x=4$
$\Rightarrow$ $x+1=4+1=5$
& $x+2=4+2=6$

Result:
Thus required numbers are 4, 5 and 6.

Q.3 The sum of five times a number and the square of the numbers is 204. (A.B + K.B)

Solution:

Let required number = x
Five times of the number $=5x$
According to given condition:
$x^2+5x=204$
$x^2+5x-204=0$
$x^2+17x-12x-204=0$
$x(x+17)-12(x+17)=0$
$(x+17)(x-12)=0$
Either
$x+17=0$ or $x-12=0$
$x=-17$ $x=12$

Result:
Thus required number is either –17 or 12.

Q.4 The product of five less than three times a certain number and one less than four times the number is7. Find the number. (A.B + K.B)

Solution:

Let the required number is x
Five less than three times the number $=3x-5$
One less than four times a number $=4x-1$
According to given condition
$(3x-5)(4x-1)=7$
$12x^2-3x-20x+5-7=0$
$12x^2-23x-2=0$
$12x^2-24x+x-2=0$
$12x(x-2)+1(x-2)=0$
$12x(x-2)+1(x-2)=0$
$(x-2)(12x+1)=0$
Either
$x-2=0$ or $12x+1=0$
$x=2$ $12x=-1$
$x=\frac{-1}{12}$

Result:
Thus, required number is either 2 or $-\frac{1}{12}$.

Q.5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number. (A.B + K.B)

Solution:

Let, required number is x
Reciprocal of the number $=\frac{1}{x}$
Difference of the numbers $=\frac{15}{4}$
According to given condition
$x-\frac{1}{x}=\frac{15}{4}$
$\frac{x^2-1}{x}=\frac{15}{4}$
By cross multiplication

$4x^2 - 4 = 15x$

$4x^2 - 15x - 4 = 0$

$4x^2 - 16x + x - 4 = 0$

$4x(x-4) + 1(x-4) = 0$

$(x-4)(4x+1) = 0$

Either

$x - 4 = 0$ or $4x + 1 = 0$

$x = 4$ $4x = -1$

$x = \frac{-1}{4}$

Result:

Thus, required number is either 4 or $\frac{-1}{4}$

Q.6 The sum of a number of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number. **(A.B + K.B)**

Solution:

Let unit's digit = x

And ten's digit = y

$\therefore$ Required number $= 10y + x$

According to given condition (I)

$x^2 + y^2 = 65 \longrightarrow$ (i)

According to condition (II)

$10y + x = 9(x + y)$

$10y + x = 9x + 9y$

$10y - 9y = 9x - x$

$y = 8x \longrightarrow$ (ii)

Put in equation (i)

$x^2 + (8x)^2 = 65$

$x^2 + 64x^2 = 65$

$65x^2 = 65$

$x^2 = 1$

Taking positive square root

$x = 1$

Put in equation (ii)

$y = 8(1)$

$y = 8$

Required number $= 10y + x$

$= 10(8) + 1$

$= 80 + 1$

$= 81$

Result:

Thus, required number is 81

Q.7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point. **(A.B + K.B)**

Solution:

Let required point is (x, y)

According to condition I

$x + y = 9 \longrightarrow$ (i)

According to condition II

$x^2 + y^2 = 45 \longrightarrow$ (ii)

Equation (i) $\Rightarrow$

$x = 9 - y \longrightarrow$ (iii)

Put in equation (ii)

$(9 - y)^2 + y^2 = 45$

$81 - 18y + y^2 + y^2 = 45$

$2y^2 - 18y + 81 - 45 = 0$

$2y^2 - 18y + 36 = 0$

$2(y^2 - 9y + 18) = 0$

$y^2 - 9y + 18 = 0$

$y^2 - 6y - 3y + 18 = 0$

$y(y-6) - 3(y-6) = 0$

$(y-6)(y-3) = 0$

Either

$y - 6 = 0$ or $y - 3 = 0$

$y = 6$ $y = 3$

Put in equation (iii)

$x = 9 - 6$ $x = 9 - (3)$

$x = 3$ $x = 9 - 3$

$x = 6$

Result:

Thus, required point is either

(3,6) or (6,3)

Q.8 Find two integers whose sum is 9 and the difference of their squares is also 9. (A.B + K.B)

Solution:

Let two integer are x and y

According to condition I

$x + y = 9 \longrightarrow (i)$

According to condition (ii)

$x^2 - y^2 = 9 \longrightarrow (ii)$

Equation (i) $\Rightarrow y = 9 - x \longrightarrow (iii)$

Put in equation (ii)

$x^2 - (9 - x)^2 = 9$

$x^2 - (81 - 18x + x^2) = 9$

$x^2 - 81 + 18x - x^2 = 9$

$18x = 9 + 81$

$\Rightarrow \quad x = 5$

Put in equation (iii)

$y = 9 - 5 = 4$

Result:

Thus, required numbers are 5 and 4.

Q.9 Find two integers whose difference is 4 and whose squares differ by 72. (A.B + K.B)

Solution:

Let the integers are x and y

According to condition-I

$x - y = 4 \longrightarrow (i)$

According to condition-II

$x^2 - y^2 = 72 \longrightarrow (ii)$

Equation (i) $\Rightarrow x = y + 4 \longrightarrow (iii)$

Put in equation (ii)

$(y + 4)^2 - y^2 = 72$

$y^2 + 8y + 16 - y^2 = 72$

$8y = 72 - 16$

$8y = 56$

$\Rightarrow y = 7$

Put in equation (iii)

$x = 7 + 4$

$x = 11$

Result:

Thus, required integers are 11 and 7.

Q.10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is $375cm^2$ (K.B + A.B)

Solution:

Let length of rectangle = x cm

And width of rectangle = y cm

According to condition-I

$2(x + y) = 80$

$\because$ perimeter=2(length+ width)

$x + y = 40 \longrightarrow (i)$

$\because$ Area = length × width

According to condition-II

$xy = 375 \longrightarrow (ii)$

From equation (i)

$y = 40 - x \longrightarrow (iii)$

Put in (ii)

$x(40 - x) = 375$

$40x - x^2 - 375 = 0$

$-x^2 - 40 - 375 = 0$

$x^2 - 40x + 375 = 0$

$x^2 - 25x - 15x + 375 = 0$

$x(x - 25) - 15(x - 25) = 0$

$(x - 25)(x - 15) = 0$

Either $x - 25 = 0$ or $x - 15 = 0$

$x = 25$ $\qquad$ $x = 15$

Put in equation (iii)

$y = 40 - 25$ $\qquad$ $y = 40 - 15$

$y = 15$ $\qquad$ $y = 25$

Result:

Dimension of rectangle are either 25cm by 15cm or 15cm by 25cm.