Some Important Sets

Natural Numbers (D.G.K 2018) (K.B)

The numbers $1,2,3,...$ which we use for counting certain objects are called natural numbers or positive integers.
It is denoted by N.
i.e. $N=\{1,2,3,...\}$

Whole Numbers (RWP 2019) (K.B)

If we include 0 in the set of natural number, the resulting set is called set of Whole Numbers.
It is denoted by W.
i.e., $W=\{0,1,2,3,...\}$

Integers (RWP 2019) (K.B)

The set of integers consist of positive counting numbers, 0 and negative counting numbers.
It is denoted by Z.
i.e. $Z=\{...,-3,-2,-1,0,1,2,3,...\}$
or $Z=\{0,\pm1,\pm2,\pm3,...\}$

Rational Numbers (K.B)

All numbers of the form $\frac{p}{q}$ where p,q are integers and q is not zero are called rational numsbers. **For example**, $\frac{2}{3},-\frac{5}{4}$ etc.
It is denoted by Q.
i.e. $Q=\left\{\frac{p}{q}\mid p,q\in Z\wedge q\neq 0\right\}$

Irrational Numbers (K.B)

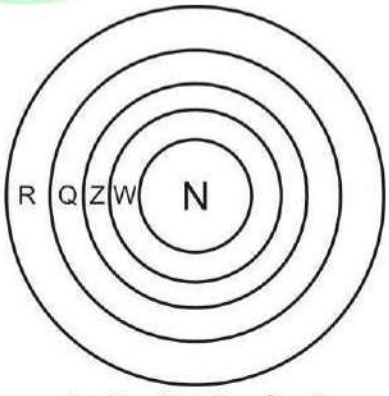
The numbers which cannot be expressed as $\frac{p}{q}$, where p and q are integers are called irrational numbers. **For example**, $\pi,\sqrt{3}$ etc.
It is denoted by Q'.
i.e., $Q'=\left\{x\mid x\neq\frac{p}{q},p,q\in Z\wedge q\neq 0\right\}$

Real Numbers (K.B)

The union of the set of rational numbers and irrational numbers is known as the set of real numbers.
It is denoted by R,
i.e., $R=Q\cup Q'$

Note (U.B)

(i) $N\subset W\subset Z\subset Q$



i.e. $N\subset W\subset Z\subset Q\subset R$

a. are disjoint sets

(ii) For each prime number p, $\sqrt{p}$ is an irrational number

(iii) Square roots of all positive non-square integers are irrational

Types of Decimal Fraction (K.B)

There are three types of decimal fractions:

(i) Terminating decimal fractions

(ii) Recurring and non-terminating decimal fractions

(iii) Non-terminating and non- recurring decimal fraction

Types of Rational Numbers (K.B)

There are two types of rational numbers:

(i) Terminating decimal fractions

(ii) Recurring and non-terminating decimal fractions

Terminating Decimal Fractions (K.B)

The decimal fraction in which there are finite number of digits in its decimal part is called a **terminating decimal** fraction.

For example: $\frac{2}{5} = 0.4$, $\frac{3}{8} = 0.375$ etc.

Recurring and Non-terminating Decimal Fractions (K.B)

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called **recurring and non-terminating** decimal fraction.

For example:

$\frac{2}{9} = 0.2222..., \frac{4}{11} = 0.363636...$ etc.

Non-Recurring and Non-terminating Decimal Fractions (K.B)

The decimal fraction (non-terminating) in which some digits are not repeated again and again in the same order in its decimal part is called **non-recurring and non-terminating** decimal fraction.

These numbers are also called **irrational numbers**.

For example:

$\sqrt{2} = 1.414213..., \pi = 3.141592...$ etc.

Representation of Real Numbers on Number Line (K.B)

The real number are represented geometrically by points on a number line ℓ Such that each real number 'a' corresponds to one and only one point on number line ℓ and to each point p on number line ℓ there corresponding precisely one real number.

Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers? **(U.B)**

Part #	Number	Type
(i)	$\sqrt{3}$	Irrational number
(ii)	$\frac{1}{6}$	Rational number
(iii)	π	Irrational number
(iv)	$\frac{15}{2}$	Rational number
(v)	7.25	Rational number
(vi)	$\sqrt{29}$	Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$ **(U.B)**

Solution: $\frac{17}{25}$

$$25\overline{)170}\ (0.68$$
$$-150$$
$$200$$
$$-160$$
$$40$$

$\Rightarrow \frac{17}{25} = 0.68$ **Ans**

(ii) $\frac{19}{4}$ **(A.B)**

Solution: $\frac{19}{4}$

$$4\overline{)19.000}\ (4.75$$
$$16$$
$$30$$
$$28$$
$$20$$
$$20$$
$$0$$

$\Rightarrow \frac{19}{4} = 4.75$ **Ans**

(iii) $\frac{57}{8}$ **(A.B)**

Solution: $\frac{57}{8}$

$$8\overline{)57}\ (7.125$$
$$-56$$
$$10$$
$$8$$
$$20$$
$$-16$$
$$40$$
$$40$$
$$0$$

$\Rightarrow \frac{57}{8} = 7.125$ **Ans**

(iv) $\frac{205}{18}$ **(A.B)**

Solution: $\frac{205}{18}$

$$18\overline{)205.000}\ (11.388$$
$$25$$
$$18$$
$$70$$
$$-54$$
$$160$$
$$-144$$
$$160$$
$$-144$$
$$16$$
$$208$$
$$18$$

$\Rightarrow \frac{205}{18} = 11.3889$ **Ans**

(v) $\frac{5}{8}$ **(A.B)**

Solution: $\frac{5}{8}$

$$8\overline{)5.000}\quad 0.625$$
$$48$$
$$20$$
$$-16$$
$$40$$
$$-40$$
$$0$$

$\Rightarrow \frac{5}{8} = 0.625$ **Ans**

(vi) $\frac{25}{38}$ **(A.B)**

Solution: $\frac{25}{38}$

$$38\overline{)250}\quad 0.65789\ldots$$
$$-228$$
$$220$$
$$-190$$
$$300$$
$$-266$$
$$340$$
$$-304$$
$$360$$
$$-342$$
$$18$$

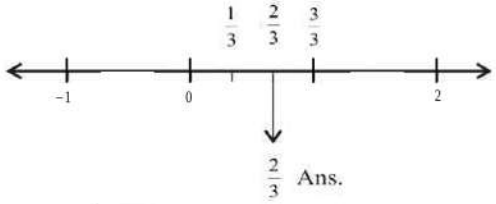
$\Rightarrow \frac{25}{38} = 0.65789$ **Ans**

Q.3 Which of the following statements are true and which are false? **(U.B)**

Part	Statement	T/F
(i)	$\frac{2}{3}$ is an irrational number	False
(ii)	π is an irrational number	True
(iii)	$\frac{1}{9}$ is a terminating fraction	False
(iv)	$\frac{3}{4}$ is a terminating fraction	True
(v)	$\frac{4}{5}$ is a recurring fraction	False

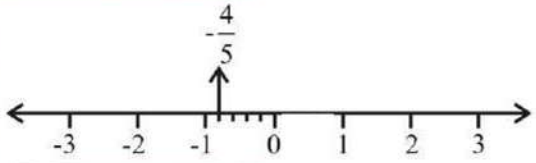
Q.4 Represent the following numbers on the number line.

(i) $\frac{2}{3}$ **(A.B)**

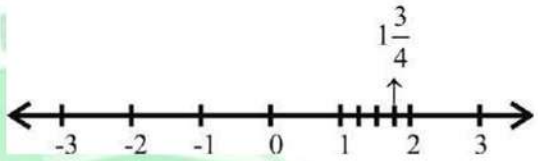


$\frac{2}{3}$ Ans.

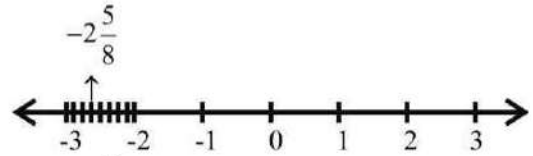
(ii) $-\frac{4}{5}$ **(A.B)**



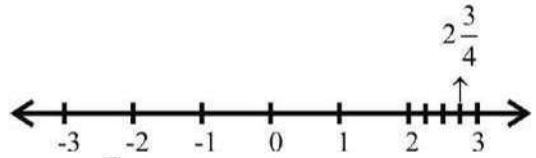
(iii) $1\frac{3}{4}$ **(A.B)**



(iv) $-2\frac{5}{8}$ **(A.B)**



(v) $2\frac{3}{4}$ **(SWL 2019)** **(A.B)**

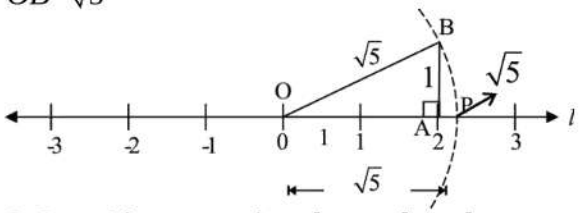


(vi) $\sqrt{5}$ **(A.B)**

$= \sqrt{4+1} = \sqrt{2^2+1^2}$

$(\text{Hypoteneus})^2 = (\text{Base})^2 + (\text{Perpencicular})^2$

$\overline{OB} = \sqrt{5}$



Q.5 Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$ **(A.B)**

(LHR 2019, SGD 2017)

Solution:

Rational number between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

$$= \left[\frac{3}{4} + \frac{5}{9}\right] \div 2$$

$$= \left[\frac{27+20}{36}\right] \div 2$$

$$= \frac{47}{36} \times \frac{1}{2}$$

$$= \frac{47}{72}$$

Q.6 Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integer and $q \neq 0$.

(i) $0.\overline{5}$ **(A.B)**

Solution:

Suppose

$x = 0.\overline{5}$

$x = 0.555...$

Multiplying both sides by 10

$10 \times x = 10 \times 0.555...$

$10x = 5.555...$

$10x = 5 + 0.555...$

$10x = 5 + x$

$10x - x = 5$

$9x = 5$

$x = \frac{5}{9}$

$\therefore \quad 0.\overline{5} = \frac{5}{9}$

(ii) $0.\overline{13}$ **(RWP 2019, D.G.K 2017)** **(A.B)**

Solutions:

Suppose

$x = 0.\overline{13}$

$x = 0.131313...$

Multiplying both sides by 100

$100x = 100 \times 1.131313...$

$100x = 13.1313...$

$100x = 13 + 0.1313...$

$100x = 13 + x$

$100x - x = 13$

$99x = 13$

$x = \frac{13}{99}$

$\therefore \quad 0.\overline{13} = \frac{13}{99}$

(iii) $0.\overline{67}$ **(A.B)**

Solutions:

Suppose

$x = 0.\overline{67}$

$x = 0.676767...$

Multiplying both sides by 100

$100 \times x = 100 \times 0.676767...$

$100x = 67.6767...$

$100x = 67 + 0.6767...$

$100x = 67 + x$

$100x - x = 67$

$99x = 67$

$x = \frac{67}{99}$

$\therefore \quad 0.\overline{67} = \frac{67}{99}$