Mathematics-10
Unit 4 – Exercise 4.1

Fraction **(K.B)**

(MTN 2017, BWP 2014, 15, 17, RWP 2016, D.G.K 2015, 17)

The quotient of two numbers or algebraic expressions is called a fraction. The quotient is indicated by a bar $(-)$. The dividend is written on the top of the bar and divisor below the bar.

For example: $\frac{2}{3}, \frac{x^2+4}{x-2}$ where $x \neq 2$

Note **(K.B + U.B)**

If $x=2$ in second example then the fraction is not defined because $x=2$ makes the denominator zero.

Rational Fraction **(K.B)**

(LHR 2014, 16, GRW 2016, FSD 2015, SGD 2015, 16, MTN 2015, D.G.K 2016)

An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients is called a rational fraction. The polynomial $D(x) \neq 0$

For example $\frac{x^2+4}{x-2}$ where $x \neq 2$

Types of Fractions **(K.B + U.B)**

There are two types of fractions.

(i) Proper Fraction

(ii) Improper Fraction

Proper Fraction **(K.B)**

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq$ 0 is called proper fraction, if degree of the polynomial N(x) is less than degree of the polynomial D(x)

For example: $\frac{2}{x+1}, \frac{5x-3}{x^2+4}$ etc.

Improper Fraction **(U.B + K.B)**

(LHR 2014, 15, GRW 2014, 17, FSD 2015, SGD 2017, RWP 2017, MTN 2015)

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq$ 0 is called an improper fraction, if degree of the polynomial N(x) is greater than or equal to degree of the polynomial D(x).

For example: $\frac{5x}{x+2}, \frac{6x^4}{x^3+1}$ etc.

Identity **(K.B)**

(GRWP 2014, 15, 17, RWP 2016, SGD 2016, D.G.K 2015, 17)

An identity is an equation, which is satisfied by all the values of the variables involved

For example: $(x+3)^2 = x^2+6x+9$,

$2(x+1) = 2x+2$ etc.

Conditional Equation **(K.B)**

An equation which is true for some specific value(s) of the variable involved.

For example: $x+2=3$ is true only for $x=1$.

Partial Fraction **(K.B)**

(LHR 2014, 16, 17, GRW 2015, FSD 2015, 17, RWP 2015, 16, BWP 2015,)

Decomposition of resultant fraction into its components or into different fractions is called partial fraction.

Note **(K.B + U.B)**

General method applicable to resolve all rational fractions of the form $\frac{N(x)}{D(x)}$, is as follows:

- The numerator $N(x)$ must be of lower degree than the denominator $D(x)$.
- Make substitutions constant accordingly.
- Multiply both sides by L.C.M.
- Arrange terms on both sides by decreasing order.
- Make the equations and solve to find constants.

Resultant Fraction **(K.B)**

Sum of two or more than two proper fractions in the form of a single fraction is called the resultant fraction.

For example:

$\frac{1}{x-1}-\frac{2}{x+1}=\frac{-x+3}{(x-1)(x+1)}$ is resultant fraction.

Example 2: (Page # 78) **(A.B)**

Resolve $\frac{1}{3+x-2x^2}$ into partial fractions.

Solution:

$\frac{1}{3+x-2x^2}$ can be written as for convenience $\frac{-1}{2x^2-x-2}$

The denominator

$$D(x)=2x^2-x-3=2x^2-3x+2x-3$$
$$=x(2x-3)+1(2x-3)=(x+1)(2x-3)$$

Let,

$$\frac{-1}{2x^2-x-3}=\frac{-1}{(x+1)(2x-3)}=\frac{A}{x+1}+\frac{B}{2x-3}\rightarrow(i)$$

multiplying both the sides by $(x+1)(2x-3)$, we get

$$-1=A(2x-3)+B(x+1)\rightarrow(ii)$$

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$$-1=A[2(-1)-3]$$
$$-1=-5A$$
$$\Rightarrow A=\frac{1}{5}$$

Put $2x-3=0 \Rightarrow x=\frac{3}{2}$ in equation (ii)

$$-1=B\left(\frac{3}{2}+1\right)$$
$$-1=\frac{5}{2}B$$
$$\Rightarrow B=-\frac{2}{5}$$

Thus, $\frac{1}{3+x-2x^2}=\frac{1}{5(x+1)}-\frac{2}{5(2x-3)}$

Note

There are two methods to resolve into partial fraction:

(i) Zero Method

(ii) Equating coefficient

Exercise 4.1

Resolve into partial fractions.

Q.1 $\frac{7x-9}{(x+1)(x-3)}$ **(FSD 2015)** **(A.B)**

Solution:

Let $\frac{7x-9}{(x+1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}\rightarrow(i)$

Multiplying with $(x+1)(x-3)$

$$\frac{7x-9}{(x+1)(x-3)}\times(x+1)(x-3)$$
$$=\frac{A}{x+1}\times(x+1)(x-3)+\frac{B}{x-3}\times(x+1)(x-3)$$
$$\Rightarrow 7x-9=A(x-3)+B(x+1)\rightarrow(ii)$$

Put $x+1=0\Rightarrow x=-1 in\, eq.(ii)$

$$7(-1)-9=A(-1-3)+B(0)$$
$$-7-9=-4(A)+0$$
$$-16=-4A$$

Or $A=\frac{-16}{-4}$

A = 4

Put $x-3=0$ or $x=3$ in $eq.(ii)$

$$7(3)-9=A(o)+B(3+1)$$

21-9 = 0 + B (4)

12 = 4B

Or 4B = 12

B = $\frac{12}{4}$

B = 3

Putting the values in equation. (i)

$$\Rightarrow\frac{7x-9}{(x+1)(x-3)}=\frac{4}{x+1}+\frac{3}{x-3}$$

Q.2 $\frac{x-11}{(x-4)(x+3)}$ **(A.B)**

Solution:

Let $\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3} \rightarrow (\text{i})$

Multiplying equation (i) by (x-4) (x+3)

$$\frac{x-11}{(x-4)(x-3)} \times (x-4)(x-3)$$

$$= \frac{A}{(x-4)} \times (x-4)(x+3) + \frac{B}{(x-3)}(x-4)(x-3)$$

$$x-11 = A(x+3) + B(x-4) \rightarrow (ii)$$

Put $x-4=0 \Rightarrow x=4$ *in eq.*(ii)

$$4-11 = A(4+3) + B(0)$$
$$-7 = A(7) + 0$$
$$-7 = 7A$$

Or $7A = -7$

$$A = \frac{-7}{7}$$
$$\Rightarrow A = -1$$

Put x+3=0 or x = -3 in eq. (ii)

$$-3-11 = A(-3+3) + B(-3-4)$$
$$-14 = A(0) + B(-7)$$
$$-14 = 0 + (-7B)$$

Or $-7B = -14$

$$B = \frac{-14}{-7}$$
$$\Rightarrow B = 2$$

Putting the values of A and B in equation (i)

$$\therefore \frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

Q.3 $\frac{3x-1}{x^2-1}$ **(A.B)**

(GRWP 2017, SWL 2014, BWP 2017, D.G.K 2014)

Solution:

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x+1)(x-1)}$$

Let $\frac{3x-1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

Multiplying by (x+1) (x-1)

$$\frac{3x-1}{(x+1)(x-1)} \times (x+1)(x-1)$$

$$= \frac{A}{(x+1)}(x+1)(x-1) + \frac{B}{(x-1)}(x+1)(x-1)$$

$$3x-1 = A(x-1) + B(x+1) \rightarrow (\text{ii})$$

Put $x-1=0 \Rightarrow x=1 \text{ in eq}(\text{ii})$

$$3(1)-1 = A(1-1) + B(1+1)$$
$$3-1 = 0 + 2B$$
$$2 = 2B$$

Or $2B = 2$

$$B = \frac{2}{2}$$
$$B = 1$$

Put $x+1=0 \Rightarrow x=-1 \, in\, eq(ii)$

$$3(-1)-1 = A(-1-1) + B(0)$$
$$-3-1 = A(-2)$$
$$-4 = -2A$$
$$-2A = -4$$
$$A = \frac{-4}{-2}$$
$$A = 2$$

Now putting the values in eq (i)

$$\Rightarrow \frac{3x-1}{x^2-1} = \frac{2}{x+1} + \frac{1}{x-1}$$

Q.4 $\frac{x-5}{x^2+2x-3}$ **(A.B)**

(FSD 2015, MTN 2016, SGD 2015)

Solution:

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2-x+3x-3}$$
$$= \frac{x-5}{x(x-1)+3(x-1)}$$
$$= \frac{x-5}{(x-1)(x+3)}$$

Let $\frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} \rightarrow (i)$

Multiplying by $(x-1)(x+3)$, we get

$$x-5 = A(x+3) + B(x-1) \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ *in eq* (ii)

$$1-5=A(1+3)+B(1-1)$$
$$-4=A(4)+B(0)$$
$$-4=4A$$
$$4A=-4$$
$$A=\frac{-4}{4}$$
A = -1

Put $x+3=0\Rightarrow x=-3$ in eq (ii)

$$-3-5=A(-3+3)+B(-3-1)$$
$$-8=-4B$$
$$-4B=-8$$
$$B=\frac{-8}{-4}$$
B=2

Now putting values in eq (i)

$$\frac{x-5}{(x-1)(x+3)}=\frac{-1}{x-1}+\frac{2}{x+3}$$
$$\Rightarrow\frac{x-5}{x^2+2x-3}=\frac{-1}{x-1}+\frac{2}{x+3}$$

Q.5 $\frac{3x+3}{(x-1)(x+2)}$ **(A.B)**

(GRW 2015, 16, FSD 2016, 17, BWP 2015, 16, SGD 2015)

Solution:

Let $\frac{(3x+3)}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}\rightarrow(i)$

Multiplying equation (i) by (x-1) (x+2)

$$\frac{3x+3}{(x-1)(x+2)}\times(x-1)(x+2)$$
$$=\frac{A}{x-1}\times(x-1)(x+2)+\frac{B}{x+2}\times(x-1)(x+2)$$
$$3x+3=A(x+2)+B(x-1)\rightarrow(ii)$$

Put $x-1=0$ or $x=1$ in eq (ii)

$$3(1)+3=A(1+2)+B(1-1)$$
$$3+3=3(A)+0$$
$$6=3A$$
$$3A=6$$
$$A=\frac{6}{3}$$
A = 2

Put $x+2=0\Rightarrow x=-2$ in eq (ii)

$$3(-2)+3=A(-2+2)+B(-2-1)$$
$$-6+3=A(0)+B(-3)$$
$$-3=0-3B$$
$$3B=3$$
$$B=\frac{3}{3}$$
B = 1

Now putting values in eq (i)

$$\Rightarrow\frac{3x+3}{(x-1)(x+2)}=\frac{2}{x-1}+\frac{1}{x+2}$$

Q.6 $\frac{(7x-25)}{(x-4)(x-3)}$ **(A.B)**

(RWP 2016, SGD 2017, D.G.K 2014, 17)

Solution:

Let $\frac{(7x-25)}{(x-4)(x-3)}=\frac{A}{x-4}+\frac{B}{x-3}\rightarrow(i)$

Multiplying by (x-4) (x-3)

$$7x-25=A(x-3)+B(x-4)\rightarrow(ii)$$

Put $x-4=0$ or $x=4$ in eq (ii)

$$7(4)-25=A(4-3)+B(4-4)$$
$$28-25=A(1)+B(0)$$
$$3=A$$
Or A = 3

Put $x-3=0$ or $x=3$ in eq (ii)

$$7(3)-25=A(0)+B(3-4)$$
$$21-25=B(-1)$$
$$-B=-4$$
B = 4

Now putting values in equation (ii)

$$\Rightarrow\frac{7x-25}{(x-4)(x-3)}=\frac{3}{x-4}+\frac{4}{x-3}$$

Q.7 $\frac{x^2+2x+1}{(x-2)(x+3)}$ **(A.B + K.B)**

Solution:

$$\frac{x^2+2x+1}{(x-2)(x+3)}=\frac{x^2+2x+1}{x^2+x-6}\text{ (improper)}$$
$$\because x^2+x-6=x^2+3x-2x-6$$
$$=x(x+3)-2(x+3)$$
$$=(x+3)(x-2)$$

$$x^2+x-6\overline{\smash{)}\,x^2+2x+1}\quad 1$$
$$\underline{-x^2 \pm x \mp 6}$$
$$x+7$$

$$\frac{x^2+2x+1}{x^2+x-6} = 1+\frac{x+7}{x^2+x-6}$$
$$= 1+\frac{x+7}{(x+3)(x-2)} \quad \to(i)$$

Consider

$$\frac{x+7}{(x+3)(x-2)} = \frac{A}{x+3}+\frac{B}{x-2} \quad \to(ii)$$

Multiplying by $(x+3)(x-2)$

$$x+7 = A(x-2)+B(x+3)\to(iii)$$

Put $x+3=0$ *or* $x=-3$ *in eq* (iii)

$$-3+7 = A(-3-2)+B(-3+3)$$
$$4 = A(-5)+B(0)$$
$$4=-5A$$
$$A=\frac{4}{-5}$$
$$A=-\frac{4}{5}$$

Put $x-2=0$ *or* $x=2$ in equation (ii)

$$2+7 = A(2-2)+B(2+3)$$
$$9 = A(0)+B(5)$$
$$9=5B$$
$$5B = 9$$
$$B=\frac{9}{5}$$

Putting the values of A and B in equation (ii)

$$\frac{x+7}{(x+3)(x-2)} = \frac{9/5}{x-2}+\frac{-4/5}{x+3}$$
$$= \frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Now putting the values in equation (i)

$$\Rightarrow \frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\frac{6x^3+5x^2-7}{3x^2-2x-1}$ **(A.B + K.B)**

Solution:

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} \quad \text{(improper fraction)}$$

$$3x^2-2x-1\overline{\smash{)}\,6x^3+5x^2-7}\quad 2x+3$$
$$\underline{\pm 6x^3 \mp 4x^2 \quad \mp 2x}$$
$$9x^2+2x-7$$
$$\underline{\pm 9x^2 \mp 6x \mp 3}$$
$$8x-4$$

$$\Rightarrow \frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{8x-4}{3x^2-2x-1}$$
$$= 2x+3+\frac{8x-4}{3x^2-3x+x-1}$$
$$= 2x+3+\frac{8x-4}{3x(x-1)+1(x-1)}$$
$$= 2x+3+\frac{8x-4}{(x-1)(3x+1)} \to(i)$$

Consider

$$\frac{8x-4}{(x-1)(3x+1)} = \frac{A}{x-1}+\frac{B}{3x+1}\to(ii)$$

Multiplying by $(x-1)(3x+1)$

$$8x-4 = A(3x+1)+B(x-1)\to(iii)$$

Put $x-1=0$ or $x=1$ in equation (iii)

$$8-4 = A(3+1)+B(0)$$
$$4 = A(4)+0$$
$$4=4A$$
$$\Rightarrow A=1$$

Put $3x+1=0$ or $x=-\frac{1}{3}$ in equation (iii)

$$8\left(-\frac{1}{3}\right)-4 = A\left[3\left(-\frac{1}{3}\right)+1\right]+B\left(-\frac{1}{3}-1\right)$$
$$\frac{-8-12}{3} = A(0)+B\frac{(-1-3)}{3}$$
$$-\frac{20}{3} = 0+B\left(-\frac{4}{3}\right)$$

$$-\frac{20}{3}=-\frac{4}{3}B$$

$$-\frac{20}{3}\left(-\frac{3}{4}\right)=B$$

$$5=B$$

Or $B=5$

Now putting values in equation (ii)

$$\frac{8x-4}{(x-1)(3x+1)}=\frac{1}{x-1}+\frac{5}{3x+1}$$

Putting the values in equation (i)

$$\Rightarrow\frac{6x^3+5x^2-7}{3x^2-2x-1}=2x+3+\frac{1}{x-1}+\frac{5}{3x+1}$$