Mathematics-9
Exercise 4.4

Monomial Surd (GRW 2017) **(K.B)**

A surd which contains a Single term is called a monomial Surd e.g., $\sqrt{2}\sqrt{3}$ etc.

Binomial Surd **(K.B)**

A surd containing two terms is called binomial surd.

Or

A Surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

For example:

$\sqrt{3}+\sqrt{7}$, $\sqrt{2}+5$ etc.

Conjugate surds **(K.B)**

Such pairs of binomial surds whose product is a rational number are called conjugate of each other.

Or

Two binomial surds of second order differing only in sign connecting their terms are called conjugate surds.

$\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ are conjugate surds of each other.

$\because \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$

$=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b$

The conjugate of $x+\sqrt{y}$ is $x-\sqrt{y}$.

For example:

$\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)=(3)^2-\left(\sqrt{5}\right)^2$

$=9-5=4$

which is a rational number.

Rationalizing a Denominator **(K.B)**

Rationalizing Real Numbers of the types

$\frac{1}{a+b\sqrt{x}}, \frac{1}{\sqrt{x}+\sqrt{y}}$

Example # 3 **(A.B)**

Simplify: $\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

Solution:

First we shall rationalize the denominators and then simplify

$\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

$$=\frac{6}{2\sqrt{3}-\sqrt{6}}\times\frac{2\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$=\frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{\left(2\sqrt{3}\right)^2-\left(\sqrt{6}\right)^2}+\frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{4\sqrt{3}\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{12-6}+\frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{3-2}-\frac{4\sqrt{3}\left(\sqrt{6}+\sqrt{2}\right)}{6-2}$$

$$=\frac{12\sqrt{3}+6\sqrt{6}}{12-6}+\frac{\sqrt{6}\times\sqrt{3}-\sqrt{2}\sqrt{6}}{1}-\frac{4\sqrt{3}\sqrt{6}+4\sqrt{3}\sqrt{2}}{4}$$

$$=2\sqrt{3}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}-3\sqrt{2}-\sqrt{6}$$

$$=0$$

Example # 5 **(A.B)**

If $x=3+\sqrt{8}$, then evaluate

(i) $x+\frac{1}{x}$ and (ii) $x^2+\frac{1}{x^2}$

Solution:

Here

$$x=3+\sqrt{8}$$

$$\frac{1}{x}=\frac{1}{3+\sqrt{8}}$$

Rationalizing the denominator

$$=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}$$

$$=\frac{3-\sqrt{8}}{9-8}$$

$$=3-\sqrt{8}$$

(i) $x+\frac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)$

$$=3+\sqrt{8}+3-\sqrt{8}$$

$$=6$$

(ii) Consider

$$x+\frac{1}{x}=6$$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=6^2$$

$$x^2+\frac{1}{x^2}+2=36$$

$$x^2+\frac{1}{x^2}=36-2$$

$$\Rightarrow x^2+\frac{1}{x^2}=34$$

Exercise 4.4

Rationalize the denominator of the following

Q.1

(i) $\frac{3}{4\sqrt{3}}$ **(A.B)**

Solution:

$$\frac{3}{4\sqrt{3}}$$

$$=\frac{3}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{3\left(\sqrt{3}\right)}{4\left(\sqrt{3}\right)^2}$$

$$=\frac{3\sqrt{3}}{4\times3}$$

$$=\frac{\sqrt{3}}{4}$$

(ii) $\frac{14}{\sqrt{98}}$ **(A.B)**

Solution:

$$\frac{14}{\sqrt{98}}$$

$$=\frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$$

$$=\frac{14\left(\sqrt{98}\right)}{\left(\sqrt{98}\right)^2}$$

$$=\frac{14\left(\sqrt{7\times7\times2}\right)}{98}$$

$$=\frac{14\times7\times\sqrt{2}}{98}$$

$$=\frac{\cancel{98}\times\sqrt{2}}{\cancel{98}}$$

$$=\sqrt{2}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$ **(A.B)**

Solution:

$$\frac{6}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6}{\sqrt{2^2\times2}\sqrt{3^2\times3}}$$

$$=\frac{6}{2\times3\sqrt{2}.\sqrt{3}}$$

$$=\frac{1}{\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{6}}{6}$$

(iv) $\frac{1}{3+2\sqrt{5}}$ **(A.B)**

Solution:

$$\frac{1}{3+2\sqrt{5}}$$

$$=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-\left(2\sqrt{5}\right)^2}$$

$$=\frac{3-2\sqrt{5}}{9-4(5)}$$

$$=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$

$$=\frac{-3+2\sqrt{5}}{11}$$

(v) $\frac{15}{\sqrt{31}-4}$ **(A.B)**

Solution:

$$\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^2}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$

(vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$ **(RWP 2016)** **(A.B)**

Solution:

$$\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$

(vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ **(BWP 2017)** **(A.B)**

Solution:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+(1)^2}{2}$$

$$=\frac{3-2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$=\frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}$$

$$=2-\sqrt{3}$$

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **(A.B)**

Solution:

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{5-3}$$

$$=\frac{5+2\sqrt{15}+3}{2}$$

$$=\frac{8+2\sqrt{15}}{2}$$

$$=\frac{\cancel{2}\left(4+\sqrt{15}\right)}{\cancel{2}}$$

$$=4+\sqrt{15}$$

Q.2 Find the conjugate of $x+\sqrt{y}$ **(K.B)**

(i) $3+\sqrt{7}$

Conjugate $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Conjugate $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Conjugate $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Conjugate $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Conjugate $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Conjugate $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Conjugate $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Conjugate $9-\sqrt{2}$

Q.3 **(GRW 2017, SWL 2017)**

(i) If $x=2-\sqrt{3}$, find $\frac{1}{x}$

Solution:

Given that

$x=2-\sqrt{3}$

$\Rightarrow \frac{1}{x}=\frac{1}{2-\sqrt{3}}$

Rationalizing the denominator

$=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$=\frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$

$=\frac{2+\sqrt{3}}{4-3}$

$=\frac{2+\sqrt{3}}{1}$

$\frac{1}{x}=2+\sqrt{3}$

(ii) If $x=4-\sqrt{17}$, find $\frac{1}{x}$ **(LHR 2014)**

Solution:

Given that $x=4-\sqrt{17}$

$\Rightarrow \frac{1}{x}=\frac{1}{4-\sqrt{17}}$

$=\frac{1}{4-\sqrt{17}}\times\frac{4+\sqrt{17}}{4+\sqrt{17}}$

$=\frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$

$=\frac{4+\sqrt{17}}{16-17}$

$=\frac{4+17}{-1}$

$=-1\left(4+\sqrt{17}\right)$

$\Rightarrow \frac{1}{x}=-4-\sqrt{17}$

(iii) If $x=\sqrt{3}+2$, find $x+\frac{1}{x}$

Solution: Given that $x=\sqrt{3}+2$ **(A.B)**

$\frac{1}{x}=\frac{1}{\sqrt{3}+2}$

$=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$

$=\frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$

$=\frac{\sqrt{3}-2}{3-4}$

$=\frac{\sqrt{3}-2}{-1}$

$=-\left(\sqrt{3}-2\right)$

$$= -\sqrt{3}+2$$

Now

$$x+\frac{1}{x}=\left(\sqrt{3}+2\right)+\left(-\sqrt{3}+2\right)$$
$$=\sqrt{3}+2-\sqrt{3}+2$$
$$=2+2$$
$$x+\frac{1}{x}=4$$

Q.4 Simplify

(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ **(A.B)**

Solution:

$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$
$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)+\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$
$$\because \left(a+b\right)\left(a-b\right)=a^2-b^2$$
$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$
$$=\frac{2\sqrt{5}-2\sqrt{6}}{5-3}$$
$$=\frac{\cancel{2}\left(\sqrt{5}-\sqrt{6}\right)}{\cancel{2}}$$
$$=\sqrt{5}-\sqrt{6}$$

(ii) $\dfrac{1}{2+\sqrt{3}}+\dfrac{2}{\sqrt{5}-\sqrt{3}}+\dfrac{1}{2+\sqrt{5}}$ **(A.B)**

Solution:

$$\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$$
$$=\left(\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)+\left(\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$
$$+\left(\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$
$$=\left(\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)$$
$$+\left(\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}\right)$$
$$=\left(\frac{2-\sqrt{3}}{4-3}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right)$$
$$=\left(\frac{2-\sqrt{3}}{1}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)+\left(\frac{2-\sqrt{5}}{-1}\right)$$
$$=2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$
$$=\cancel{2}-\cancel{2}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}+\sqrt{5}+\sqrt{5}$$
$$=\sqrt{5}+\sqrt{5}$$
$$=2\sqrt{5}$$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$ **(A.B)**

Solution:

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$
$$=\left(\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$$
$$-\left(\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}\right)$$
$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}\right)$$
$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}\right)$$
$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{1}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}\right)$$
$$=\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}-\cancel{\sqrt{5}}+\cancel{\sqrt{2}}$$
$$=0$$

Q.5 **If** $x=2+\sqrt{3}$, **then find the value of** $x-\frac{1}{x}$ **and** $\left(x-\frac{1}{x}\right)^2$ **(A.B)**

Solution:

Given that $x=2+\sqrt{3}$

$$\Rightarrow \frac{1}{x}=\frac{1}{2+\sqrt{3}}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$=\frac{2-\sqrt{3}}{4-3}$$

$$=\frac{2-\sqrt{3}}{1}$$

$$=2-\sqrt{3}$$

Now

$$x-\frac{1}{x}=(2+\sqrt{3})-(2-\sqrt{3})$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=\sqrt{3}+\sqrt{3}$$

$$=2\sqrt{3}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(2\sqrt{3})^2$$

$$=4(\sqrt{3})^2$$

$$=4(3)$$

$$=12$$

(i) **If** $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, **find the value of** $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ **and** $x^3+\frac{1}{x^3}$

Solution: **(A.B)**

Given that $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$\Rightarrow \frac{1}{x}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

Now

$$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{(\sqrt{5}-\sqrt{2})^2+(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$$\because (a+b)^2+(a-b)^2=2(a^2+b^2),$$

$$(a+b)(a-b)=a^2-b^2$$

$$=\frac{2\left[(\sqrt{5})^2+(\sqrt{2})^2\right]}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$=\frac{2(5+2)}{5-2}$$

$$\Rightarrow x+\frac{1}{x}=\frac{14}{3}$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$$\because (a+b)^2=a^2+b^2+2ab$$

$$x^2+\frac{1}{x^2}+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=\frac{196}{9}$$

$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

Consider

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking cube on both sides

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$\because (a+b)^3=a^3+b^3+3ab(a+b)$$

$$x^3+\frac{1}{x^3}+3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+14=\frac{2744}{24}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2366}{27}$$

Q.6 Determine the rational numbers a **and** b **if** $\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$

(A.B)

Solution:

Given that

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

Or $a+b\sqrt{3}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$=\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$\because$ $(a+b)^2+(a-b)^2=2\left(a^2+b^2\right)$,

$(a+b)(a-b)=a^2-b^2$

$$=\frac{2\left[\left(\sqrt{3}\right)^2+(1)^2\right]}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2(3+1)}{3-1}$$

$$=\frac{2(4)}{2}$$

$$a+b\sqrt{3}=4$$

$$a+b\sqrt{3}=4+0\sqrt{3}$$

Comparing both sides

$a=4 \quad b\sqrt{3}=0\sqrt{3}$

$$b=\frac{0\sqrt{3}}{\sqrt{3}}$$

$$b=0$$