Example: (Page # 80) **(A.B)**

Resolve $\frac{11x+3}{(x-3)(x^2+9)}$ into partial fractions.

Solution:

Let, $\frac{11x+3}{(x-3)(x^2+9)}=\frac{A}{(x-3)}+\frac{Bx+C}{x^2+9}$

Multiplying both the sides by $(x-3)(x^2+9)$ on both sides

$\Rightarrow \quad 11x+3=A(x^2+9)+(Bx+C)(x-3)$

$11x+3=A(x^2+9)+B(x^2-3x)+C(x-3)\rightarrow(i)$

Since (i) is an identity, we have on substituting $x=3$

$33+3=A(9+9)$

$\Rightarrow 18A=36$

$\Rightarrow \quad A=2$

Comparing the coefficients of x^2 and x on both sides of (*i*), we get

$A+B=0$

$\Rightarrow B=-2$

$-3B+C=11$

$\Rightarrow -3(-2)+C=11$

$\Rightarrow C=5$

Therefore, the partial fractions are

$\frac{2}{x-3}+\frac{-2x+5}{x^2+9}$

Thus, $\frac{11x+3}{(x-3)(x^2+9)}=\frac{2}{x-3}+\frac{-2x+5}{x^2+9}$

Exercise 4.3

Resolve into partial fractions.

Q.1 $\frac{3x-11}{(x+3)(x^2+1)}$ **(A.B)**

Solution

Let $\frac{3x-11}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}\rightarrow(\text{i})$

Multiplying by $(x+3)(x^2+1)$

$3x-11=A(x^2+1)+(Bx+C)(x+3)\rightarrow(\text{ii})$

$3x-11=Ax^2+A+Bx^2+3Bx+3C+Cx$

$3x-11=Ax^2+Bx^2+3Bx+Cx+A+3C$

$0x^2+3x-11=(A+B)x^2+(3B+C)x+(A+3C)$

By comparing co-efficient of alike powers of x

$0=A+B$ ________ (iii)

$3=3B+C$ ________ (iv)

$-11=A+3C$ ________ (v)

Put $x+3=0\Rightarrow x=-3$ in equation (ii)

$3(-3)-11=A[(-3)^2+1]+(B(-3)+C)(-3+3)$

$-9-11=A(9+1)+(Bx+C)(0)$

$-20=A(10)+0$

Or A = – 2

Put in equation (iii)

$0=-2+B$

$B=2$

Put in equation (iv)

$3=3(2)+C$

$3=6+C$

$3-6=C$

$-3=C$

$C=-3$

Now putting in equation (i)

$$\frac{3x-11}{(x+3)(x^2+1)} = -\frac{2}{x+3} + \frac{2x-3}{x^2+1}$$

Or $\frac{3x-11}{(x+3)(x^2+1)} = \frac{2x-3}{x^2+1} - \frac{2}{x+3}$

Q.2 $\frac{3x+7}{(x^2+1)(x+3)}$ **(A.B)**

(SWL 2015, MTN 2015)

Solution:

Let $\frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+3} \rightarrow \text{(i)}$

Multiplying by $(x^2+1)(x+3)$

$$3x+7 = (Ax+B)(x+3) + C(x^2+1) \rightarrow \text{(ii)}$$

$$0x^2+3x+7 = Ax^2+3Ax+Bx+3B+Cx^2+C$$
$$= Ax^2+Cx^2+Bx+3Ax+3B+C$$
$$= (A+C)x^2+(B+3A)x+(3B+C)$$

By comparing coefficients of alike powers of x

$0 = A+C$ __________ (iii)
$3 = 3A+B$ __________ (iv)
$7 = 3B+C$ __________ (v)

Put $x+3=0 \Rightarrow x=-3$ in equation (ii)

$$3(-3)+7 = (A(-3)+B)(0)+C\left[(-3)^2+1\right]$$
$$-9+7 = C(9+1)$$
$$-2 = 0+C(10)$$
$$-\frac{2}{10} = C$$
$$\Rightarrow C = -\frac{1}{5}$$

Put in equation (iii)

$$0 = A - \frac{1}{5}$$

Or $A = \frac{1}{5}$

Put in equation (iv)

$$3 = 3\left(\frac{1}{5}\right)+B$$
$$3 = \frac{3}{5}+B$$
$$3-\frac{3}{5} = B$$
$$\frac{15-3}{5} = B$$
$$\frac{12}{5} = B$$
$$\Rightarrow B = \frac{12}{5}$$

Now putting values in equation (i)

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{\frac{1}{5}x+\frac{12}{5}}{x^2+1} + \frac{-\frac{1}{5}}{x+3}$$
$$= \frac{\frac{1}{5}(x+12)}{x^2+1} - \frac{1}{5(x+3)}$$
$$\Rightarrow \frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$$

Q.3 $\frac{1}{(x+1)(x^2+1)}$ **(A.B)**

Solution:

Let $\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ __(i)

Multiplying by $(x+1)(x^2+1)$

$$1 = A(x^2+1)+(Bx+C)(x+1) \text{ \_\_ (ii)}$$
$$= Ax^2+A+Bx^2+Bx+C+Cx$$
$$= Ax^2+Bx^2+Bx+Cx+A+C$$
$$0x^2+0x+1 = (A+B)x^2+(B+C)x+(A+C)$$

By comparing coefficients of alike powers of x

$0 = A+B$ __________ (iii)
$0 - B+C$ __________ (iv)
$1 = A+C$ __________ (v)

Put $x+1=0 \rightarrow x=-1$ in equation (ii)

$$1 = A\left[(-1)^2+1\right]+(B(-1)+C)(0)$$
$$1 = A(1+1)+0$$
$$1 = A(2)$$

Or $A = \frac{1}{2}$

Put in equation (iii)

$$0=\frac{1}{2}+B$$

$$B=-\frac{1}{2}$$

Put in equation (iv)

$$0=-\frac{1}{2}+C$$

$$\Rightarrow C=\frac{1}{2}$$

$$\frac{1}{(x+1)(x^2+1)}=\frac{\frac{1}{2}}{2(x+1)}+\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}$$

$$\frac{1}{(x+1)(x^2+1)}=\frac{1}{2(x+1)}+\frac{-\frac{1}{2}(x-1)}{x^2+1}$$

$$\Rightarrow\frac{1}{(x+1)(x^2+1)}=\frac{1}{2(x+1)}-\frac{x-1}{2(x^2+1)}$$

Q.4 $\dfrac{9x-7}{(x+3)(x^2+1)}$ **(A.B)**

Solution:

Let $\dfrac{9x-7}{(x+3)(x^2+1)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}$ ___ (i)

Multiplication by $(x+3)(x^2+1)$

$$9x-7=A(x^2+1)+(Bx+C)(x+3) \text{ ___ (ii)}$$

$$=Ax^2+A+Bx^2+Cx+3Bx+3C$$

$$=Ax^2+Bx^2+3Bx+Cx+A+3C$$

$$0x^2+9x-7=(A+B)x^2+(3B+C)x+(A+3C)$$

By comparing coefficients of alike powers of x

$$0=A+B \text{ __________(iii)}$$

$$9=3B+C \text{ __________(iv)}$$

$$-7=A+3C \text{ __________(v)}$$

Put $x+3=0\Rightarrow x=-3$ in equation (ii)

$$9(-3)-7=A\left[(-3)^2+1\right]+\left[B(-3)+C\right](-3+3)$$

$$-27-7=A(9+1)+0$$

$$-34=A(10)$$

$$A=-\frac{34}{10}$$

$$A=-\frac{17}{5}$$

Put in equation (iii)

$$0=-\frac{17}{5}+B$$

$$B=\frac{17}{5}$$

Put in equation (iv)

$$9=3\left[\frac{17}{5}\right]+C$$

$$9=\frac{51}{5}+C$$

$$9-\frac{51}{5}=C$$

$$\frac{45-51}{5}=C$$

$$-\frac{6}{5}=C$$

$$C=-\frac{6}{5}$$

Now putting the values in equation (i)

$$\frac{9x-7}{(x+3)(x^2+1)}=-\frac{\frac{17}{5}}{(x+3)}+\frac{\frac{17}{5}x+\left(-\frac{6}{5}\right)}{x^2+1}$$

$$=-\frac{17}{5(x+3)}+\frac{\frac{1}{5}(17x-6)}{x^2+1}$$

$$=-\frac{17}{5(x+3)}+\frac{17x-6}{5(x^2+1)}$$

$$\Rightarrow\frac{9x-7}{(x+3)(x^2+1)}=\frac{17x-6}{5(x^2+1)}-\frac{17}{5(x+3)}$$

Q.5 $\dfrac{3x+7}{(x+3)(x^2+4)}$ **(A.B)**

(SWL 2015, MTN 2015)

Solution:

Let $\dfrac{3x+7}{(x+3)(x^2+4)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4}$ ___ (i)

Multiplying by $(x+3)(x^2+4)$

$$3x+7=A(x^2+4)+(Bx+C)(x+3) \_\_\_ \text{(ii)}$$
$$=Ax^2+4A+Bx+3Bx+Cx+3C$$
$$=Ax^2+Bx^2+3Bx+Cx+4A+3C$$
$$0x^2+3x+7=(A+B)x^2+(3B+C)x+(4A+3C)$$

By comparing coefficients of alike powers of x,

$$0=A+B \_\_\_\_\_\_\_\_\_\_ \text{(iii)}$$
$$3=3B+C \_\_\_\_\_\_\_\_\_\_ \text{(iv)}$$
$$7=4A+3C \_\_\_\_\_\_\_\_\_\_ \text{(v)}$$

Put $x+3=0 \Rightarrow x=-3$ in equation (ii)

$$3(-3)+7=A\left[(-3)^2+4\right]+(B(-3)+C)(-3+3)$$
$$-9+7=A(9+4)+0$$
$$-2=A(13)$$
$$A=-\frac{2}{13}$$

Put in equation (iii)

$$0=-\frac{2}{13}B$$
$$B=\frac{2}{13}$$

Put in equation (iv)

$$3=3\left[\frac{2}{13}\right]+C$$
$$3=\frac{6}{13}+C$$
$$3-\frac{6}{13}+C$$
$$\frac{39-6}{13}=C$$
$$\frac{33}{13}=C$$
$$C=\frac{33}{13}$$

Now putting the values in equation (i)

$$\frac{3x+7}{(x+3)(x^2+4)}=\frac{-\frac{2}{13}}{13(x+3)}+\frac{\frac{2}{13}x+\frac{33}{13}}{x^2+4}$$
$$=-\frac{2}{13(x+3)}+\frac{\frac{1}{13}(2x+33)}{x^2+4}$$
$$\Rightarrow \frac{3x+7}{(x+3)(x^2+4)}=\frac{2x+33}{13(x^2+4)}-\frac{2}{13(x+3)}$$

Q.6 $\quad \dfrac{x^2}{(x+2)(x^2+4)}$ **(A.B)**

Solution:

Let $\dfrac{x^2}{(x+2)(x^2+4)}=\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$ __ (i)

Multiplying by $(x+2)(x^2+4)$

$$x^2=A(x^2+4)+(Bx+C)(x+2) \_\_ \text{(ii)}$$
$$=Ax^2+4A+Bx^2+2Bx+Cx+2C$$
$$=Ax^2+Bx^2+2Bx+Cx+4A+2C$$
$$x^2+0x+0=(A+B)x^2+(2B+C)x+(4A+2C)$$

By comparing coefficients of alike powers of x

$$1=A+B \_\_\_\_\_\_\_\_\_\_ \text{(iii)}$$
$$0=2B+C \_\_\_\_\_\_\_\_\_\_ \text{(iv)}$$
$$0=4A+2C \_\_\_\_\_\_\_\_\_\_ \text{(v)}$$

Put $x+2=0 \Rightarrow x=-2$ in equation (ii)

$$(-2)^2=A\left[(-2)^2+4\right]+\left[B(-2)+C\right](0)$$
$$4=A(4+4)+0$$
$$4=A(8)$$
$$4=8A$$
$$A=\frac{1}{2}$$

Put in equation (iii)

$$1=\frac{1}{2}+B$$
$$1-\frac{1}{2}=B$$
$$\frac{2-1}{2}=B$$

$B=\frac{1}{2}$

Put in equation (iv)

$0=2\left(\frac{1}{2}\right)+C$

$0=1+C$

$C=-1$

Now putting values of A,B,C in equation (i)

$$\frac{x^2}{(x+2)(x^2+4)}=\frac{\frac{1}{2}}{(x+2)}+\frac{\frac{1}{2}x+(-1)}{x^2+4}$$

$$=\frac{1}{2(x+2)}+\frac{\frac{1}{2}(x-2)}{(x^2+4)}$$

$$\Rightarrow \frac{x^2}{(x+2)(x^2+4)}=\frac{1}{2(x+2)}+\frac{x-2}{2(x^2+4)}$$

Q.7 $\frac{1}{x^3+1}$ **(K.B + A.B)**

Solution:

$$\frac{1}{x^3+1}=\frac{1}{(x+1)(x^2-x+1)}$$

Let $\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$ ___(i)

Multiplying by $(x+1)(x^2-x+1)$

$1=A(x^2-x+1)+(Bx+C)(x+1)$ ____ (ii)

$1=Ax^2-Ax+A+Bx^2+Bx+Cx+C$

$1=Ax^2+Bx^2+Bx+Cx-Ax+A+C$

$0x^2+0x+1=(A+B)x^2+(B+C-A)x+(A+C)$

By comparing coefficients of alike power of x

$0=A+B$ __________ (iii)

$0=B+C-A$ __________ (iv)

$1=A+C$ __________ (v)

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$1=A\left[(-1)^2-(-1)+1\right]+\left[B(-1)+C\right](-1+1)$

$1=A(1+1+1)+(Bx+C)(0)$

$1=A(3)+0$

$A=\frac{1}{3}$

Put in equation (iii)

$0=\frac{1}{3}+B$

$B=-\frac{1}{3}$

Put $A=\frac{1}{3}$ and $B=-\frac{1}{3}$ in equation (iv)

$0=-\frac{1}{3}+C-\frac{1}{3}$

$C=\frac{1}{3}+\frac{1}{3}$

$C=\frac{2}{3}$

Now putting values in equation (i)

$$\frac{1}{(x+1)(x^2-x+1)}=\frac{\frac{1}{3}}{(x+1)}+\frac{-\frac{1}{3}x+\frac{2}{3}}{x^2-x+1}$$

$$=\frac{1}{3(x+1)}+\frac{-\frac{1}{3}(x-2)}{x^2-x+1}$$

$$\Rightarrow \frac{1}{(x+1)(x^2-x+1)}=\frac{1}{3(x+1)}-\frac{x-2}{3(x^2-x+1)}$$

Q.8 $\frac{x^2+1}{x^3+1}$ **(K.B + A.B)**

Solution:

$$\frac{x^2+1}{x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)}$$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\rightarrow$(i)

Multiplying by $(x+1)(x^2-x+1)$

$x^2+1=A(x^2-x+1)+(Bx+C)(x+1)$ __ (ii)

$=Ax^2-Ax+A+Bx^2+Bx+Cx+C$

$=Ax^2+Bx^2-Ax+Bx+Cx+A+C$

$x^2+0x+1=(A+B)x^2+(B+C-A)x+(A+C)$

By comparing coefficients of power of x

$1=A+B$ __________ (iii)

$0=B+C-A$ __________ (iv)

$1=A+C$ __________ (v)

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$(-1)^2+(1)=A\left[(-1)^2-(-1)+1\right]+\left[B(-1)+C\right](0)$

$1+1=A(1+1+1)+0$

$2=A(3)$

$\frac{2}{3}=A$

$A=\frac{2}{3}$

Put in equation (iii)

$1=\frac{2}{3}+B$

$1-\frac{2}{3}=B$

$\frac{3-2}{3}=B$

$\frac{1}{3}+B$

$B=\frac{1}{3}$

Put $P=\frac{2}{3}$ in equation (v)

$1=\frac{2}{3}+C$

$1-\frac{2}{3}+C$

$C=\frac{1}{3}$

Putting the values in equation (i)

$\frac{x^2+1}{x^3+1}=\frac{\frac{2}{3}}{3(x+1)}+\frac{\frac{1x}{3}+\frac{1}{3}}{x^2-x+1}$

$=\frac{2}{3(x+1)}+\frac{\frac{1}{3}(x+1)}{x^2-x+1}$

$\Rightarrow\frac{x^2+1}{x^3+1}=\frac{2}{3(x+1)}+\frac{x+1}{3(x^2-x+1)}$