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Mathematics-9
Exercise 2.4

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LAW OF EXPONENTS / INDICES

Base and Exponent **(K.B+U.B)**

In the exponential notation a^n (read as a to the nth power) we call 'a' as the base and 'n' as the exponent or the power to which the base is raised.

Laws of Exponents **(K.B+U.B)**

If $a, b \in R$ and m, n are positive integers, then

(i) $a^m.a^n = a^{m+n}$

(ii) $\left(a^m\right)^n = a^{mn}$

(iii) $(ab)^n = a^n b^n$

(iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

(v) $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

(vi) $a^0 = 1 \text{ where } a \neq 0$

(vii) $a^{-n} = \frac{1}{a^n} \text{ where } a \neq 0$

Example # 2 **(A.B)**

(ii) Simplify: $\frac{4(3)^n}{3^{n+1} - 3^n}$

Solution:

$$\frac{4(3)^n}{3^{n+1} - 3^n}$$

$$= \frac{4(3)^n}{3^n \times 3 - 3^n} \quad \because a^m.a^n = a^{m+n}$$

$$= \frac{4\cancel{(3)^n}}{\cancel{3^n}(3-1)} \quad \because \frac{a^n}{a^n} = 1$$

$$= \frac{4}{2} = 2$$

$$\Rightarrow \frac{4(3)^n}{3^{n+1} - 3^n} = 2$$

Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i) $\dfrac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$ **(A.B)**

Solution:

$$\frac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$$

$$=\frac{(3^5)^{-\frac{2}{3}}\times(2^5)^{-1/5}}{\sqrt{\left[(14)^2\right]^{-1}}} \quad \textbf{(Factorization)}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{\sqrt{\left[(14)^{-1}\right]^2}} \quad \because (a^m)^n=a^{mn}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{(14)^{-1}} \quad \because \sqrt[n]{a^n}=a$$

$$=\frac{7\times \cancel{2}}{(3)^{\frac{10}{3}}\times\cancel{2}} \quad \because a^{-n}=\frac{1}{a^n},\ \frac{a^n}{a^n}=1$$

$$=\frac{7}{3^{\frac{10}{3}}}$$

$$=\frac{7}{\sqrt[3]{3^{10}}} \quad \because \sqrt[n]{a}=(a)^{1/n}$$

$$=\frac{7}{\sqrt[3]{\overline{3\times3\times3}\times\overline{3\times3\times3}\times\overline{3\times3\times3}\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3\times3^3\times3^3\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3}} \quad \because \sqrt[n]{ab}=\sqrt[n]{a}\ \sqrt[n]{b}$$

$$=\frac{7}{3\times3\times3\times\sqrt[3]{3}} \quad \because \sqrt[n]{a^n}=a$$

$$=\frac{7}{27\sqrt[3]{3}}$$

$$\Rightarrow \frac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}}=\frac{7}{27\sqrt[3]{3}}$$

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$ **(A.B)**

Solution:

$$(2x^5y^{-4})(-8x^{-3}y^2)$$

$$=2(-8)x^5.x^{-3}.y^{-4}.y^2$$

$$=-16x^{5-3}y^{-4+2} \quad \because a^m.a^n=a^{m+n}$$

$$=-16x^2y^{-2}$$

$$=\frac{-16x^2}{y^2} \quad \because a^{-n}=\frac{1}{a^n}$$

$$\Rightarrow (2x^5y^{-4})(-8x^{-3}y^2)=\frac{-16x^2}{y^2}$$

(iii) $\left[\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right]^{-3}$ **(A.B)**

Solution:

$$\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right]^{-3}$$

$$=\left[x^{-2-4}y^{-1+3}z^{-4-0}\right]^{-3} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=(x^{-6}y^{+2}z^{-4})^{-3}$$

$$=(x^{-6})^{-3}(y^2)^{-3}(y^{-4})^{-3} \because (ab)^n=a^nb^n$$

$$=x^{18}y^{-6}z^{12} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=\frac{x^{18}z^{12}}{y^6} \quad \because a^{-n}=\frac{1}{a^n}$$

$$\Rightarrow \left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right]^{-3}=\frac{x^{18}z^{12}}{y^6}$$

(iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$ **(A.B)**

Solution:

$$\frac{(81)^n .3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

$$= \frac{(3^4)^n .3^5 - 3^{4n}.3^{-1}.3^5}{(3^2)^{2n}.3^3} \quad \text{(factorization)}$$

$$= \frac{3^{4n}.3^5 - 3^{4n}.3^{-1+5}}{3^{4n}.3^3} \quad \because (a^m)^n = a^{mn}$$

$$= \frac{3^{4n}.3^5 - 3^{4n}.3^4}{3^{4n}.3^3} \quad \because a^m.a^n = a^{m+n}$$

$$= \frac{3^{4n}.3^4(3-1)}{3^{4n}.3^3} \quad \text{(taking common)}$$

$$= 3^{4n-4n}.3^{4-3}.(2) \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= 3^0.3^1.2$$
$$= 1 \times 3 \times 2$$
$$= 6$$

Q.2 Show that

$$\left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a} = 1$$

(K.B+A.B+U.B)

(LHR 2018, 19, SGD 2017, SWL 2017)

Proof:

$$\text{L.H.S} = \left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a}$$

$$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \quad \because (a^m)^n = a^{mn}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-a^2+c^2-a^2} \quad \because a^m.a^n = a^{m+n}$$

$$= x^0 \quad \because a^0 = 1$$

$$= 1 = \text{R.H.S}$$

Proved

Q.3 Simplify

(i) $$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$ **(A.B)**

Solution:

$$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (2\times2\times3\times5)^{\frac{1}{2}}}{(2\times2\times3\times3\times5)^{\frac{1}{2}} \times (2^2)^{-\frac{1}{3}} \times (3^2)^{\frac{1}{4}}} \quad \text{(factorization)}$$

$$= \frac{2^{\frac{1}{3}} \times 3 \times (2^2)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}} \quad \because (a^m)^n = a^{mn},\ (ab)^n = a^n b^n$$

$$= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= 2^{\frac{1}{3}+1-1+\frac{2}{3}} \times 3^{1+\frac{1}{2}-1-\frac{1}{2}} \times 5^{\frac{1}{2}-\frac{1}{2}} \quad \because \frac{a^m}{a^n} = a^{m-n}, \because a^m.a^n = a^{m+n}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 3^0 \times 5^0$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 1 \times 1 \quad \because a^0 = 1$$

$$= 2^{\frac{3}{3}}$$

$$= 2$$

$$\Rightarrow \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}} = 2$$

(ii) Simplify: $$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$ **(A.B)**

(GRW 2019, RWP 2018, 19)

Solution:

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

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$$=\sqrt{\frac{\left(6^2\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}\quad \text{(factorization)}$$

$$=\sqrt{\frac{6^2\times 5}{\left(\frac{^{25}\cancel{100}}{\cancel{4}}\right)^{\frac{1}{2}}}}\quad \because a^{-n}=\frac{1}{a^n},\left(a^m\right)^n=a^{mn}$$

$$=\sqrt{\frac{6^2\times 5}{(25)^{\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^2\times 5}{\left(5^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}}}\quad \because \left(a^m\right)^n=a^{mn}$$

$$=\sqrt{\frac{6^2\times 5}{5}}\quad \because \frac{a^n}{a^n}=1$$

$$=\sqrt{6^2}$$

$$=6\quad \because \sqrt[n]{a^n}=a$$

$$\Rightarrow \sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}=6$$

(iii) $5^{2^3}\div\left(5^2\right)^3$ **(A.B)**

(LHR 2018, 21, GRW 2017, 21, SWL 2019, FSD 2021, SGD 2017, 21)

Solution:

$$5^{2^3}\div\left(5^2\right)^3$$

$$=5^8\div 5^6\quad \because \left(a^m\right)^n=a^{mn}$$

$$=5^{8-6}\quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=5^2$$

$$=25$$

$$\Rightarrow 5^{2^3}\div\left(5^2\right)^3=25$$

(iv) $\left(x^3\right)^2\div x^{3^2}, x\neq 0$ **(A.B)**

(LHR 2017, FSD 2017, SWL 2017, D.G.K 2018)

Solution:

$$\left(x^3\right)^2\div x^{3^2}$$

$$=x^6\div x^9\quad \because \left(a^m\right)^n=a^{mn}$$

$$=x^{6-9}\quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=x^{-3}$$

$$=\frac{1}{x^3}\quad \because a^{-n}=\frac{1}{a^n}$$

$$\Rightarrow \left(x^3\right)^2\div x^{3^2}=\frac{1}{x^3}$$