Q.1 Multiple Choice Questions

Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) **The identity** $(5x+4)^2 = 25x^2 + 40x + 16$ **is true for;** (SWL 2015) **(K.B + A.B)**

(a) One value of x (b) Two values of x

(c) All values of x (d) None of these

(ii) **A function of the form** $f(x) = \frac{N(x)}{D(x)}$, **with** $D(x) \neq 0$, **where** $N(x)$ **and** $D(x)$ **are polynomials in** x **is called;** **(K.B + A.B)**

(a) An identity (b) An equation

(c) A fraction (d) Algebraic relation

(iii) **A fraction in which the degree of numerator is greater or equal to the degree of denominator is called;** (GRW 2017, RWP 2015, 17, FSD 2015) **(K.B + A.B)**

(a) A proper fraction (b) An improper fraction

(c) An equation (d) Algebraic relation

(iv) **A fraction in which the degree of numerator is less than the degree of the denominator is called** (GRW 2014, 15, 16, FSD 2017) **(K.B + A.B)**

(a) An equation (b) An improper fraction

(c) An identity (d) A proper fraction

(v) $\frac{2x+1}{(x+1)(x-1)}$ **is;** **(K.B + A.B)**

(a) An improper fraction (b) An equation

(c) A proper fraction (d) None of these

(vi) $(x+3)^2 = x^2 + 6x + 9$ **is;** (LHR 2014, 15, 16, MTN 2015, SWL 2015, D.G.K 2015) **(K.B + A.B)**

(a) A linear equation (b) An equation

(c) An identity (d) None of these

(vii) $\frac{x^3+1}{(x-1)(x+2)}$ **is;** **(K.B + A.B)**

(a) A proper fraction (b) An improper fraction

(c) An identity (d) A constant term

(viii) **Partial fraction of** $\frac{x-2}{(x-1)(x+2)}$ **are of the form;** **(K.B + A.B)**

(a) $\frac{A}{x-1} + \frac{B}{x+2}$ (b) $\frac{Ax}{x-1} + \frac{B}{x+2}$

(c) $\frac{A}{x-1} + \frac{Bx+C}{x+2}$ (d) $\frac{Ax+B}{x-1} + \frac{C}{x+2}$

(ix) **Partial fraction of** $\frac{x+2}{(x+1)(x^2+2)}$ **are of the form;** (GRW 2014, RWP 2017) **(K.B + A.B)**

(a) $\frac{A}{x+1}+\frac{B}{x^2+2}$ **(b)** $\frac{A}{x+1}+\frac{Bx+C}{x^2+2}$

(c) $\frac{Ax+B}{x+1}+\frac{C}{x^2+2}$ (d) $\frac{A}{x+1}+\frac{Bx}{x^2+2}$

(x) **Partial fraction of** $\frac{x^2+1}{(x+1)(x-1)}$ **are of the form;** **(K.B + A.B)**

(a) $\frac{A}{x+1}+\frac{B}{x-1}$ (b) $1+\frac{A}{x+1}+\frac{Bx+C}{x-1}$

(c) $1+\frac{A}{x+1}+\frac{B}{x-1}$ (d) $1+\frac{Ax+B}{(x+1)}+\frac{C}{x-1}$

ANSWER KEY

i	ii	iii	iv	v	Vi	vii	viii	ix	x
c	c	b	d	c	c	b	a	b	c

Q.2 Write short answers of the following questions.

(i) Define a rational fraction. **(K.B)**

Ans:

Rational Fraction

An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients is called a rational fraction. The polynomial $D(x)\neq 0$

For example $\frac{x^2+4}{x-2}$ where $x\neq 2$

(ii) What is a proper fraction? **(K.B)**

Ans:

Proper Fraction

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq$ 0 is called proper fraction, if degree of the polynomial N(x) is less than degree of the polynomial D(x)

For example: $\frac{2}{x+1},\frac{5x-3}{x^2+4}$ etc.

(iii) What is an improper fraction? **(K.B)**

Ans:

Improper Fraction

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq$ 0 is called an improper fraction, if degree of the polynomial N(x) is greater than or equal to degree of the polynomial D(x).

For example: $\frac{5x}{x+2},\frac{6x^4}{x^3+1}$ etc.

(iv) What are partial fractions? **(K.B)**

Ans:

Partial Fraction

(LHR 2016, FSD 2016, BWP 2014, RWP 2016, 17, MTN 2016, 17, SWL 2017, SGD 2017)

Decomposition of resultant fraction into its components or into different fractions is called partial fraction.

(v) **How can we make partial fractions of** $\frac{x-2}{(x+2)(x+3)}$**?** **(K.B)**

Solution:

Let $\frac{x-2}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$ $\rightarrow$(i)

Multiply by $(x+2)(x+3)$

$x-2=A(x+3)+B(x+2)$ $\rightarrow$(ii)

Put $x+2=0$ or $x=-2$ in equation (ii)

$-2-2=A(-2+3)+B(0)$

$-4=A(1)+0$

$\Rightarrow A=-4$

Put $x+3=0$ or $x=-3$ in equation (ii)

$-3-2=A(0)+B(-3+2)$

$-5=0+B(-1)$

$-5=-B$

$\Rightarrow B=5$

Now putting the values in equation (i)

$$\frac{x-2}{(x+2)(x+3)}=\frac{-4}{x+2}+\frac{5}{x+3}$$

(vi) Resolve $\frac{1}{x^2-1}$ into partial fractions.

(A.B + K.B)

Solution:

(LHR 2016, 17, GRW 2016, 17, SWL 2017, RWP 2017, BWP 2015, 16, 17, MTN 2017, SGD 2014, 15, 16, 17)

$$\frac{1}{x^2-1}=\frac{1}{(x+1(x-1)}$$

Let $\frac{1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$ $\rightarrow$ (i)

Multiply $(x+1)(x-1)$ on both sides

$1=A(x-1)+B(x+1)$ $\rightarrow$ (ii)

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$1=A(-1-1)+B(0)$

$1=-2A+0$

$1=-2A$

$\Rightarrow A=\frac{-1}{2}$

Put $x-1=0\Rightarrow x=1$ in equation (ii)

$1=A(0)+B(1+1)$

$1=0+2B$

$1=2B$

$\Rightarrow B=\frac{1}{2}$

Now putting the values in equation $\rightarrow$ (i)

$$\frac{1}{(x+1)(x-1)}=\frac{\frac{-1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}$$

$$\Rightarrow\frac{1}{x^2-1}=-\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$$

(vii) Find partial fractions of $\frac{3}{(x+1)(x-1)}$.

(BWP 2015, SGD 2014)

Solution:

Let $\frac{3}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$ $\rightarrow$ (i)

Multiply by $(x+1)(x-1)$ on both sides

$3=A(x-1)+B(x+1)$ $\rightarrow$ (ii)

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$3=A(-1-1)+B(0)$

$3=-2A+0$

$3=-2A$

$\frac{3}{-2}=A$

Or $A=\frac{-3}{2}$

Put $x-1=0$ or $x=1$ in equal (ii)

$3=A(0)+B(1+1)$

$3=0+2B$

$3=2B$

$\frac{3}{2}=B$

Or $B=\frac{3}{2}$

Now put the values in equation (i)

$$\frac{3}{(x+1)(x-1)}=\frac{\frac{-3}{2}}{x+1}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{3}{(x+1)(x-1)}=-\frac{3}{2(x+1)}+\frac{3}{2(x-1)}$$

Or $\frac{3}{(x+1)(x-1)}=\frac{3}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$

(viii) Resolve $\frac{x}{(x-3)^2}$ into partial fractions.

(A.B + K.B)

(FSD 2015, RWP 2014, D.G.K 2014)

Solution:

Let $\frac{x}{(x-3)^2}=\frac{A}{x-3}+\frac{B}{(x-3)^2}$ $\rightarrow$ (i)

Multiply by $(x-3)^2$ on both sides

$x=A(x-3)+B$

$x=Ax-3A+B$

$x+0=Ax-3A+B$

By comparing coefficient of alike powers of "x"

$1=A \quad \rightarrow(\text{ii})$

$0=-3A+B \quad \rightarrow(\text{iii})$

$A=1$ Put in equation (iii)

$0=-3(1)+B$

$0=-3+B$

Or $B=3$

Now putting the values in equation (i)

$$\frac{x}{(x-3)^2}=\frac{1}{x-3}+\frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$?

(A.B + K.B + U.B)

Solution:

Let $\frac{x}{(x+a)(x-a)}=\frac{A}{x+a}+\frac{B}{x-a}$

Multiplying by L.C.M i.e. $(x+a)(x-a)$

$$(x+a)(x-a)\frac{x}{(x+a)(x-a)}$$

$$=(x+a)(x-a)\frac{A}{x+a}+(x+a)(x-a)\frac{B}{x-a}$$

$x=A(x-a)+B(x+a)\rightarrow(\text{i})$

Put $x=-a$ in equation (i)

$-a=A(-a-a)+B(-a+a)$

$-a=A(-2a)+B(0)$

$-a=-2aA$

$A=\frac{-a}{-2a}$

$A=\frac{1}{2}$

Put $x=a$ in equation (i)

$a=A(a-a)+B(a+a)$

$a=0+B(2a)$

$a=2aB$

$B=\frac{a}{2a}$

$B=\frac{1}{2}$

$$\therefore \frac{x}{(x+a)(x-a)}=\frac{1}{2(x+a)}+\frac{1}{2(x-a)}$$

(x) Whether $(x+3)^2=x^2+6x+9$ is an identity? (A.B + K.B + U.B)

$(x+3)^2=x^2+6x+9$

Let $x=1$

$(1+3)^2=(1)^2+6(1)+9$

$(4)^2=1+6+9$

$16=16 \quad (True)$

Let $x=25$

$(2+3)^2=(2)^2+6(2)+9$

$(5)^2=4+12+9$

$25=25 \quad (True)$

Hence:

Given equation is an identity because it is true for all values of variable 'x'