Mathematics-9
Exercise 2.2

Properties of Real Numbers under Addition

(i) Closure property (K.B)

$\forall\ a,b \in R$

$a+b \in R$

For example:

If -3 and $5 \in R$

Then $-3+5=2 \in R$

(ii) Commutative Property (K.B)

$\forall\ a,b \in R$

$a+b=b+a$

For example:

If 2 and $3 \in R$

Then $2+3=3+2$

or $5=5$

(iii) Associative Property (K.B)

$\forall\ a,b,c \in R$

$a+b+c=a+(b+c)$

For example:

If $5,7,3 \in R$

Then $(5+7)+3=5+(7+3)$

Or $12+3=5+10$

$15=15$

(iv) Additive Identity (K.B)

The exists a unique real number 0, called additive identity such that

$a+0=a=0+a,\ \forall a \in R$

For example:

If $5,0 \in R$

Then $5+0=0+5=5$

(v) Additive Inverse (K.B)

For ever $a \in R$ there exists a unique real number –a, called additive inverse of a such that

$a+(-a)=0=(-a)+a$

For example:

Additive inverse of 3 is –3

Since

$3+(-3)=0=(-3)+3$

Properties of Real Numbers under Multiplication

(i) Closure property (K.B)

$\forall\ a,b \in R$

$ab \in R$

For example:

If -3 and $5 \in R$

Then $(-3)(5)=-15 \in R$

(ii) Commutative Property (K.B)

$\forall\ a,b \in R$

$ab=ba$

For example:

If $\frac{1}{3},\frac{3}{2} \in R$

Then $\left(\frac{1}{3}\right)\left(\frac{3}{2}\right)=\left(\frac{3}{2}\right)\left(\frac{1}{3}\right)$

Or $\frac{1}{2}=\frac{1}{2}$

(iii) Associative Property (K.B)

$\forall\ a,b,c \in R$

$(ab)c=a(bc)$

For example:

If $2,3,5 \in R$

Then $(2\times3)\times5=2\times(3\times5)$

Or $6\times5=2\times15$

Or $30=30$

(iv) Multiplicative Identity (K.B)

The exists a unique real number 1, called the multiplicative identity such that

$a.1 = a = 1,\ \forall a \in R$

For example:

If $5, 1 \in R$

Then $5\times1 = 1\times5 = 5$

(v) Multiplicative Inverse (K.B)

For every non zero real number, there exists a unique real number a^{-1} or $\frac{1}{a}$, called multiplicative inverse of a, such that

$aa^{-1} = 1 = a^{-1}a$

$a\times\frac{1}{a} = 1 = \frac{1}{a}\times a$

For example:

If $5, \frac{1}{5} \in R$

Then $5\times\frac{1}{5} = 1 = \frac{1}{5}\times5$

So 5 and $\frac{1}{5}$ are multiplicative inverse of each other.

Multiplication is Distributive over Addition and Subtraction (K.B)

$\forall\ a, b, c \in R$

$a(b+c) = ab + ac$ (Left distributive law)

$(a+b)c = ac + bc$ (Right distributive law)

For example:

If $2, 3, 5 \in R$ then

$2(3+5) = 2\times3 + 2\times5$

Or $2(8) = 6 + 10$

Or $16 = 16$

And for all $a, b, c \in R$

$a(b-c) = ab - ac$ (Left distributive law)

$(a-b)c = ac - bc$ (Right distributive law)

For example:

If $2, 5, 3 \in \text{R}$ then

$2(5-3) = 2\times5 - 2\times3$

Or $2\times2 = 10 - 6$

Or $4 = 4$

Note (K.B +U.B)

(i) The symbol $\forall$ means "for all"

(ii) a is the multiplicative inverse of a^{-1}

i.e. $a = \left(a^{-1}\right)^{-1}$

(iii) If a, b are real number their sum is written as $a+b$ and product as ab or $a\times b$ or $a.b$ or (a)(b).

Properties of Equality of Real Number

(i) Reflexive Property (K.B)

$a = a \quad \forall a \in R$

For example:

2=2

(ii) Symmetric Property (K.B)

$\forall a, \text{b} \in \text{R}$

If $a = b$, then $b = a$

For example:

If $2 = x$, then $x = 2$

(iii) Transitive Property (K.B)

$\forall\ a, \text{b}, \text{c} \in \text{R}$

If $a = b$ and $b = c$ then $a = c$

For example:

If $x = 2$ and $y = 2$ then $x = y$

(iv) Additive Property (K.B)

$\forall a, b, c \in R$

If $a = b$, then $a + c = b + c$

For example:

If $2 = 2$, then $2 + 3 = 2 + 3$

(v) Multiplicative Property (K.B)

$\forall a, b, c \in R$

If $a = b$, then $ac = bc$

For example:

If $2 = 2$, then $2\times3 = 2\times3$

(vi) Cancellation Property for Addition (K.B)

$\forall a,b,c \in R$

If $a+c=b+c$, then $a=b$

For example:

If $x+2=y+2$, then $x=y$

(vii) Cancellation Property of Multiplication (K.B)

$\forall a,b,c \in R$

If $ac=bc, c\neq 0$ then $a=b$

For example:

If $2x=8$, then $x=4$

Properties of Inequalities of Real Numbers

(i) Trichotomy Property (K.B)

$\forall\ a,\mathrm{b}\in \mathrm{R}$

$a<b$ or $a=b$ or $a>b$

For example:

$x<0$ or $x=0$ or $x>0$

Any one statement is true, not all.

(ii) Transitive Property (K.B)

$\forall\ a,\mathrm{b},\mathrm{c}\in \mathrm{R}$

(a) $a<b$ and $b<c \Rightarrow a<c$

(b) $a>b$ and $b>c \Rightarrow a>c$

For example:

$5<6$ and $6<8 \Rightarrow 5<8$

Or $8>6$ and $6>5 \Rightarrow 8>5$

(iii) Additive Property (K.B)

$\forall\ a,\mathrm{b},\mathrm{c}\in \mathrm{R}$

(a) $a<b \Rightarrow a+c<b+c$ and
$a<b \Rightarrow c+a<c+b$

(b) $a>b \Rightarrow a+c>b+c$
$a>b \Rightarrow c+a>c+b$

For example:

$5<6 \Rightarrow 5+10<6+10$

Or $20>10 \Rightarrow 20+5>10+5$

(iv) Multiplicative Property (K.B)

$\forall\ a,\mathrm{b},\mathrm{c}\in \mathrm{R}$

Case: 1 $c>0$

(a) $a>b \Rightarrow ca>cb$
$a>b \Rightarrow ac>cb$

(b) $a<b \Rightarrow ac<bc$
$a<b \Rightarrow ca<cb$

For example:

$5>2 \Rightarrow 5\times 4>2\times 4$

Case: 2 $c<0$

(a) $a>b \Rightarrow ac<bc$
$a>b \Rightarrow ca<bc$

(b) $a<b \Rightarrow ac>bc$
$a<b \Rightarrow ca>cb$

For example:

$5>2 \Rightarrow -4\times 5<-4\times 2$

i.e $-20<-8$

(v) Multiplicative Inverse Property (K.B)

$\forall a,b\in R$ and $a\neq 0, b\neq 0$

(a) $a<b \Leftrightarrow \frac{1}{a}>\frac{1}{b}$

(b) $a>b \Leftrightarrow \frac{1}{a}<\frac{1}{b}$

For example:

$1<5 \Leftrightarrow \frac{1}{1}>\frac{1}{5}$

i.e. $1>0.2$

Exercise 2.2

Q.1 Identify the property used in the following: (A.B+K.B+U.B)

Solution:

Part #	Statement	Property
(i)	$a+b=b+a$	Commutative Property *w.r.t* +
(ii)	$(ab)c=a(bc)$	Associative Property *w.r.t* $\times$
(iii)	$7\times1=7$	Multiplicative Identity
(iv)	$x>y$ or $x=y$ or $x<y$	Trichotomy
(v)	$ab=ba$	Commutative *w.r.t* $\times$
(vi)	$a+c=b+c \Rightarrow a=b$	Cancellation Property of +
(vii)	$5+(-5)=0$	Additive Inverse
(viii)	$7\times\frac{1}{7}=1$	Multiplicative Inverse
(ix)	$a>b \Rightarrow ac>bc\,(c>0)$	Multiplicative

Q.2 Fill in the following blanks by stating the properties of real numbers used. (K.B+U.B)

Solution:

$3x+3(y-x)$

$=3x+3y-3x$ **Distributive property**

$=3x-3x+3y$ **Commutative *w.r.t* +**

$=0+3y$ **Additive Inverse**

$=3y$ **Additive identity**

Q.3 Give the name of property used in the following:

Solution:

(i) $\sqrt{24}+0=\sqrt{24}$ **(A.B)**

Ans. Additive Identity

(ii) $-\frac{2}{3}\left[5+\frac{7}{2}\right]=\left[-\frac{2}{3}\right](5)+\left[-\frac{2}{3}\right]\left[\frac{7}{2}\right]$ **(A.B)**

Ans. Distributive Property

(iii) $\pi+(-\pi)=0$ **(A.B)**

Ans. Additive Inverse

(iv) $\sqrt{3}.\sqrt{3}$ is a real number. **(A.B)**

Ans. Closure property w.r.t $\times$.

(v) $\left[-\frac{5}{8}\right]\left[-\frac{8}{5}\right]=1$ **(A.B)**

Ans. Multiplicative Inverse