SURDS AND THEIR APPLICATION

SURD

An irrational radical with radicand is called a surd.

For example: **(K.B)**

$\sqrt{3}, \sqrt{\frac{2}{5}}, \sqrt[3]{7}, \sqrt[4]{10}$

Note **(U.B+K.B)**

Hence the radical $\sqrt[n]{a}$ is a surd if

(i) a is rational

(ii) The result $\sqrt[n]{a}$ is irrational.

Order of a Surd **(K.B)**

If $\sqrt[n]{a}$ is an irrational number then n is called index or the order of the surd and the rational number 'a' is called the radicand.

For example:

In $\sqrt[3]{7}$, order of order surd is 3.

Note

- $\sqrt{\pi}$ and $\sqrt{2+\sqrt{17}}$ are not surds because π and $2+\sqrt{17}$ are not rational.
- Every surd is an irrational number but every irrational number is not a surd. e.g., the surd $\sqrt[3]{5}$ is an irrational number but the irrational number $\sqrt{\pi}$ is not a surd.

OPERATOINS ON SURDS

(a) **Addition and Subtracting of Surd** **(U.B+K.B)**

Similar surds (i.e. surds having same irrational factors) can be added or subtracted into a single term.

Example (Page # 89) **(K.B)**

$$\sqrt[3]{128} - \sqrt[3]{250} + \sqrt[3]{432}$$

$$= \sqrt[3]{64\times 2} - \sqrt[3]{125\times 2} + \sqrt[3]{216\times 2}$$

$$= \sqrt[3]{4^3\times 2} - \sqrt[3]{5^3\times 2} + \sqrt[3]{6^3\times 2}$$

$$= \sqrt[3]{(4)^3}\times\sqrt[3]{2} - \sqrt[3]{(5)^3}\times\sqrt[3]{2} + \sqrt[3]{(6)^3}\times\sqrt[3]{2}$$

$$= 4\sqrt[3]{2} - 5\sqrt[3]{2} + 6\sqrt[3]{2}$$

$$= (4-5+6)\sqrt[3]{2}$$

$$= 5\sqrt[3]{2}$$

(b) **Multiplications and Division of Surds** **(K.B)**

We can multiply and divide surds of the same order by making use of the following laws of surds.

$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

and the result obtained will be a surd of the same order. If surds to be multiplied or divided are not of the same order, they must be reduced to the surds of the same order.

Example (Page # 89) **(A.B)**

(ii) $\frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$

We have

For $\sqrt{3}\ \sqrt[3]{2}$ the L.C.M of 2 and 3 is 6

$$\sqrt{3} = (3)^{\frac{1}{2}} = (3)^{\frac{3}{3}\times\frac{1}{2}} = (3)^{\frac{3}{6}} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$\sqrt[3]{2} = (2)^{\frac{1}{2}} = (2)^{\frac{2}{2}\times\frac{1}{3}} = 2^{\frac{2}{6}} = \sqrt[6]{2^2} = \sqrt[6]{4}$$

Hence

$$\frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}} = \frac{\sqrt[6]{12}}{\sqrt[6]{27}\times\sqrt[6]{4}} = \frac{\sqrt[6]{12}}{\sqrt[6]{27\times 4}} = \sqrt[6]{\frac{12}{108}} = \sqrt[6]{\frac{1}{9}}$$

It simplest form is

$$\sqrt[6]{\left(\frac{1}{3}\right)^2}=\left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{6}}=\left[\frac{1}{3}\right]^{2\times\frac{1}{3}}=\left(\frac{1}{3}\right)^{\frac{1}{3}}=\sqrt[3]{\frac{1}{3}}$$

Method II

$$\frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$$

$$=\frac{(12)^{1/6}}{(3)^{1/2}(2)^{1/3}} \quad \text{(in exponential form)}$$

$$=\frac{\left(2^2\times 3\right)^{1/6}}{(3)^{1/2}(2)^{1/3}}$$

$$=\frac{\left(2^2\right)^{1/6}\times(3)^{1/6}}{(3)^{1/2}(2)^{1/3}} \quad \because (ab)^n=a^nb^n$$

$$=\frac{2^{1/3}\times 3^{1/6}}{3^{1/2}\times 2^{1/3}} \quad \because \left(a^m\right)^n=a^{mn}$$

$$=\frac{3^{1/6}}{3^{1/2}} \quad \because \frac{a^n}{a^n}=1$$

$$=\frac{1}{3^{1/2-1/6}} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=\frac{1}{3^{2/6}}$$

$$=\frac{1}{3^{1/3}}$$

$$=\frac{1}{\sqrt[3]{3}} \quad \text{(in radical form)}$$

Exercise 4.3

Q.1 Express each of the following surd in the simplest form:

(i) $\sqrt{180}$ **(A.B)**

Solution:

$$\sqrt{180}$$

$$=\sqrt{2^2\times 3^2\times 5}$$

$$=\sqrt{2^2}\times\sqrt{3^2}\times\sqrt{5} \quad \because \sqrt[n]{ab}=\sqrt[n]{a}\times\sqrt[n]{b}$$

$$=2\times 3\times\sqrt{5}$$

$$=6\sqrt{5}$$

(ii) $3\sqrt{162}$ **(A.B)**

Solution:

$$3\sqrt{162}$$

$$=3\left(\sqrt{9^2\times 2}\right)$$

$$=3\left(\sqrt{9^2}\times\sqrt{2}\right)$$

$$=3\times 9\sqrt{2}$$

$$=27\sqrt{2}$$

(iii) $\frac{3}{4}\sqrt[3]{128}$ **(A.B)**

Solution:

$$\frac{3}{4}\sqrt[3]{128}$$

$$=\frac{3}{4}\left(\sqrt[3]{4^3\times 2}\right)$$

$$=\frac{3}{4}\left[\sqrt[3]{4^3}\times\sqrt[3]{2}\right]$$

$$=\frac{3}{4}\times 4\times\sqrt[3]{2}$$

$$=3\sqrt[3]{2}$$

(iv) $\sqrt[5]{96x^6y^7z^8}$ **(A.B)**

Solution:

$$\sqrt[5]{96x^6y^7z^8}$$

$$=\sqrt[5]{2^5\times 3\times x^5y^5z^5\times xy^2z^3}$$

$$=\sqrt[5]{2^5x^5y^5z^5}\times\sqrt[5]{3xy^2z^3}$$

$$=\sqrt[5]{2^5}\times\sqrt[5]{x^5}\times\sqrt[5]{y^5}\times\sqrt[5]{z^5}\times\sqrt[5]{3xy^2z^3}$$

$$=2xyz\sqrt[5]{3xy^2z^3}$$

Q.2 Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$ **(BWP 2014)** **(A.B)**

Solution:

$$\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$$

$$=\sqrt{\frac{3^2\times 2}{3\times 2}}$$

$$\because \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}},\ \sqrt{a}\times\sqrt{b} = \sqrt{ab}$$
$$= \sqrt{3}$$

(ii) $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$ **(A.B)**

(LHR 2017, FSD 2016, MTN 2016, SWL 2017, SGD 2017, D.G.K 2017)

Solution:

$$\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$$
$$= \sqrt{\frac{21\times 9}{63}} \quad \because \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}},\ \sqrt{a}\times\sqrt{b} = \sqrt{ab}$$
$$= \sqrt{\frac{7\times 3\times 3^2}{7\times 3^2}}$$
$$= \sqrt{3}$$

(iii) $= \sqrt[5]{243x^5y^{10}z^{15}}$ **(A.B)**

Solution:

$$\sqrt[5]{243x^5y^{10}z^{15}}$$
$$= \sqrt[5]{3^5x^5\left(y^2\right)^5\left(z^3\right)^5}$$
$$= \sqrt[3]{3^5}\times\sqrt[5]{x^5}\times\sqrt[5]{\left(y^2\right)^5}\times\sqrt[3]{\left(z^3\right)^5}$$
$$= 3\times x\times y^2\times z^3$$
$$= 3xy^2z^3$$

(iv) $\dfrac{4}{5}\sqrt[3]{125}$ **(MTN 2013, SGD 2013)** **(A.B)**

Solution:

$$\frac{4}{5}\sqrt[3]{125}$$
$$= \frac{4}{5}\sqrt[3]{5\times 5\times 5}$$
$$= \frac{4}{5}\sqrt[3]{5^3}$$
$$= \frac{4}{\cancel{5}}\times\cancel{5}$$
$$= 4$$

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$ **(A.B)**

Solution:

$$\sqrt{21}\times\sqrt{7}\times\sqrt{3}$$
$$= \sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$$
$$= \sqrt{7\times 3\times 7\times 3}$$
$$= \sqrt{7^2\times 3^2}$$
$$= \sqrt{7^2}\times\sqrt{3^2}$$
$$= 7\times 3$$
$$= 21$$

Q.3 Simplify by combining similar terms.

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ **(A.B)**

(D.G.K 2017)

Solution:

$$\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$$
$$= \sqrt{9\times 5} - 3\sqrt{5\times 4} + 4\sqrt{5}$$
$$= \sqrt{3^2}\times\sqrt{5} - 3\sqrt{2^2}\times\sqrt{5} + 4\sqrt{5}$$
$$= 3\sqrt{5} - 3\times 2\sqrt{5} + 4\sqrt{5}$$
$$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$$
$$= \sqrt{5}(3 - 6 + 4)$$
$$= \sqrt{5}(3 - 2)$$
$$= \sqrt{5}(1)$$
$$= \sqrt{5}$$

(ii) $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$ **(A.B)**

Solution:

$$4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$$
$$= 4\sqrt{2^2\times 3} + 5\sqrt{3^2\times 3} - 3\sqrt{5^2\times 3} + \sqrt{10^2\times 3}$$
$$= 4\times 2\sqrt{3} + 5\times 3\sqrt{3} - 3\times 5\sqrt{3} + 10\sqrt{3}$$
$$= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3}$$
$$= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3}$$
$$= \sqrt{3}\left(8 + \cancel{15} - \cancel{15} + 10\right)$$
$$= \sqrt{3}(8 + 10)$$
$$= \sqrt{3}(18)$$
$$= 18\sqrt{3}$$

(iii) $\sqrt{3}\left(2\sqrt{3} + 3\sqrt{3}\right)$ **(A.B)**

Solution:

$\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$

$=\sqrt{3}\times\sqrt{3}(2+3)$

$=\left(\sqrt{3}\right)^2\times(5)$

$=3(5)$

$=15$

(iv) $2\left(6\sqrt{5}-3\sqrt{5}\right)$ **(LHR 2016)** **(A.B)**

Solution:

$2\left(6\sqrt{5}-3\sqrt{5}\right)$

$=2\times\sqrt{5}(6-3)$

$=2\times\sqrt{5}(3)$

$=6\sqrt{5}$

Q.4 Simplify

(FSD 2016, SGD 2013, BWP 2014)

(i) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

Solution: **(A.B)**

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=(3)^2-\left(\sqrt{3}\right)^2$

$=9-3$

$=6$

(ii) $\left(\sqrt{5}+\sqrt{3}\right)^2$ **(A.B)**

Solution:

$\left(\sqrt{5}+\sqrt{3}\right)^2$

$\because (a+b)^2=a^2+2ab+b^2$

$=\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$

$=5+2\sqrt{5\times3}+3$

$=8+2\sqrt{15}$

(iii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$ **(A.B)**

Solution:

$\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$

$=5-3$

$=2$

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$ **(A.B)**

(BWP 2014)

Solution:

$\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=\left(\sqrt{2}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2$

$=2-\frac{(1)^2}{\left(\sqrt{3}\right)^2}$

$=2-\frac{1}{3}$

$=\frac{6-1}{3}$

$=\frac{5}{3}$

(v) $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

(A.B)

Solution:

$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=\left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right](x+y)\left(x^2+y^2\right)$

$=(x-y)(x+y)\left(x^2+y^2\right)$

$=\left[(x)^2-(y)^2\right]\left(x^2+y^2\right)$

$=\left(x^2-y^2\right)\left(x^2+y^2\right)$

$=\left[\left(x^2\right)^2-\left(y^2\right)^2\right]=x^4-y^4$