Mathematics-9
Exercise 4.2

Algebraic Formulae and their uses

(K.B+U.B)

(i) $(a+b)^2+(a-b)^2=2(a^2+b^2)$

(ii) $(a+b)^2-(a-b)^2=4ab$

(iii) $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

(iv) $(a+b)^3=a^3+3ab(a+b)+b^3$

(v) $(a-b)^3=a^3-3ab(a-b)-b^3$

(vi) $a^3\pm b^3=(a\pm b)(a^2\pm ab+b^2)$

Example (Page # 83) **(A.B)**

If a + b = 7 and a – b = 3 then find the value of **(a)** a^2+b^2 **(b)** ab

Solution:

We are given $a+b=7$ and $a-b=3$

(a) Formula

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Substituting the values, we get

$$(7)^2+(3)^2=2(a^2+b^2)$$

$\Rightarrow$ $49+9=2(a^2+b^2)$

$\Rightarrow$ $58=2(a^2+b^2)$

$\Rightarrow$ $29=a^2+b^2$

Or $a^2+b^2=29$

(b) Formula

$$(a+b)^2-(a-b)^2=4ab$$

Putting the values

$\Rightarrow$ $(7)^2-(3)^2=4ab$

$\Rightarrow$ $49-9=4ab$

$\Rightarrow$ $40=4ab$

$\Rightarrow$ $10=ab$

Or $ab=10$

Hence $a^2+b^2=29$ and $ab=10$

Example (Page # 84) **(A.B)**

If $2x-3y=10$ and $xy=2$ then find the value of $8x^3-27y^3$.

Solution:

Here $2x-3y=10$

Taking cube on both sides

$\Rightarrow$ $(2x-3y)^3=(10)^3$

$$8x^3-27y^3-3(2x)(3y)(2x-3y)=1000$$

$\because$ $(a-b)^3=a^3-3ab(a-b)-b^3$

Putting the values

$\Rightarrow$ $8x^3-27y^3-18(2)(10)=1000$

$\Rightarrow$ $8x^3-27y^3-360=1000$

$\Rightarrow$ $8x^3-27y^3=1000+360$

Hence $8x^3-27y^3=1360$

Example 1 (Page # 86) **(A.B)**

Factorize: $64x^3+343y^3$

Solution:

We have

$64x^3+343y^3$

$=(4x)^3+(7y)^3$

$\because$ $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(4x+7y)\left[(4x)^2-(4x)(7y)+(7y)^2\right]$

$=(4x+7y)(16x^2-28xy+49y^2)$

Example # 4 **(A.B)**

Find the product

$$\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\frac{16x^2}{25}+\frac{25}{16x^2}+1\right]$$

Solution:

$$\left[\frac{4}{5}x-\frac{5}{4x}\right]\left[\frac{16}{25}x^2+\frac{25}{16x^2}+1\right]$$
$$=\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\frac{16x^2}{25}+1+\frac{25}{16x^2}\right]$$
$$=\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\left(\frac{4x}{5}\right)^2+\left(\frac{4x}{5}\right)\left(\frac{5}{4x}\right)+\left(\frac{5}{4x}\right)^2\right]$$
$$\because\ (a-b)(a^2+ab+b^2)=a^3-b^3$$
$$=\left[\frac{4x}{5}\right]^3-\left[\frac{5}{4x}\right]^3$$
$$=\frac{16x^3}{125}-\frac{125}{64x^3}$$

Exercise 4.2

Q.1

(i) If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2) (A.B)

Solution:

Formula
$$2(a^2+b^2)=(a+b)^2+(a-b)^2$$
Putting the values
$$2(a^2+b^2)=(10)^2+(6)^2$$
$$2(a^2+b^2)=100+36$$
$$2(a^2+b^2)=136$$
$$(a^2+b^2)=\frac{\cancel{136}^{68}}{\cancel{2}}$$
$$(a^2+b^2)=68$$

(ii) If $a+b=5, a-b=\sqrt{17}$, then find the value of ab. (A.B)

Solution:
$$4ab=(a+b)^2-(a-b)^2$$
Putting the values
$$4ab=(5)^2-\left(\sqrt{17}\right)^2$$
$$4ab=25-17$$
$$4ab=8$$
$$ab=\frac{8}{4}$$
$$ab=2$$

Q.2 If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$. (A.B)

Solution:

We know that
$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$
Putting the values
$$(-1)^2=45+2(ab+bc+ca)$$
$$1=45+2(ab+bc+ca)$$
$$1-45=2(ab+bc+ca)$$
$$-44=2(ab+bc+ca)$$
$$\frac{\cancel{-44}^{22}}{\cancel{2}}=(ab+bc+ca)$$
$$(ab+bc+ca)=-22$$

Q.3 If $m+n+p=10$ and $mn+np+np=27$, find the value of $m^2+n^2+p^2$ (A.B)

Solution:

We know that
$$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$$
$$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$$
Putting the values
$$100=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46$$

Q.4 If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, find the value of $x+y+z$. (A.B)

Solution:

We know that
$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$$
$$(x+y+z)^2=78+2(xy+yz+zx)$$
Putting the values
$$(x+y+z)^2=78+2(59)$$

$(x+y+z)^2 = 78+118$

$(x+y+z)^2 = 196$

Taking square root at both sides

$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$

$x+y+z = \pm 14$

Q.5 If $x+y+z=12$ and $x^2+y^2=64$, find the value of $xy+yz+zx$. **(A.B)**

Solution:

We know that

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

$(12)^2 = 64+2(xy+yz+zx)$

$144-64 = 2(xy+yz+zx)$

$80 = 2(xy+yz+zx)$

$\frac{\cancel{80}^{40}}{\cancel{2}} = (xy+yz+zx)$

$40 = xy+yz+zx$

$xy+yz+zx = 40$ **Ans**

Q.6 If $x+y=7$ and $xy=12$, then find the value of x^3+y^3 **(A.B)**

Solution:

We know that

$(x+y)^3 = x^3+y^3+3xy(x+y)$

$(7)^3 = x^3+y^3+3(12)(7)$

$343 = x^3+y^3+252$

$343-252 = x^3+y^3$

$91 = x^3+y^3$

$x^3+y^3 = 91$ **Ans**

Q.7 If $3x+4y=11$ and $xy=12$, then find the value of $27x^3+64y^3$. **(A.B)**

Solution:

We know that

$(3x+4y)^3 = (3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$

$\because (a+b)^3 = a^3+3ab(a+b)+b^3$

$(3x+4y)^3 = 27x^3+64y^3+36xy(3x+4y)$

Putting the values

$(11)^3 = 27x^3+64y^3+36(12)(11)$

$1331 = 27x^3+64y^3+4752$

$1331-4752 = 27x^3+64y^3$

$-3421 = 27x^3+64y^3$

$27x^3+64y^3 = -3421$ Ans

Q.8 If $x-y=4$ and $xy=21$, then find the value of x^3-y^3 **(A.B)**

Solution:

We know that

$(x-y)^3 = x^3-y^3-3xy(x-y)$

$(4)^3 = x^3-y^3-3(21)(4)$

$64 = x^3-y^3-252$

$64+252 = x^3-y^3$

$316 = x^3-y^3$

$x^3-y^3 = 316$ **Ans**

Q.9 If $5x-6y=13$ and $xy=6$, then find the value of b$125x^3-216y^3$ **(A.B)**

Solution:

We know that

$(x-y)^3 = x^3-y^3-3xy(x-y)$

$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$

$(5x-6y)^3 = 125x^3-216y^3-90xy(5x-6y)$

Putting the values

$(13)^3 = 125x^3-216y^3-90(6)(13)$

$2197 = 125x^3-216y^3-7020$

$2197+7020 = 125x^3-216y^3$

$9217 = 125x^3-216y^3$

$125x^3-216y^3 = 9217$ **Ans**

Q.10 If $x+\frac{1}{x}=3$ **then find the value of** $x^3+\frac{1}{x^3}$ **(A.B)**

Solution:

We know that

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

Putting the values

$$(3)^3=x^3+\frac{1}{x^3}+3(3)$$

$$27=x^3+\frac{1}{x^3}+9$$

$$27-9=x^3+\frac{1}{x^3}$$

$$18=x^3+\frac{1}{x^3}$$

$$x^3+\frac{1}{x^3}=18\textbf{ Ans}$$

Q.11 If $x-\frac{1}{x}=7$, **then find the value of** $x^3-\frac{1}{x^3}$ **(A.B)**

Solution:

We know that

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$$

Putting the values

$$(7)^3=x^3-\frac{1}{x^3}-3(7)$$

$$343=x^3-\frac{1}{x^3}-21$$

$$343+21=x^3-\frac{1}{x^3}$$

$$364=x^3-\frac{1}{x^3}$$

$$x^3-\frac{1}{x^3}=364\textbf{ Ans}$$

Q.12 If $\left[3x+\frac{1}{3x}\right]=5$, **then find the value of** $\left[27x^3+\frac{1}{27x^3}\right]$ **(A.B)**

Solution:

We know that

$$\left(3x+\frac{1}{3x}\right)^3=(3x)^3+\left(\frac{1}{3x}\right)^3+3(\cancel{3x})\left(\frac{1}{\cancel{3x}}\right)\left(3x+\frac{1}{3x}\right)$$

Putting the values

$$(5)^3=27x^3+\frac{1}{27x^3}+3(5)$$

$$125=27x^3+\frac{1}{27x^3}+15$$

$$125-15=27x^3+\frac{1}{27x^3}$$

$$110=27x^3+\frac{1}{27x^3}$$

$$27x^3+\frac{1}{27x^3}=110$$

Q.13 If $\left(5x-\frac{1}{5x}\right)=6$, **then find the value of** $\left(125x^3-\frac{1}{125x^3}\right)$ **(A.B)**

Solution:

We know that

$$\left(5x-\frac{1}{5x}\right)^3=(5x)^3-\left(\frac{1}{5x}\right)^3-3(\cancel{5x})\left(\frac{1}{\cancel{5x}}\right)\left(5x-\frac{1}{5x}\right)$$

$$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$$

$$216=125x^3-\frac{1}{125x^3}-18$$

$$216+18=125x^3-\frac{1}{125x^3}$$

$$234=125x^3-\frac{1}{125x^3}$$

$$125x^3-\frac{1}{125x^3}=234\textbf{ Ans}$$

Q.14 Factorize

(i) x^3-y^3-x+y **(GRW 2015)** **(A.B)**

Solution: x^3-y^3-x+y

$$=(x)^3-(y)^3-1(x-y)$$

$$=(x-y)(x^2+xy+y^2)-1(x-y)$$

$$=(x-y)(x^2+xy+y^2-1)\ \textbf{Ans}$$

(ii) $8x^3-\dfrac{1}{27y^3}$ **(A.B)**

(FSD 2015, MTN 2013, SWL 2013, BWP 2016)

Solution: $8x^3-\dfrac{1}{27y^3}$

$$=(2x)^3-\left(\frac{1}{3y}\right)^3$$

$$=\left[2x-\frac{1}{3y}\right]\left[(2x)^2+(2x)\left(\frac{1}{3y}\right)+\left(\frac{1}{3y}\right)^2\right]$$

$$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)\ \textbf{Ans}$$

Q.15 Find the products, using formula.

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$ **(A.B)**

Solution: $(x^2+y^2)(x^4-x^2y^2+y^4)$

$$=(x^2+y^2)\left[(x^2)^2-(x^2)(y^2)+(y^2)^2\right]$$

$$\left[(x^2)^3+(y^2)^3\right]$$

$$=x^6+y^6\ \textbf{Ans}$$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$ **(A.B)**

Solution: $(x^3-y^3)(x^6+x^3y^3+y^6)$

$$(x^3-y^3)\left[(x^3)^2+(x^3)(y^3)+(y^3)^2\right]$$

$$=(x^3)^3-(y^3)^3$$

$$=x^9-y^9\ \textbf{Ans}$$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$
$(x^2+xy+y^2)(x^4-x^2y^2+y^4)$

(A.B)

Solution:

$$(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$$
$$(x^2+xy+y^2)(x^4-x^2y^2+y^4)$$

$$=\left[(x-y)(x^2+xy+y^2)\right]\left[(x+y)(x^2-xy+y^2)\right]$$
$$\left[(x^2+y^2)(x^4-x^2y^2+y^4)\right]$$

$$=\left[(x^3-y^3)(x^3+y^3)\right]\left[(x^2)^3+(y^2)^3\right]$$

$$=\left[(x^3)^2-(y^3)^2\right]\left[(x^6+y^6)\right]$$

$$=\left[(x^6-y^6)(x^6+y^6)\right]$$

$$=\left[(x^6)^2-(y^6)^2\right]$$

$$=x^{12}-y^{12}$$

(iv) $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

Solution:

$$(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$$

$$=\left[(2x^2-1)(4x^4+2x^2+1)\right]\left[(2x^2+1)(4x^4-2x^2+1)\right]$$

$$=\left[(2x^2)^3-(1)^3\right]\left[(2x^2)^3+(1)^3\right]$$

$$=(8x^6-1)(8x^6+1)$$

$$=(8x^6)^2-(1)^2$$

$$=64x^{12}-1\ \textbf{Ans}$$