# Mathematics-9
## Exercise 2.6



## Basic Operations on Complex Numbers

### (i) Addition (K.B)

Let $z = a + ib$ *and* $z_2 = c + id$ be two complex numbers and $a, b, c, d \in R$

Then

$$z_1 + z_2 = (a + bi) + (c + di)$$
$$= (a + c) + (b + d)i$$

i.e., the sum of two complex numbers is the sum of the corresponding real and the imaginary parts.

**For example:**

$$(3 - 8i) + (5 + 2i)$$
$$= (3 + 5) + (-8 + 2)i$$
$$= 8 - 6i$$

### (ii) Multiplication (K.B)

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d \in R$.

Then

**(a) Multiplication of a complex number with a scalar**

If $k \in R$, then $kz_1 = k(a + bi) = ka + kbi$

**For example:**

If $z = 23 - 2i$

Then $5z = 5(3 - 2i)$

$$= 15 - 10i$$

**(b) Multiplication of two complex numbers**

$$z_1 z_2 = (a + bi)(c + di)$$
$$= (ac - bd) + (ad + bc)i$$

The multiplication of any two complex numbers $(a + bi)$ and $(c + di)$ is explained as

$$z_1 z_2 = (a + bi)(c + di) = a(c + di) + bi(c + di)$$
$$= ac + adi + bci + bdi^2$$
$$= ac + adi + bci + bd(-1) \because i^2 = -1$$
$$= (ac - bd) + (ad + bc)i \quad \text{(combining like terms)}$$

**For example:**

$$(2 - 3i)(4 + 5i)$$
$$= 8 + 10i - 12i - 15i^2$$

$= 23 - 2i \qquad \because i^2 = -1$

**(iii) Subtraction** **(K.B)**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers

$z_1 - z_2 = (a + bi) - (c + di)$

$= (a - c) + (b - d)i$

i.e., the difference of two complex number is the difference of the corresponding real and imaginary parts.

**For example:**

$(-2 + 3i) - (2 + i)$

$= (-2 - 2) + (3 - 1)i$

$= -4 + 2i$

**(iv) Division** **(K.B+A.B)**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers such that $z_2 \neq 0$

Then

$$\frac{z_1}{z_2} = \frac{a + bi}{c + di}$$

(Multiplying the numerator and denominator by $c - di$, the complex conjugate of $c + di$)

$$= \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$$

$$= \frac{ac + bci - adi - bdi^2}{c^2 - (di)^2}$$

$$= \frac{ac + bci - adi + bd}{c^2 + d^2}, \quad \because i^2 = -1$$

$$= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + \left(\frac{bc - ad}{c^2 + d^2}\right)i$$

**Example # 4**

Solve $(3 - 4i)(x + yi) = 1 + 0.i$ for real numbers $x$ and $y$, where $i = \sqrt{-1}$ **(A.B)**

**Solution:**

Here

$(3 - 4i)(x + yi) = 1 + 0.i$

$(3 - 4i)(x + yi) = 1 + 0 = 1$

$$x+yi=\frac{1}{3-4i}$$

$$x+yi=\frac{1}{3-4i}\times\frac{3+4i}{3+4i}$$

$$=\frac{3-4i}{(3)^2-(4i)^2}$$

$$=\frac{3-4i}{9-16(-1)} \quad \because i^2=-1$$

$$=\frac{3-4i}{9+16}$$

$$=\frac{3-4i}{25}$$

$$=\frac{3}{25}-\frac{4}{25}i$$

Equating the real and imaginary part, we obtain

$$x=\frac{3}{25} \quad \text{and} \quad y=\frac{4}{25}$$

$\because$ Solution Set $=\left\{\left(\frac{3}{25},\frac{4}{25}\right)\right\}$

## Exercise 2.6

**Q.1 Identify the following statement as true or false.**

**(i)** $\sqrt{-3}\sqrt{-3}=3$ **False** **(K.B)**

**(ii)** $i^{73}=-i$ **False** **(K.B)**

**(iii)** $i^{10}=-1$ **True** **(K.B)**

**(iv)** **Complex conjugate of** $\left(-6i+i^2\right)$ **is** $\left(-1+6i\right)$ **True** **(K.B)**

**(v)** **Difference of a complex number** $z=a+bi$ **and its conjugate is a real number. False** **(K.B)**

**(vi)** **If** $(a-1)-(b+3)i=5+8i,$ **then** $a=6$ **and** $b=-11.$ **True** **(K.B)**

**(vii)** **Product of a complex number and its conjugate is always a non-negative real number. True** **(K.B)**

**Q.2 Express the each complex number in the standard form** $a+bi$**, where** $a$ **and** $b$ **are real number.**

**(i)** $(2+3i)+(7-2i)$ **(K.B)**

**Solution:**

$$(2+3i)+(7-2i)$$
$$=2+3i+7-2i$$
$$=2+7+3i-2i$$
$$=9+i$$
$$\Rightarrow (2+3i)+(7-2i)=9+i$$

**(ii)** $2(5+4i)-3(7+4i)$ **(A.B)**

**Solution:**

$$2(5+4i)-3(7+4i)$$
$$=10+8i-21-12i$$
$$=10-21+8i-12i$$
$$=-11-4i$$
$$\Rightarrow 2(5+4i)-3(7+4i)=-11-4i$$

**(iii)** $(-3+5i)-(4+9i)$ **(A.B)**

**Solution:**

$$(-3+5i)-(4+9i)$$
$$=+3-5i-4-9i$$
$$=3-4-5i-9i$$
$$=-1-14i$$
$$\Rightarrow (-3+5i)-(4+9i)=-1-14i$$

**(iv)** $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$ **(A.B)**

**Solution:**

$$2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$$
$$=2(-1)+6i^2-i+3\left(i^2\right)^8-6\left(i^2\right)^9.i+4\left(i^2\right)^{12}.i$$
$$\because i^2=-1$$
$$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$$
$$=-2-6i+3-6(-1)i+4(+1)i$$
$$=1-\cancel{6i}+\cancel{6i}+4i$$
$$=1+4i$$
$$\Rightarrow 2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}=1+4i$$

**Q.3 Simplify and write your answer in the form** $a+bi$

**(i)** $(-7+3i)(-3+2i)$ **(A.B)**

**(GRW 2017, FSD 2019, SWL 2017, BWP 2019, D.G.K 2017)**

**Solution:**

$$(-7+3i)(-3+2i)$$
$$=-7(-3+2i)+3i(-3+2i)$$
$$=21-14i-9i+6i^2$$
$$=21-23i+6(-1)$$
$$=21-23i-6$$
$$=21-6-23i$$
$$=15-23i$$
$$\Rightarrow (-7+3i)(-3+2i)=15-23i$$

**(ii)** $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$ **(A.B)**

**Solution:**

$$\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$$
$$=\left(2-\sqrt{4\times-1}\right)\left(3-\sqrt{4\times-1}\right)$$
$$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$$
$$=\left(2-\sqrt{2^2i^2}\right)\left(3-\sqrt{2^2i^2}\right)$$
$$=(2-2i)(3-2i)$$
$$=2(3-2i)-2i(3-2i)$$
$$=6-4i-6i+4i^2$$
$$=6-10i+4(-1)$$
$$=6-10i-4$$
$$=2-10i$$

$$\Rightarrow \left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)=2-10i$$

**(iii)** $\left(\sqrt{5}-3i\right)^2$ **(A.B)**

**Solution:**

$$\left(\sqrt{5}-3i\right)^2$$
$$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$$
$$=5+9i^2-6\sqrt{5}i$$
$$=5+9(-1)-6\sqrt{5}i$$
$$=5-9-6\sqrt{5}i$$
$$=-4-6\sqrt{5}i$$
$$\Rightarrow \left(\sqrt{5}-3i\right)^2=-4-6\sqrt{5}i$$

**(iv)** $(2-3i)\left(\overline{3-2i}\right)$ **(A.B)**

**Solution:**

$$(2-3i)\left(\overline{3-2i}\right)$$
$$=(2-3i)(3+2i)$$
$$=2(3+2i)-3i(3+2i)$$
$$=6+4i-9i-6i^2$$
$$=6-5i-6(-1)$$
$$=6-5i+6$$
$$=6+6-5i$$
$$=12-5i$$
$$\Rightarrow (2-3i)\left(\overline{3-2i}\right)=12-5i$$

**Q.4 Simplify and write your answer in the form** $a+bi$.

**(i)** $\frac{-2}{1+i}$ **(FSD 2019, D.G.K 2017) (A.B)**

**Solution:**

$$\frac{-2}{1+i}$$
$$=\frac{-2}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{-2(1-i)}{(1)^2-(i)^2}$$
$$=\frac{-2+2i}{1-i^2}$$
$$=\frac{-2+2i}{1-(-1)}$$
$$=\frac{-2+2i}{1+1}$$
$$=\frac{-2+2i}{2}$$
$$=-\frac{2}{2}+\frac{2i}{2}$$
$$=-1+i$$
$$\Rightarrow \frac{-2}{1+i}=-1+i$$

**(ii)** $\frac{2+3i}{4-i}$ **(A.B)**

**Solution:**

$$\frac{2+3i}{4-i}$$
$$=\frac{2+3i}{4-i}\times\frac{4+i}{4+i}$$
$$=\frac{(2+3i)(4+i)}{(4)^2-(i)^2}$$
$$=\frac{2(4+i)+3i(4+i)}{16-(-1)}$$
$$=\frac{8+2i+12i+3i^2}{16+1}$$
$$=\frac{8+4i+3(-1)}{17}$$
$$=\frac{8+14i-3}{17}$$
$$=\frac{8-3+14i}{17}$$
$$=\frac{5+14i}{17}$$
$$=\frac{5}{17}+\frac{14}{17}i$$
$$\Rightarrow \frac{2+3i}{4-i}=\frac{5}{17}+\frac{14}{17}i$$

**(iii)** $\frac{9-7i}{3+i}$ (GRW 2021, MTN 2018) **(A.B)**

**Solution:**

$$\frac{9-7i}{3+i}$$
$$=\frac{9-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{(9-7i)(3-i)}{(3)^2-(i)^2}$$
$$=\frac{9(3-i)-7i(3-i)}{9-(-1)}$$
$$=\frac{27-9i-21i+7i^2}{9+1}$$
$$\frac{27-30i+7(-1)}{10}$$
$$=\frac{27-30i-7}{10}$$
$$=\frac{27-7-30i}{10}$$
$$=\frac{20-30i}{10}$$
$$=\frac{20}{10}-\frac{30i}{10}$$
$$=2-3i$$
$$\Rightarrow \frac{9-7i}{3+i}=2-3i$$

**(iv)** $\frac{2-6i}{3+i}-\frac{4+i}{3+i}$ **(A.B)**

**Solution:**

$$\frac{2-6i}{3+i}-\frac{4+i}{3+i}$$
$$=\frac{2-6i-(4+i)}{3+i}$$
$$=\frac{2-6i-4-i}{3+i}$$
$$=\frac{2-4-6i-i}{3+i}$$
$$=\frac{-2-7i}{3+i}$$
$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$$
$$=\frac{-6+2i-21i+7i^2}{9-(-1)}$$
$$=\frac{-6-19i+7(-1)}{9+1}$$
$$=\frac{-6-19i-7}{10}$$
$$=\frac{-6-7-19i}{10}$$
$$=\frac{-13-19i}{10}$$
$$=\frac{-13}{10}-\frac{19i}{10}$$
$$\Rightarrow \frac{2-6i}{3+i}-\frac{4+i}{3+i}=\frac{-13}{10}-\frac{19i}{10}$$

**(v)** $\left[\frac{1+i}{1-i}\right]^2$ **(A.B)**

**Solution:**

$$\left[\frac{1+i}{1-i}\right]^2$$
$$=\frac{(1+i)^2}{(1-i)^2}$$
$$=\frac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$$
$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$
$$=\frac{1+(-1)+2i}{+1+(-1)-2i}$$
$$=\frac{\not{1}-\not{1}+2i}{\not{1}-\not{1}-2i}$$

$$=\frac{2i}{-2i}$$

$$=-1$$

$$\Rightarrow\left[\frac{1+i}{1-i}\right]^2=-1+0i$$

**(vi)** $\frac{1}{(2+3i)(1-i)}$ **(A.B)**

**Solution:**

$$\frac{1}{(2+3i)(1-i)}$$

$$=\frac{1}{2(1-i)+3i(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$

$$=\frac{1}{2+i-3(-1)}$$

$$=\frac{1}{2+i+3}$$

$$=\frac{1}{2+3+i}$$

$$=\frac{1}{5+i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$

$$=\frac{1(5-i)}{(5)^2-(i)^2}$$

$$=\frac{5-i}{25-(-1)}$$

$$=\frac{5-i}{25+1}$$

$$=\frac{5-i}{26}$$

$$=\frac{5}{26}-\frac{1i}{26}$$

$$\Rightarrow\frac{1}{(2+3i)(1-i)}=\frac{5}{26}-\frac{1i}{26}$$

**Q.5 Calculate** $(a)\,\bar{z}\,(b)\,z+\bar{z}\,(c)\,z-\bar{z}\,(d)\,z\bar{z}$ **for each of the following.**

**(i)** $z=-i$ **(A.B)**

**Solution:** $z=-i$

(a) $\bar{z}=i$

(b) $z+\bar{z}=-i+i$
$=0$

(c) $z-\bar{z}=(-i)-(i)$
$=-2i$

(d) $z\bar{z}=(-i)(i)$
$=-i^2$
$=-(-1)$
$=1$

**(ii)** $z=2+i$ **(A.B)**

**Solution:**

$z=2+i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=(2+i)+(2-i)$
$=2+i+2-i$
$=2+2$
$=4$

(c) $z-\bar{z}=(2+i)-(2-i)$
$=2+i-2+i$
$=i+i$
$=2i$

(d) $z\bar{z}=(2+i)(2-i)$
$=(2)^2-(i)^2$
$=4-i^2$
$=4-(-1)$
$=4+1$
$=5$

**(iii)** $z=\frac{1+i}{1-i}$ **(A.B)**

**Solution:**

$$z=\frac{1+i}{1-i}$$

$$z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$$

$$=\frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$$

$$=\frac{1+i+i+(-1)}{(1)^2-(i)^2}$$

$$=\frac{1+2i+(-1)}{1-(-1)}$$

$$=\frac{\cancel{1}+2i-\cancel{1}}{1+1}$$

$$=\frac{\cancel{2}i}{\cancel{2}}$$

$\Rightarrow z=i$

(a) $\bar{z}=-i$

(b) $z+\bar{z}=i+(-i)$

$=\cancel{i}-\cancel{i}$

$=0$

(c) $z-\bar{z}=i-(-i)$

$=i+i$

$=2i$

(d) $z\bar{z}=(i)(-i)$

$=-i^2$

$=-(-1)$

$=1$

**(iv)** $z=\frac{4-3i}{2+4i}$ **(A.B)**

**Solution:**

$$z=\frac{4-3i}{2+4i}$$

$$z=\frac{4-3i}{2+4i}\times\frac{2-4i}{2-4i}$$

$$=\frac{4(2-4i)-3i(2-4i)}{(2+4i)(2-4i)}$$

$$=\frac{8-16i-6i+12i^2}{(2)^2-(4i)^2}$$

$$=\frac{8-22i+12(-1)}{4-16i^2}$$

$$=\frac{8-22i-12}{4-16(-1)}$$

$$=\frac{8-12-22i}{4+16}$$

$$=\frac{-4-22i}{20}$$

$$=\frac{-4}{20}-\frac{22}{20}i$$

$$\Rightarrow z=-\frac{1}{5}-\frac{11}{10}i$$

**(a)** $\bar{z}=\frac{-1}{5}+\frac{11}{10}i$ **(A.B)**

**(b)** $z+\bar{z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)+\left(-\frac{1}{5}+\frac{11}{10}i\right)$ **(A.B)**

$$=-\frac{1}{5}-\cancel{\frac{11}{10}i}-\frac{1}{5}+\cancel{\frac{11}{10}i}$$

$$=-\frac{1}{5}-\frac{1}{5}$$

$$=\frac{-1-1}{5}$$

$$=-\frac{2}{5}$$

**(c)** $z-\bar{z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)-\left(-\frac{1}{5}+\frac{11}{10}i\right)$ **(A.B)**

$$=-\cancel{\frac{1}{5}}-\frac{11}{10}i+\cancel{\frac{1}{5}}-\frac{11}{10}i$$

$$=-\frac{11}{10}i-\frac{11}{10}i=\frac{-11i-11i}{10}$$

$$=-\frac{\cancel{22}^{11}i}{\cancel{10}^{5}}$$

$$=-\frac{11}{5}i$$

**(d)** $z\bar{z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)\left(-\frac{1}{5}+\frac{11}{10}i\right)$ **(A.B)**

$$=\left(-\frac{1}{5}\right)^2-\left(\frac{11}{10}i\right)^2$$

$$=\frac{1}{25}-\frac{121}{100}i^2$$

$$=\frac{1}{25}-\frac{121}{100}(-1)$$

$$=\frac{1}{25}+\frac{121}{100}$$
$$=\frac{4+121}{100}$$
$$=\frac{125}{100}$$
$$=\frac{5}{4}$$

**Q.6** If $z=2+3i$ and show that. **(A.B)**

**(i)** $\overline{z+w}=\bar{z}+\bar{w}$

**Proof:** L.H.S $=\overline{z+w}$

$$z+w=2+3i+5-4i$$
$$=2+5+3i-4i$$
$$=7-i$$
$$\Rightarrow \overline{z+w}=\overline{7-i}$$
$$=7+i \quad \ldots(i)$$

R. H. S $=\bar{z}+\bar{w}$

$$=\left(\overline{2+3i}\right)+\left(\overline{5-4i}\right)$$
$$=2-3i+5+4i$$
$$=2+5-3i+4i$$
$$=7+i \quad \ldots(ii)$$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z+w}=\bar{z}+\bar{w}$

**Hence proved**

**(ii)** $\overline{z-w}=\bar{z}-\bar{w}$ **(A.B)**

**Proof:** L.H.S. $=\overline{z-w}$

$$z-w=(2+3i)-(5-4i)$$
$$=2+3i-5+4i$$
$$=2-5+3i+4i$$
$$=-3+7i$$
$$\Rightarrow \overline{z-w}=\overline{-3+7i}$$
$$=-3-7i \quad \ldots(i)$$

R.H.S $=\bar{z}-\bar{w}$

$$=\left(\overline{2+3i}\right)-\left(\overline{5-4i}\right)$$
$$=2+3i-(5+4i)$$
$$=2-3i-5-4i$$
$$=-3-7i$$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z-w}=\overline{z-w}$

**Hence proved**

**(iii)** $\overline{zw}=\bar{z}\,\bar{w}$ **(A.B)**

**Proof:** L.H.S $=\overline{zw}$

$$zw=(2+3i)(5+4i)$$
$$=2(5-4i)+3i(5-4i)$$
$$=10-8i+15i-12i^2$$
$$=10+7i-12(-1)$$
$$=10+7i+12$$
$$=22+7i$$
$$\Rightarrow \overline{zw}=\overline{22+7i}$$
$$=22-7i$$

R.H.S $=\overline{zw}$

$$=\left(\overline{2+3i}\right)\left(\overline{5-4i}\right)$$
$$=(2-3i)(5+4i)$$
$$=2(5+4i)-3i(5+4i)$$
$$=10+8i-15i-12i^2$$
$$=10-7i-12(-1)$$

$= 10 - 7i + 12$

$= 22 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{zw} = \overline{z}\,\overline{w}$

**Hence proved**

**(iv)** $\overline{\left[\frac{z}{w}\right]} = \frac{\overline{z}}{\overline{w}}$, where $\neq 0$ **(A.B)**

**Proof: L.H.S.=** $\overline{\left[\frac{z}{w}\right]}$

$$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$$

$$= \frac{2(5+4i)+3i(5+4i)}{(5-4i)(5+4i)}$$

$$= \frac{10+8i+15i+12i^2}{(5)^2-(4i)^2}$$

$$= \frac{10+23i+12(-1)}{25-16i^2}$$

$$= \frac{10+23i-12}{25-(6(-1))}$$

$$= \frac{10+23i-12}{25+16}$$

$$= \frac{-2+23i}{41}$$

Now

$$\overline{\left[\frac{z}{w}\right]}$$

$$= \overline{\left(\frac{-2+23i}{41}\right)}$$

$$= \frac{-2}{41} - \frac{23}{41}i \quad \rightarrow (i)$$

$$\text{R.H.S} = \frac{\overline{z}}{\overline{w}}$$

$$= \frac{\left(\overline{2+3i}\right)}{\overline{(5-4i)}}$$

$$= \frac{2-3i}{5+4i}$$

$$= \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i}$$

$$= \frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$

$$= \frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$

$$= \frac{10-23i+12(-1)}{25-16i^2}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-23i-12}{25+16}$$

$$= \frac{-2-23i}{41}$$

$$= \frac{-2}{41} - \frac{23}{41}i \quad \rightarrow (ii)$$

From (i) and (ii) we get

L.H.S=R.H.S

**Hence Proved**

$\frac{1}{2}\left(z+\overline{z}\right)$ is the real part of $z$.

**Proof:**

$$\frac{1}{2}\left(z+\overline{z}\right)$$

$$= \frac{1}{2}\left[(2+3i)+\left(\overline{2+3i}\right)\right]$$

$$=\frac{1}{2}\left[(2+3i)+(2-3i)\right]$$

$$=\frac{1}{2}\left[2+\cancel{3i}+2-\cancel{3i}\right]$$

$$=\frac{1}{2}[2+2]$$

$$=\frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$

$$=2=\text{Re}(z)$$

Hence,

$\frac{1}{2}(z+\bar{z})$ is the real part of $z$.

**Proved**

**(v)** $\frac{1}{2}(z-\bar{z})$ **is the imaginary part of** $z$. **(A.B)**

**Proof:**

$$\frac{1}{2}(z-\bar{z})$$

$$=\frac{1}{2}\left[(2+3i)-(\overline{2+3i})\right]$$

$$=\frac{1}{2}\left[(2+3i)-(2-3i)\right]$$

$$=\frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$

$$=\frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$

$$=3i$$

$$=\text{Imaginary}(z)$$

Hence,

$\frac{1}{2}(z-\bar{z})$ is the imaginary part of $z$.

**Proved**

**Q.7 Solve the following equations for real $x$ and $y$.**

**(i)** $(2-3i)(x+yi)=4+i$ **(A.B)**

**Solution:** $(2-3i)(x+yi)=4+i$

$$x+yi=\frac{4+i}{2-3i}$$

$$x+yi=\frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$

$$=\frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$

$$=\frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$

$$=\frac{8+14i+3(-1)}{4-9i^2}$$

$$=\frac{8+14i-3}{4-9(-1)}$$

$$=\frac{8-3+14i}{4+9}$$

$$=\frac{5+14i}{13}$$

$$x+yi=\frac{5}{13}+\frac{14}{13}i$$

By comparing real and imaginary parts, we get

$$\Rightarrow x=\frac{5}{13},\ y=\frac{14}{13}$$

$$\therefore \text{Solution Set}=\left\{\left(\frac{5}{13},\frac{14}{13}\right)\right\}$$

**(ii)** $(3-2i)(x+yi)=2(x-2yi)+2i-1$ **(A.B)**

**Solution:**

$$(3-2i)(x+yi)=2(x-2yi)+2i-1$$

$$3(x+yi)-2i(x+yi)=2x-4yi+2i-1$$
$$3x+3yi-2xi-2yi^2=(2x-1)+i(2-4y)$$
$$3x+(3y-2x)i-2y(-1)=(2x-1)+i(2-4y)$$
$$3x+(3y-2x)i+2y=(2x-1)+i(2-4)$$
$$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$$

By comparing the real and imaginary parts.

$$3x+2y=2x-1 \quad , \quad 3y-2x=2-4y$$
$$3x-2x+2y=-1 \quad , \quad 3y-2x=2-4y$$
$$x+2y=-1 \rightarrow (i) \quad , \quad -2x+3y+4y=2$$
$$-2x+7y=2 \rightarrow (ii)$$

Multiply equation (i) with 2

$$2(x+2y)=-1\times 2$$
$$2x+4y=-2 \rightarrow (iii)$$

By adding equation (ii) and (iii)

$$\cancel{2x}+4y=\cancel{-2}$$
$$\underline{\cancel{-2x}+7y=\cancel{2}}$$
$$11y=0$$
$$y=\frac{0}{11}$$
$$y=0$$

Putting $y=0$ in equation (i)

$$x+2y=-1$$
$$x+2(0)=-1$$
$$x+0=-1$$
$$x=-1$$

$\therefore$ Solution Set $=\{(0,-1)\}$

**(iii)** $(3+4i)^2-2(x-yi)=x+yi$ **(A.B)**

**Solution:** $(3+4i)^2-2(x-yi)=x+yi$

$$(3+4i)^2-2(x-yi)=x+yi$$
$$9+24i+16i^2-2x+2yi=x+yi$$
$$9+24i+16(-1)-2x+2yi=x+yi$$
$$9+24i-16-2x+2yi=x+yi$$
$$9+24i-16-2x=x+2yi-yi=0$$
$$9+24i-16-3x+yi=0$$
$$-3x+yi=-9-24i+16$$
$$-3x+yi=16-9-24i$$
$$-3x+yi=7-24i$$

By comparing the real and imaginary parts, we get

$$-3x=7 \quad \text{and} \quad y=-24$$
$$\Rightarrow x=\frac{-7}{3} \qquad y=-24$$

$\therefore$ Solution Set $=\left\{\left(\frac{-7}{3},-24\right)\right\}$