Mathematics-9

Exercise 2.5

Need of Complex Numbers **(K.B)**

Since square of a real number is non-negative. So the solution of the equation $x^2+1=0$ or $x^2=-1$ does not exist in R. To overcome this inadequacy of real numbers, we need a number whose square is –1. Thus the mathematicians were tempted to introduce a larger set of numbers called the set of complex numbers which contains R and every number whose square is negative. They invented a new number $\sqrt{-1}$, called the imaginary unit, and denoted it by the letter *i* (iota) having the property that $i^2=-1$.

Invention of iota (Complex Number) **(K.B)**

The swiss mathematician Leonard Euler $(1707-1783)$ was the first to use the symbol i for the number $\sqrt{-1}$.

Note **(U.B+K.B)**

Number like $\sqrt{-1}, \sqrt{-5}$ etc are called pure imaginary numbers

Definition of a Complex Number **(U.B)**

A number of the form $a+bi$ where a and b are real number and $i=\sqrt{-1}$, is called a complex number and is represented by *z*.

i.e. $z=a+ib$

Set of Complex Number **(U.B)**

The set of all complex number is denoted by C.

$$C=\left\{z \mid z=a+bi, where\ a,b \in R\ and\ i=\sqrt{-1}\right\}$$

Note **(U.B+K.B)**

(i) The numbers a and b, called the real and imaginary parts of Z are denoted as $a=\text{Re}(Z)$ and $b=Im(Z)$

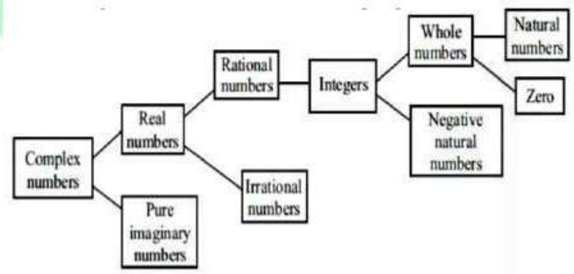
(ii) Every $a \in R$ may be identified with complex numbers of the form $a+0i$ taking b = 0. Therefore, every real number is also a complex number. Thus $R \subset C$.

(iii) Every complex number is not a real number.

(iv) If a = 0, then a + bi reduces to a purely imaginary number bi. The set of purely imaginary numbers is also contained in C.

(v) If $a=b=0$, then $z=0+i0$ is called the complex number 0.

The set of complex numbers is shown in the following diagram: **(U.B+K.B)**



Conjugate of a Complex Number **(U.B+K.B)**

If we change i to $-i$ in $z=a+bi$ we obtain another complex number $a-bi$ called the complex conjugate of *z* and is denoted by $\bar{z}$ (read z bar)

Thus if $z=-1-i$, then $\bar{z}=-1+i$

The number $a+bi$ and $a-bi$ are called conjugates of each other.

Note **(U.B+K.B)**

(i) $\bar{\bar{z}} = z$

(ii) The conjugate of a real number $z = a + 0i$ coincides with the number itself since $\bar{z} = \overline{a + 0i} = a - 0i = a$.

The Equality of Complex Number

(U.B)

For all $a, b, c, d \in R$,

$a + bi = c + di$ If and only if $a = c$ and $b = d$

e.g., $2x + y^2 i = 4 + 9i$ if and only if

$2x = 4$ and $y^2 = 9$, i.e, $x = 2$ and $y = \pm 3$

Properties of Complex Number

(U.B)

Properties of real number R are also valid for the set of Complex numbers

(i) $z_1 = z_1$ **(Reflexive Law)**

(ii) If $z_1 = z_2$ then $z_2 = z_1$ **(Symmetric Law)**

(iii) If $z_1 = z_2$ and $z_2 = z_3$ then $z_1 = z_3$ **(Transitive Law)**

Exercise 2.5

Q.1 Evaluate

(i) i^7 **(A.B)**

Solution:

i^7
$= i^6.i$
$= (i^2)^3.i$
$= (-1)^3.i \quad \because i^2 = -1$
$= -1 \times i$
$= -i$

(ii) i^{50} **(A.B)**
(LHR 2021, FSD 2017, 21, SWL 2018, SGD 2018, BWP 2017, 21)

Solution:

i^{50}
$= (i^2)^{25}$
$= (-1)^{25} \quad \because i^2 = -1$
$= -1$

(iii) i^{12} **(LHR 2017)** **(A.B)**

Solution:

i^{12}
$= (i^2)^6$
$= (-1)^6 \quad \because i^2 = -1$
$= 1$

(iv) $(-i)^8$ **(A.B)**
(RWP 2021, MTN 2021, SWL 2021, D.G.K 2017)

Solution:

$(-i)^8$
$= i^8$
$= (i^2)^4$
$= (-1)^4 \quad \because i^2 = -1$
$= 1$

(v) $(-i)^5$ **(FSD 2017)** **(A.B)**

Solution:

$(-i)^5$
$= -i^5$
$= -i^4.i$
$= -(i^2)^2.i$
$= -(-1)^2.i \quad \because i^2 = -1$
$= -(1)(i)$
$= -i$

(vi) i^{27} **(A.B)**

Solution:

i^{27}
$= i^{26}.i$
$= (i^2)^{13}.i$
$= (-1)^{13}.i \quad \because i^2 = -1$
$= -1.i$
$= -i$

Q.2 Write the conjugate of the following numbers. **(K.B)**

Part #	Complex Number	Conjugate of Number
(i)	$2+3i$	$2-3i$
(ii)	$3-5i$	$3+5i$
(iii)	$-i$	i
(iv)	$-3+4i$	$-3-4i$
(v)	$-4-i$	$-4+i$
(vi)	$i-3$	$-i-3$

Q.3 Write the real and imaginary part of the following numbers. **(K.B)**

Part #	Complex Number	Real Part	Imaginary Part
(i)	$1+i$	1	1
(ii)	$-1+2i$	-1	2

(iii)	$-2-2i$	-2	-2
(iv)	$-3i$	0	-3
(v)	$2+0i$	2	0

Q.4 Find the value of x and y if $x+iy+1=4-3i$ **(A.B)**

(GRW 2017, FSD 2016, RWP 2017, 18, MTN 2019, D.G.K 2016, BWP 2021)

Solution:

Here

$x+iy+1=4-3i$

$x+iy=4-3i-1$

$x+iy=3-3i$

By comparing real and imaginary part, we get

$x=3$ and $y=-3$