Mathematics-9
Review Exercise - 4

(i) **is an algebraic …** **(K.B)**

(a) Expression (b) Sentence
(c) Equation (d) In-equation

(ii) **The degree of polynomial $4x^4+3x^2y$ is** **(K.B)**

(FSD 2014, SGD 2017, MTN 2016, BWP 2016)

(a) 1 (b) 2
(c) 3 (d) 4

(iii) a^3+b^3 **is equal to** **(U.B)**

(a) $(a-b)(a^2+ab+b^2)$ (b) $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$ (d) $(a-b)(a^2+ab+b^2)$

(iv) $(3+\sqrt{2})(3-\sqrt{2})$ **is equal to** **(A.B)**

(a) 7 (b) -7
(c) -1 (d) 1

(v) **Conjugate of surd $a+\sqrt{b}$ is;** **(U.B)**

(a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$
(c) $\sqrt{a}+\sqrt{b}$ (d) $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{a-b}-\frac{1}{a+b}$ **is equal to** **(A.B)**

(LHR 2015, FSD 2015, MTN 2014, BWP 2017, D.G.K 2013, 15)

(a) $\frac{2a}{a^2-b^2}$ (b) $\frac{2b}{a^2-b^2}$
(c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ **is equal to** **(FSD 2016, SWL 2013, BWP 2014, RWP 2013, 17)** **(U.B)**

(a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to** **(U.B)**

(a) a^2+b^2 (b) a^2-b^2
(c) $a-b$ (d) $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	Viii
a	d	b	a	b	b	d	C

Q.2 Fill in the blanks

(i) **The degree of polynomial** $x^2y^2+3xy+y^3$ **is** __________ **(U.B)**

(ii) x^2-4__________ **(U.B)**

(iii) $x^3+\frac{1}{x^3}=\left[x+\frac{1}{x}\right](\_\_\_\_\_\_\_\_\_\_)$ **(U.B)**

(iv) $2(a^2+b^2)=(a+b)^2+(\_\_\_\_\_\_\_\_\_\_)^2$ **(U.B)**

(v) $\left[x-\frac{1}{x}\right]^2=$ __________ **(U.B)**

(vi) **Order of surd** $\sqrt[3]{x}$ **is** __________ **(U.B)**

(vii) $\frac{1}{2-\sqrt{3}}=$ ______________ **(U.B)**

ANSWER KEY

(i) 4

(ii) $(x-2)(x+2)$

(iii) $x^2-1+\frac{1}{x^2}$

(iv) $a-b$

(v) $x^2+\frac{1}{x^2}-2$

(vi) 3

(vii) $2+\sqrt{3}$

Q.3 **If** $x+\frac{1}{x}=3,$ **find** **(A.B)**

(i) $x^2+\frac{1}{x^2}$ **(ii)** $x^4+\frac{1}{x^4}$

Solution:

Given that $x+\frac{1}{x}=3$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=3^2$$

$$\because (a+b)^2=a^2+b^2+2ab$$

$$(x)^2+\left(\frac{1}{x}\right)^2+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=9$$

$$x^2+\frac{1}{x^2}+2=9$$

$$x^2+\frac{1}{x^2}=9-2$$
$$x^2+\frac{1}{x^2}=7$$

(ii) **For** $x^4+\frac{1}{x^4}$

Here $x^2+\frac{1}{x^2}=7$

Squaring both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=7^2$$
$$x^4+\frac{1}{x^4}+2=49$$
$$x^4+\frac{1}{x^4}=49-2$$
$$x^4+\frac{1}{x^4}=47$$

Q.4 **If** $x-\frac{1}{x}=2$ **find**

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$ **(A.B)**

Solution (i)

Given that $x-\frac{1}{x}=2$

Squaring both sides

$$\left(x-\frac{1}{x}\right)^2=2^2$$
$$\because (a+b)^2=a^2+b^2+2ab$$

Putting the values

$$\left(x-\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2-2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)$$
$$(2)^2=x^2+\frac{1}{x^2}-2$$
$$4+2=x^2+\frac{1}{x^2}$$
$$x^2+\frac{1}{x^2}=6$$

Solution (ii)

Here $x^2+\frac{1}{x^2}=6$

Again squaring both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=6^2$$
$$x^4+\frac{1}{x^4}+2\left(\cancel{x^2}\right)\left(\frac{1}{\cancel{x^2}}\right)=36$$
$$x^4+\frac{1}{x^4}+2=36$$
$$x^4+\frac{1}{x^4}=36-2$$
$$x^4+\frac{1}{x^4}=34$$

Q.5 **Find the value of** x^3+y^3 **and** xy **if** $x+y=5$ **and** $x-y=3$. **(A.B)**

Solution:

Formula

$$(x+y)^2-(x-y)^2=4xy$$

Putting the values

$$(5)^2-(3)^2=4xy$$
$$4xy=25-9$$
$$4xy=16$$
$$xy=\frac{\cancel{16}^{4}}{\cancel{4}}$$
$$xy=4$$

As we know that

$$(x+y)^3=x^3+y^3+3xy(x+y)$$

Putting the values

$$(5)^3=x^3+y^3+3\times4\times5$$
$$125=x^3+y^3+60$$
$$x^3+y^3=125-60$$
$$x^3+y^3=65$$

Q.6 **If** $P=2+\sqrt{3}$ **, find** **(A.B)**

(i) $P+\frac{1}{P}$

Solution:

Given that $P=2+\sqrt{3}$

$$\frac{1}{P}=\frac{1}{2+\sqrt{3}}$$

Rationalizing the denominator

$$\frac{1}{P}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}\quad\because(a+b)(a-b)=a^2-b^2$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{1}$$

$$\frac{1}{P}=2-\sqrt{3}$$

$$P+\frac{1}{P}=2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$$

$$P+\frac{1}{P}=4$$

(ii) **For** $P-\frac{1}{P}$

$$P-\frac{1}{P}=(2+\sqrt{3})-(2-\sqrt{3})$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=2\sqrt{3}$$

(iii) **For** $P^2+\frac{1}{P^2}$

Here $P+\frac{1}{P}=4$

Squaring both sides

$$\left(P+\frac{1}{P}\right)^2=(4)^2$$

$$\therefore(a+b)^2=a^2+b^2+2ab$$

$$(P)^2+\left(\frac{1}{P}\right)^2+2(\cancel{P})\left(\frac{1}{\cancel{P}}\right)=16$$

$$P^2+\frac{1}{P^2}+2=16$$

$$P^2+\frac{1}{P^2}=16-2$$

$$P^2+\frac{1}{P^2}=14$$

(iv) **For** $P^2-\frac{1}{P^2}$

Formula

$$P^2-\frac{1}{P^2}=\left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)$$

Putting the values

$$P^2-\frac{1}{P^2}=(4)(2\sqrt{3})$$

$$=8\sqrt{3}$$

Q.7 **If** $q=\sqrt{5}+2$ **find.** **(A.B)**

(i) $q+\frac{1}{q}$ **(ii)** $q-\frac{1}{q}$

(iii) $q^2+\frac{1}{q^2}$ **(iv)** $q^2-\frac{1}{q^2}$

Solution:

(i) **For** $q+\frac{1}{q}$

Given that $q=\sqrt{5}+2$

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}$$

Rationalizing the denominator

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$=\frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}\quad\because(a+b)(a-b)=a^2-b^2$$

$$=\frac{\sqrt{5}-2}{5-4}$$

$$=\sqrt{5}-2$$

Now

$$q+\frac{1}{q}=(\sqrt{5}+2)+(\sqrt{5}-2)$$

$$q+\frac{1}{q}=\sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$$

$$q+\frac{1}{q}=2\sqrt{5}$$

(ii) **For** $q-\frac{1}{q}$

$$q-\frac{1}{q}=\left(\sqrt{5}+2\right)-\left(\sqrt{5}-2\right)$$

$$=\cancel{\sqrt{5}}+2-\cancel{\sqrt{5}}+2$$

$$q-\frac{1}{q}=4$$

(iii) **For** $q^2+\frac{1}{q^2}$

$$q-\frac{1}{q}=4$$

Squaring both sides

$$\left(q-\frac{1}{q}\right)^2=(4)^2$$

$$q^2+\frac{1}{q^2}-2=16$$

$$q^2+\frac{1}{q^2}=16+2$$

$$q^2+\frac{1}{q^2}=18$$

(iv) **For** $q^2-\frac{1}{q^2}$ **(A.B)**

By using formula

$$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}\,2}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2}$$

Putting the values

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5}$$

Q.8 **Simplify** **(A.B)**

(i) $\dfrac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

Solution:

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

Rationalizing the denominator

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$\because (a+b)^2=a^2+2ab+b^2,\ (a+b)(a-b)=a^2-b^2$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{\left(a^2+2\right)-\left(a^2-2\right)}$$

$$=\frac{a^2+\cancel{2}+a^2-\cancel{2}+2\left(\sqrt{\left(a^2+2\right)\left(a^2-2\right)}\right)}{a^2+2-a^2+2}$$

(ii) $\dfrac{1}{a-\sqrt{a^2-x^2}}-\dfrac{1}{a+\sqrt{a^2-x^2}}$ **(A.B)**

Solution:

$$\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$$

$$=\frac{1\left(a+\sqrt{a^2-x^2}\right)-1\left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)\left(a+\sqrt{a^2-x^2}\right)}$$

$$\because (a+b)(a-b)=a^2-b^2$$

$$=\frac{a+\sqrt{a^2-x^2}-a+\sqrt{a^2-x^2}}{a^2-\left(\sqrt{a^2-x^2}\right)^2}$$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-\left(a^2-x^2\right)}$$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-a^2+x^2}$$

$$=\frac{2\sqrt{a^2-x^2}}{x^2}$$