Mathematics-9
Exercise 4.1

Algebraic Expression (LHR 2016) (K.B)

An expression consists of constants and variables connected with arithmetic operators $(+,-,\times,\div$ etc.) is called algebraic expression.

Or

When operations of addition and subtraction are applied to algebraic terms, we obtain an algebraic expression.

For example:

$5x^2-3x+\frac{2}{\sqrt{x}}$ and $3xy+\frac{3}{x}(x\neq 0)$ etc.

Types of Algebraic Expressions (K.B)

There are three types of algebraic expressions:

(i) Polynomial Expression

(ii) Rational Expression

(iii) Irrational Expression

Polynomials (K.B)

An expression consists of one or more terms in each of which exponent of variable is either 0 or positive integer is called polynomial expression.

A polynomial in the variable x is an algebraic expression of the form

$p(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+.....a_1x+a_0,\ a_n\neq 0\rightarrow(i)$

For example:

$2x+3,\ x^3-27,\ xy^2z,\ 0,\ \frac{5}{4}$ etc.

Degree of the Polynomial (K.B)

When n the highest power of variable, is a non-negative integer called the degree of the polynomial.

For example:

Degree of expressions $2x^3+5x^2+8x+1$ and $2x^4y^3+x^2y^2+8x$ is 3 and 7 respectively.

Leading Coefficient (K.B)

The coefficient a_n of the highest power of x is called the leading coefficient of the polynomial.

For example:

In the polynomial $2x^3+5x^2+8x+1$ leading coefficient is 2.

Note (U.B+K.B)

From similar properties of integers and polynomials w.r.t. addition and multiplication, we may say that polynomials behave line integers.

Rational Expression (K.B)

The quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$ where $q(x)$ is non-zero polynomial, is called a rational expression.

For example:

$\frac{2x+1}{3x+8}, 3x+8\neq 0$

Irrational Expression **(K.B)**

An expression which cannot be expressed as the quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x) \neq 0$ is called an irrational expression.

For example:

$2\sqrt{x}+3, \sqrt{x}+\frac{1}{\sqrt{x}}$ etc.

Rational Expressions behave like Rational numbers (SGD 2017) **(K.B)**

Let a and b be two integers, then $\frac{a}{b}$ is not necessarily an integer. Therefore, number system is extended and $\frac{a}{b}$ is defined as a rational number where $a, b \in Z$ and $b \neq 0$.

Similarly if $P(x)$ and $q(x)$ are two polynomial, then $\frac{P(x)}{q(x)}$ is not necessarily a polynomial where $q(x) \neq 0$.

Numerator **(K.B)**

In the rational expression $\frac{P(x)}{q(x)}, P(x)$ is called the numerator.

Denominator **(K.B)**

In the rational expression $\frac{P(x)}{q(x)}, q(x)$ is known as the denominator of the rational expression.

Note **(K.B+U.B)**

Every polynomial $P(x)$ can be regarded as a rational expression, Since We can write $P(x)$ as $\frac{P(x)}{1}$. Thus every polynomial is a rational expression but every rational expression need not be a polynomial.

Working Rule to Reduce a Rational Expression to its Lowest Terms **(K.B)**

Let the given rational expression be $\frac{p(x)}{q(x)}$.

Step I: Factorize each of the two polynomials $P(x)$ and $q(x)$

Step II: Find H.C.F. of $P(x)$ and $q(x)$.

Step III: Divide the numerator $P(x)$ and the denominator $q(x)$ by the H.C.F. of $P(x)$ and $q(x)$. The rational expression so obtained, is in its lowest terms.

In other words, an algebraic fraction can be reduced to its lowest form by first factorizing both the polynomials in the numerator and the denominator and then cancelling the common factors between them.

Example **(A.B)**

$$\frac{3x^2+18x+27}{5(x^2-9)} = \frac{3(x^2+6x+9)}{5(x^2-9)} \quad \text{(common)}$$

$$= \frac{3(x+3)(x+3)}{5(x+3)(x-3)} \quad \text{(Factorizing)}$$

$$= \frac{3(x+3)}{5(x-3)} \quad \text{(Cancelling common factors)}$$

is in the lowest form.

Sum, Difference and Product of Rational Expressions

Example # 2 **(A.B)**

Simplify

$$\frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{x^2-y^2}$$

Solution:

$$\frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{x^2-y^2}$$

$$= \frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{(x-y)(x+y)}$$

$=\frac{x+y-(x-y)+2x}{(x+y)(x-y)}$ (L.C.M of denominators)

$=\frac{x+y-x+y+2x}{(x+y)(x-y)}$

$=\frac{2x+2y}{(x+y)(x-y)}$ (Simplifying)

$=\frac{2(x+y)}{(x+y)(x-y)}$

$=\frac{2}{x-y}$ (Cancelling common factors)

Example # 2 **(A.B)**

Find the product $\frac{x+2}{2x-3y}\cdot\frac{4x^2-9y^2}{xy+2y}$ in simplified factor.

Solution:

$\frac{x+2}{2x-3y}\cdot\frac{4x^2-9y^2}{xy+2y}$

$=\frac{x+2}{2x-3y}\cdot\frac{(2x)^2-(3y)^2}{y(x+2)}$

$=\frac{(x+2)}{(2x-3y)}\times\frac{(2x-3y)(2x+3y)}{y(x+2)}$ (Factorizing)

$=\frac{2x+3y}{y}$ (Reduced to the lowest form)

Dividing a Rational Expression with Another Rational Expression

Example **(A.B)**

Simplify:

$\frac{7xy}{x^2-4x+4}\div\frac{14y}{x^2-4}$

Solution:

$\frac{7xy}{x^2-4x+4}\div\frac{14y}{x^2-4}$

$=\frac{7xy}{x^2-4x+4}\times\frac{x^2-4}{14y}$ (changing division into multiplication)

$=\frac{7xy}{(x-2)(x-2)}\cdot\frac{(x+2)(x-2)}{14y}$ ...(factorizing)

$=\frac{x(x+2)}{2(x-2)}$ ...(reduced to lowest form)

Evaluation of Algebraic Expression for some Particular Real Number

Value of the expression (K.B)

If specific numbers are substituted for the variables in an algebraic expression, the resulting number is called the value of the expression.

Example **(A.B)**

Evaluate: $\frac{3x^2\sqrt{y}+6}{5(x+y)}$ if $x=-4$ and $y=9$

Solution:

$\frac{3x^2\sqrt{y}+6}{5(x+y)}$

Putting $x=-4$ and $y=9$, we get

$=\frac{3(-4)^2\sqrt{9}+6}{5(-4+9)}$

$=\frac{150}{25}=6$

Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2+\frac{1}{x}-5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3-4x^2-x\sqrt{x}+3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2-3x+\sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

(iv) $\frac{3x}{2x-1}+8$

No (Because of $\frac{1}{2x-1}$) **Ans**

Q.2 State whether each of the following expressions is a rational expression or not. (A.B+K.B)

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3-2x^3+\sqrt{3}}{2+3x-x^2}$

Rational

(iii) $\frac{x^2+6x+9}{x^2-9}$

Rational

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational

Q.3 Reduce the following expression to the lowest form. (A.B+K.B)

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$ **(SWL 2017)**

Solution: $\frac{{}^{4}\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$=\frac{120x^2y^3z^5}{30x^3yz^2}$

$=4x^{2-3}y^{3-1}z^{5-2}$

$=4x^{-1}y^2z^3$

$=\frac{4y^2z^3}{x}$

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

(LHR 2017, FSD 2015, 16, SWL 2015, BWP 2017, MTN 2017, D.G.K 2017)

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$=\frac{{}^{4}\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$=\frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$=\frac{4a}{x-1}$

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2-4xy}{(x-y)^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\therefore (x-y)^2=x^2+y^2-2xy$

$=\frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$=\frac{x^2+y^2-2xy}{x^2+y^2-2xy}$

$=\frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$

$=1$

(iv) $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

Solution: $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$(a^3+b^3)=(a-b)(a^2+ab+b^2)$

$=\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x^3-y^3)}$

$=x^2-2xy+y^2$

$\therefore (x-y)^2=x^2-2xy+y^2$

$=(x-y)^2$

(v) $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

(GRW 2018, MTN 2013, 15, RWP 2018)

Solution: $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$=\dfrac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$

$=\dfrac{(x+2)(x-1)(x+1)}{(x+1)(x-2)(x+2)}$

$=\dfrac{(x-1)}{(x-2)}$ **Ans**

(vi) $\dfrac{x^2-4x+4}{2x^2-8}$ **(A.B+U.B)**

(LHR 2013, FSD 2014, 16, SWL 2016, SGD 2017, BWP 2013, 17, D.G.K 2015)

Solution: $\dfrac{x^2-4x+4}{2x^2-8}$

$\therefore (a-b)^2=a^2-2ab+b^2$

$\therefore a^2-b^2=(a+b)(a-b)$

$=\dfrac{(x)^2-2(x)(2)+(2)^2}{2(x^2-4)}$

$=\dfrac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$

$=\dfrac{(x-2)^2}{2(x+2)(x-2)}$

$=\dfrac{(x-2)(x-2)}{2(x+2)(x-2)}$

$=\dfrac{x-2}{2(x+2)}$ **Ans**

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

(SGD 2015, SWL 2014, MTN 2014)

Solution: $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

$=\dfrac{64x(x^4-1)}{8(x^2+1).2(x+1)}$

$=\dfrac{64\left[(x^2)^2-(1)^2\right]}{16(x^2+1)(x+1)}$

$=\dfrac{^4\,64(x^2-1)(x^2+1)}{16(x^2+1)(x+1)}$

$=\dfrac{4x(x-1)(x+1)}{(x+1)}$

$=4x(x-1)$ **Ans**

(viii) $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Solution: $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

$=\dfrac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$

$=\dfrac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$

$=\dfrac{(x^2+3x-4)(-x^2+3x+4)}{(-x^2+3x+4)}$

$=x^2+3x-4$ **Ans**

Q.4 Evaluate (K.B+A.B+U.B)

(a) $\dfrac{x^3y-2z}{xz}$ **for**

(i) $x=3, y=-1, z=-2$

(ii) $x=-1, y=-9, z=4$

(b) $\dfrac{x^2y^2-5z^4}{xyz}$ **for** $x=4, y=-2$ **and** $z=-1$

Solution for 1st part

When $x=3,\ y=-1,\ z=-2$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$

$$=\frac{27(-1)+4}{-6}$$

$$=\frac{-27+4}{-6}$$

$$=\frac{-23}{-6}$$

$$=\frac{23}{6}\ \textbf{Ans}$$

Solution for 2nd Part.

When $x=-1,\ y=-9,\ z=4$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$

$$=\frac{-1(-9)-8}{-4}$$

$$=\frac{9-8}{-4}$$

$$=\frac{1}{-4}$$

$$=-\frac{1}{4}\ \textbf{Ans}$$

(a) $\dfrac{x^2y^3-5z^4}{xyz}$ **for** $x=4, y=-2, z=-1$

(LHR 2016, GRW 2015, D.G.K 2016)

Solution: $\dfrac{x^2y^3-5z^4}{xyz}$ **(A.B)**

$$=\frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{-128-5}{8}$$

$$=-\frac{133}{8}$$

$$=-16\frac{5}{8}\ \textbf{Ans}$$

Q.5 Perform the indicated operation and simplify. (A.B+U.B)

(i) $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

Solution: $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

$$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}$$

$$=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$$

$$=\frac{15}{2x-3y}+\frac{4}{2x-3y}$$

$$=\frac{19}{2x-3y}\ \textbf{Ans}$$

(ii) $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$ **(A.B)**

Solution: $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$

$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$

$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$=\frac{4x+4x}{1-4x^2}$$

$$=\frac{8x}{1-4x^2}$$ **Ans**

(iii) $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

Solution: $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$

$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$$=\frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$=\frac{(x+5)(1)}{x^2-36}$$

$$=\frac{x+5}{x^2-36}$$ **Ans**

(iv) $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$ **(A.B)**

Solution: $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{x^2-y^2}$$

$$=\frac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$$

$$=\frac{x-y}{x+y}$$ **Ans**

(v) $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$ **(A.B)**

Solution: $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

$$=\frac{x-2}{(x)^2+2(3)(x)+3^2}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2\left[(x)^2-(3)^2\right]}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2x^2-10x+12-x^2-5x-6}{2(x+3)^2(x-3)}$$

$$=\frac{x^2-15x+6}{2(x+3)^2(x-3)}$$ **Ans**

(vi) $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$ **(A.B)**

Solution: $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$$=\frac{(x+1)-(x-1)}{(x-1)(x+1)}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{x}+1-\cancel{x}+1}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{2x^2}+2-\cancel{2x^2}+2}{(x^2)^2-(1)^2}-\frac{4}{x^4-1}$$

$$=\frac{4}{x^4-1}-\frac{4}{x^4-1}$$

$$=\frac{4-4}{x^4-1}$$

$$=\frac{0}{x^4-1}$$

$= 0$ **Ans**

Q.6 Perform the indicated operation and simplify.

(i) $(x^2-49).\dfrac{5x+2}{x+7}$ **(A.B)**

Solution: $(x^2-49).\dfrac{5x+2}{x+7}$

$$=\left[(x)^2-(7)^2\right].\frac{5x+2}{x+7}$$

$$=(x+7)(x-7)\frac{(5x+2)}{(x+7)}$$

$=(x-7)(5x+2)$ **Ans**

(ii) $\dfrac{4x-12}{x^2-9}\div\dfrac{18-2x^2}{x^2+6x+9}$ **(A.B)**

Solution: $\dfrac{4x-12}{x^2-9}\div\dfrac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$=\frac{4(x-3)}{(x^2)-(3)^2}\div\frac{2(9-x^2)}{(x+3)^2}$$

$$=\frac{4\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}\times\frac{(x+3)^2}{2(9-x^2)}$$

$$=\frac{4}{x+3}\times\frac{(x+3)^2}{2(3+x)(3-x)}$$

$$=\frac{{}^2\cancel{4}x\cancel{(x+3)^2}}{\cancel{2}\cancel{(x+3)^2}(3-x)}$$

$=\dfrac{2}{3-x}$ **Ans**

(iii) $\dfrac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$ **(A.B)**

Solution: $\dfrac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

$$=\frac{(x^2)^3-(y^2)^3}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{\cancel{(x^2-y^2)}\left[(x^2)^2+x^2y^2+(y^2)^2\right]}{\cancel{(x^2-y^2)}}\div(x^4+x^2y^2+y^4)$$

$$=\cancel{(x^4+x^2y^2+y^4)}\times\frac{1}{\cancel{(x^4+x^2y^2+y^4)}}$$

$= 1$ **Ans**

(iv) $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$ **(A.B)**

Solution: $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$

$$=\frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)}\times\frac{x+5}{-(x-1)}$$

$$=\frac{(x+1)\cancel{(x-1)}}{(x+1)^2}\times\frac{(x+5)}{-\cancel{(x-1)}}$$

$$=-\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$= -\frac{(x+5)}{x+1}$ **Ans**

(v) $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$ **(A.B)**

Solution: $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$= \frac{x(x+y)}{y(x+y)} \cdot \frac{x(x+y)}{y(x+y)} \div \frac{x(x-1)}{y(x-2)}$

$= \frac{x.x}{y.y} \times \frac{y(x-2)}{x(x-1)}$

$= \frac{x(x-2)}{y(x-1)}$ **Ans**