Miscellaneous Exercise 2

Q.1 Multiple Choice Questions

Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) **If α, β are the roots of $3x^2+5x-2=0$, then $\alpha+\beta$ is;** **(K.B + A.B)**

(LHR 2017, SWL 2014, MTN 2015, 17, SGD 2015, 17, RWP 2016, D.G.K 2016)

(a) $\frac{5}{3}$ **(b)** $\frac{3}{5}$

(c) $\frac{-5}{3}$ **(d)** $\frac{-2}{3}$

(ii) **If α, β are the roots of $7x^2-x+4=0$, then $\alpha\beta$ is;** **(K.B + U.B)**

(LHR 2014, GRW 2014, 15, FSD 2016, BWP 2016)

(a) $\frac{-1}{7}$ **(b)** $\frac{4}{7}$

(c) $\frac{7}{4}$ **(d)** $\frac{-4}{7}$

(iii) **Roots of the equation $4x^2-5x+2=0$ are;** **(K.B + A.B)**

(a) Irrational **(b)** Imaginary

(c) Rational **(d)** None of these

(iv) **Cube roots of -1 are;** **(K.B + U.B)**

(LHR 2017, GRW 2017, SWL 2017, MTN 2014, 17, SGD 2015, 16, D.G.K 2017)

(a) $-1, -\omega, -\omega^2$ **(b)** $-1, \omega, -\omega^2$

(c) $-1, -\omega, \omega^2$ **(d)** $1, -\omega, -\omega^2$

(v) **Sum of the cube roots of unity is;** **(K.B + A.B)**

(a) 0 **(b)** 1

(c) -1 **(d)** 3

(vi) **Product of cube roots of unity is;** **(K.B + A.B)**

(LHR 2016, GRW 2014, 16, SGD 2015, 17, BWP 2016, 17, RWP 2017)

(a) 0 **(b)** 1

(c) -1 **(d)** 3

(vii) **If $b^2-4ac<0$, then the roots of $ax^2+bx+c=0$ are;** **(GRW 2014)** **(K.B + A.B)**

(a) Irrational **(b)** Rational

(c) Imaginary **(d)** None of these

(viii) **If $b^2-4ac>0$, but not a perfect square then roots of $ax^2+bx+c=0$ are;** **(K.B)**

(LHR 2014, BWP 2017)

(a) Imaginary **(b)** Rational

(c) Irrational **(d)** None of these

(ix) $\frac{1}{\alpha}+\frac{1}{\beta}$ **is equal to;** **(K.B + U.B)**

(LHR 2014, 15, GRW 2016, FSD 2017, BWP 2017, RWP 2016, SGD 2017)

(a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha}-\frac{1}{\beta}$
(c) $\frac{\alpha-\beta}{\alpha\beta}$ **(d)** $\frac{\alpha+\beta}{\alpha\beta}$

(x) $\alpha^2+\beta^2$ **is equal to;** **(U.B + A.B)**
(LHR 2014, 15, GRW 2014, 17, FSD 2016, BWP 2015, RWP 2016, 17)
(a) $\alpha^2-\beta^2$ (b) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$
(c) $(\alpha+\beta)^2-2\alpha\beta$ (d) $\alpha+\beta$

(xi) **Two square roots of unity are;** **(U.B + A.B)**
(LHR 2015, 16, GRW 2014, FSD 2015, 16, MTN 2016, SGD 2016, D.G.K 2015, 16, 17)
(a) $1,-1$ (b) $1,\omega$
(c) $1,-\omega$ (d) ω,ω^2

(xii) **Roots of the equation** $4x^2-4x+1=0$ **are;** **(U.B + A.B)**
(LHR 2015, GRW 2017, FSD 2016, BWP 2015, MTN 2017, SGD 2016)
(a) Real equal (b) Real unequal
(c) Imaginary (d) Irrational

(xiii) **If** α,β **are the roots of** $px^2+qx+r=0$**, then sum of the roots** 2α **and** 2β **is;** **(K.B)**
(a) $\frac{-q}{p}$ (b) $\frac{r}{p}$
(c) $\frac{-2q}{p}$ (d) $-\frac{q}{2p}$

(xiv) **If** α,β **are the roots of** $x^2-x-1=0$**, then product of the roots** 2α **and** 2β **is;** **(U.B)**
(FSD 2014, 17, BWP 2016, D.G.K 2015, 16, 17)
(a) -2 (b) 2
(c) 4 **(d)** -4

(xv) **The nature of the roots of equation** $ax^2+bx+c=0$ **is determined by;** **(A.B)**
(GRW 2016, SWL 2015, 2017, MTN 2015)
(a) Sum of the roots (b) Product of the roots
(c) Synthetic division **(d)** Discriminant

(xvi) **The discriminant of** $ax^2+bx+c=0$ **is;** **(K.B + A.B)**
(LHR 2016, FSD 2017, SWL 2016, 17, RWP 2014, 16, SGD 2016, MTN 2015, D.G.K 2016)
(a) b^2-4ac (b) b^2+4ac
(c) $-b^2+4ac$ (d) $-b^2-4ac$

ANSWER KEY

(i)	c	(v)	a	(ix)	D	(xiii)	c
(ii)	b	(vi)	b	(x)	C	(xiv)	d
(iii)	b	(vii)	c	(xi)	A	(xv)	d
(iv)	a	(viii)	c	(xii)	A	(xvi)	a

Q.2

(i) Discuss the nature of the roots of the following equations. (A.B)

Solution:

(a) $x^2+3x+5=0$

Here $a=1, b=3, c=5$

Disc $= b^2 - 4ac$
$= (3)^2 - 4(1)(5)$
$= 9 - 20$
$= -11$
< 0

$\therefore$ Roots are complex conjugate or imaginary.

(b) $2x^2-7x+3=0$ **(A.B)**

(GRW 2016, SGD 2014, RWP 2017, D.G.K 2016)

Here $a=2, b=-7, c=3$

Disc $= b^2 - 4ac$
$= (-7)^2 - 4(2)(3)$
$= 49 - 24$
$= 25$

Since disc >0 and perfect square roots are rational and unequal.

(c) $x^2+6x-1=0$ **(A.B)**

Here $a=1, b=6, c=-1$

Disc $= b^2 - 4ac$
$= (6)^2 - 4(1)(-1)$
$= 36 + 4$
$= 40$

Since Disc. >0 and not a perfect square roots are irrational and unequal.

(d) $16x^2-8x+1=0$ **(FSD 2017)** **(A.B)**

Here $a=16, b=-8, c=1$

Disc. $= b^2-4ac$
$= (-8)^2-4(16)(1)$
$= 64 - 64$
$= 0$

Since, Disc. = 0, roots are rational and equal.

(ii) Find ω^2, if $\omega=\dfrac{-1+\sqrt{-3}}{2}$ (A.B)

(GRW 2017, SGD 2014, BWP 2016)

Solution:

Here

$$\omega=\frac{-1+\sqrt{-3}}{2}$$

Square both sides

$$(\omega)^2=\left(\frac{-1+\sqrt{-3}}{2}\right)^2$$

$$\omega^2=\frac{(-1)^2+\left(\sqrt{-3}\right)^2+2(-1)\left(\sqrt{-3}\right)}{(2)^2}$$

$$\omega^2=\frac{1+(-3)-2\sqrt{-3}}{4}$$

$$\omega^2=\frac{-2-2\sqrt{-3}}{4}$$

$$\omega^2=2\left(\frac{-1-\sqrt{-3}}{4}\right)$$

$$\omega^2=\frac{-1-\sqrt{-3}}{2}$$

(iii) Prove that the sum of all cube roots of unity is zero. (A.B + K.B)

Ans. $1+\omega+\omega^2$

(By putting values)

$$=1+\frac{-1+\sqrt{-3}}{2}+\frac{-1-\sqrt{-3}}{2}$$

$$=\frac{2+(-1)+\sqrt{-3}-1-\sqrt{-3}}{2}$$

$$=\frac{2-1+\sqrt{-3}-1-\sqrt{-3}}{2}$$

$$=\frac{0}{2}$$

$$= 0$$

(iv) Find the product of complex cube roots of unity. (A.B + K.B)

Ans. $$=(1)\times\left(\frac{-1+\sqrt{-3}}{2}\right)\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$=\frac{(-1)^2-\left(\sqrt{-3}\right)^2}{4}$$

$$=\frac{1-(-3)}{4}$$

$$=\frac{1+3}{4}$$

$$=\frac{4}{4}$$

$$= 1$$

(v) Show that: $x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$ **(A.B + K.B)**

Ans. $x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$

(SGD 2015, BWP 2016)

Proof:

R.H.S

$= (x+y)(x+\omega y)(x+\omega^2 y)$

$= (x+y)(x^2+\omega^2 xy+\omega xy+\omega^3 y^2)$

$= (x+y)\left[x^2+(\omega^2+\omega)xy+(1)y^2\right] \because \omega^3=1$

$= (x+y)\left[x^2+(-1)xy+y^2\right] \because 1+\omega+\omega^2=0$

$= (x+y)\left[x^2-xy+y^2\right]$

$= x^3+y^3$

= L.H.S

Proved

(vi) Evaluate: $\omega^{37}+\omega^{38}+1$ **(A.B + K.B)**

Solution:

$\omega^{37}+\omega^{38}+1$

$= \omega^{36}.\omega+\omega^{36}.\omega^2+1$

$= (\omega^3)^{12}.\omega+(\omega^3)^{12}.\omega^2+1$

$= (1)^{12}.\omega+(1)^{12}.\omega^2+1$

$= \omega + \omega^2 + 1$

$= 0$

$\therefore \omega^{37}+\omega^{38}+1=0$

(vii) Evaluate $(1-\omega+\omega^2)^6$ **(A.B + K.B)**

Solution:

$(1-\omega+\omega^2)^6$

$= [1+\omega^2-\omega]^6$

$= (-\omega-\omega)^6$

$= (-2\omega)^6$

$= (-2)^6\omega^6$

$= 64(\omega^3)^2$

$= 64(1)^2$

$= 64(1)$

$= 64$

(viii) If ω is cube root of unity, form an equation whose roots are 3ω and $3\omega^2$. **(A.B + K.B)**

Solution:

Roots of required equation are 3ω and $3\omega^2$.

Sum of roots = S = $3\omega + 3\omega^2$

$= 3(\omega+\omega^2)$

$= 3(-1)$

$= -3$

Product of roots = P = $(3\omega)(3\omega^2)$

$= 9\omega^3$

$= 9(1)$

$= 9$

$\therefore$ Required quadratic equation is

$x^2-Sx+P=0$

$x^2-(-3)x+9=0$

$x^2+3x+9=0$

(ix) Use synthetic division, find the remainder and quotient when $(x^3+3x^2+2)\div(x-2)$. **(A.B)**

Solution:

$P(x)=x^3+3x^2+2$

$=x^3+3x^2+0x+2$

	1	3	0	2
2	↓	2	10	20
	1	5	10	22

$\therefore$ Remainder = 22

$Q(x) = x^2+5x+10$

(x) Use synthetic division, show that $x-2$ is the factor x^3+x^2-7x+2.

Solution: **(A.B + K.B)**

$P(x)=x^3+x^2-7x+2$

	1	1	−7	2
2	↓	2	6	−2
	1	3	−1	0

Since remainder is zero, $x-2$ is a factor of given polynomial.

(xi) Find the sum and product of the roots of the equation $2Px^2+3qx-4r=0$ **(A.B + K.B)**

Solution:

$2Px^2+3qx-4r=0$

Here $a=2P, b=3q, c=-4r$

Sum of roots $= \frac{-b}{a}$

$= -\frac{3q}{2P}$

Product of roots $= \frac{c}{a}$

$= \frac{-4r}{2P}$

$= -\frac{2r}{P}$

(xii) Find $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ when α, β are of the roots of the equation
$x^2-4x+3=0$. **(A.B + K.B)**

Solution:

$x^2-4x+3=0$

Here $a=1, b=-4, c=3$

Let roots of given equation are α, β

Then sum of roots = $\alpha+\beta = -\frac{b}{a}$

$= -\frac{-4}{1}$

$= 4$

Product of roots = $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

$\alpha\beta = 3$

Consider

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$$

$$= \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(4)^2-2(3)}{(3)^2}$$

$$= \frac{16-6}{9}$$

$$\Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{10}{9}$$

(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$, find (A.B)

(a) $\alpha^2 + \beta^2$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(c) $\alpha - \beta$

Solution:

$4x^2 - 3x + 6 = 0$

Roots of given equation are α, β

$\alpha + \beta = -\frac{-3}{4}$

$= \frac{3}{4}$

$\alpha\beta = \frac{6}{4}$

$= \frac{3}{2}$

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **(A.B)**

$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)$

$= \frac{9}{16} - 3$

$= \frac{9 - 48}{16}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{-39}{16}$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ **(A.B)**

$= \frac{-39/16}{3/2}$

$= \frac{-39}{16} \times \frac{2}{3}$

$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{-13}{8}$

(c) $\alpha - \beta = \sqrt{(\alpha - \beta)^2}$ **(A.B)**

$= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)}$

$= \sqrt{\frac{9}{16} - 6}$

$= \sqrt{\frac{9 - 96}{16}}$

$= \sqrt{-\frac{87}{16}}$

$\Rightarrow \alpha - \beta = \frac{\sqrt{-87}}{4}$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are

(a) $-\alpha, -\beta$

(b) $2\alpha, 2\beta$

Solution:

$x^2 - 5x + 7 = 0$

Roots of given equation are α, β

$\alpha + \beta = -\frac{-5}{1}$

$= 5$

$\alpha\beta = \frac{7}{1}$

$\alpha\beta = 7$

(a) Roots of required equation are $-\alpha, -\beta$ (A.B)

Sum of roots = S = $-\alpha + (-\beta)$

$= -(\alpha + \beta)$

$= -5$

Prod. Of roots = P = $(-\alpha)(-\beta)$

$= \alpha\beta$

$= 7$

∴ Required equation is:

$x^2 - Sx + P = 0$

$x^2 - (-5)x + 7 = 0$

$x^2 + 5x + 7 = 0$

(b) Roots of required equation are $2\alpha, 2\beta$ (A.B)

Sum of roots = S = $2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2(5)$

$= 10$

Prod. Of roots = P = $(2\alpha)(2\beta)$

$= 4\alpha\beta$

$= 4(7)$

$= 28$

∴ Required quadratic equation is:

$x^2 - Sx + P = 0$

$x^2 - 10x + 28 = 0$

Q.3 Fill in the blanks

(i) The discriminant of $ax^2+bx+c=0$ is ________. **(K.B)**

(ii) If $b^2-4ac=0$, then roots of $ax^2+bx+c=0$ are ________. **(K.B)**

(iii) If $b^2-4ac>0$, then the roots of $ax^2+bx+c=0$ are ________. **(K.B)**

(iv) If $b^2-4ac<0$, then the root of $ax^2+bx+c=0$ are ________. **(K.B)**

(v) If $b^2-4ac<0$ and perfect square, then the roots of $ax^2+bx+c=0$ are________. **(K.B)**

(vi) If $b^2-4ac<0$ and not a perfect square, then roots of $ax^2+bx+c=0$ are________. **(K.B)**

(vii) If α,β are the roots of $ax^2+bx+c=0$, then sum of the roots is________. **(K.B)**

(viii) If α,β are the roots of $ax^2+bx+c=0$, then product of the roots is________. **(K.B)**

(ix) If α,β are the roots of $7x^2-5x+3=0$, then sum of the roots is________. **(K.B)**

(x) If α,β are the roots of $5x^2+3x-9=0$, then product of the roots is________. **(K.B)**

(xi) For a quadratic equation $ax^2+bx+c=0$, $\frac{1}{\alpha\beta}$ is equal to ________. **(K.B)**

(xii) Cube roots of unity are________. **(K.B)**

(xiii) Under usual notation sum of the roots of unity is________. **(K.B)**

(xiv) If $1,\omega,\omega^2$ are the cube roots of unity, then ω^{-7} is equal to________. **(K.B)**

(xv) If α,β are the roots of the quadratic equation, then the quadratic equation is written as ________.

(xvi) If 2ω and $2\omega^2$ are the roots of an equation, then equation is________.

ANSWER KEY

(i) b^2-4ac
(ii) Equal
(iii) Real
(iv) Imaginary
(v) Rational
(vi) Irrational (real)
(vii) $-\frac{b}{a}$
(viii) $\frac{c}{a}$
(ix) $\frac{5}{7}$
(x) $\frac{-9}{5}$
(xi) $\frac{a}{c}$
(xii) $1,\omega,\omega^2$
(xiii) Zero
(xiv) ω^2
(xv) $x^2-(\alpha+\beta)x+\alpha\beta=0$
(xvi) $x^2+2x+4=0$