Unit – 5

Factorization

Factorization   (U.B)

If a polynomial $p(x)$ can be expressed as $p(x) = g(x)h(x)$, then each of the polynomial, $g(x)$ and $h(x)$ is called a factor of $p(x)$.

For example:

$ab + ac = a(b+c)$, then a and $(b+c)$ are factors of $(ab+ac)$.

Note   (K.B)

When a polynomial has been written as a product consisting only of prime factors, then it is said to be factored completely.

Important role of Factorization in Mathematics

Factorization plays an important role in mathematics as it helps to reduce the study of a complicated expression to the study of simpler expressions.

(a) Factorization of the Expression of the Type $Ka + Kb + Kc$

Example # 1   (K.B)

Factorize $5a - 5b + 5c$

Solution: (A.B)

$$5a - 5b + 5c = 5(a - b + c)$$

Example # 2

Factorize

$$5a - 5b - 15c = 5(a - b - 3c)$$

(b) Factorization of the Expression of the Type $ac + ad + bc + bd$

Example   (K.B)

Factorize: $ac + ad + bc + bd$

Solution:

$$ac + ad + bc + bd$$
$$= (ac + ad) + (bc + db)$$
$$= a(c + d) + b(c + d)$$
$$= (c + d)(a + b)$$

Example 2 (Page # 99)

Factorize: $pqr + qr^2 - pr^2 - r^3$

Solution: (A.B)

$$pqr + qr^2 - pr^2 - r^3$$
$$= r(pq + qr - pr - r^2)$$
$$= r[q(p + r) - r(p + r)]$$
$$= r(p + r)(q - r)$$

(c) Factorization of the Expression of Type $a^2 \pm 2ab + b^2$ (K.B)

We know that

(i) $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

(ii) $a^2 - 2ab + b^2 = (a-b)^2 = (a-b)(a-b)$

Example # 1

Factorize: $25x^2 + 40x + 16$

Solution: (A.B)

$$25x^2 + 40x + 16$$
$$= (5x)^2 + 2(5x)(4) + (4)^2$$
$$\because a^2 + 2ab + b^2 = (a+b)^2$$
$$= (5x + 4)^2$$
$$= (5x + 4)(5x + 4)$$

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Example # 2   (A.B)

Factorize: $12x^2 - 36x + 27$

Solution:

$$12x^2 - 36x + 27 = 3\left[4x^2 - 12x + 9\right]$$

$$= 3\left[(2x)^2 - 2(2x)(3) + (3)^2\right]$$

$$\because a^2 - 2ab + b^2 = (a-b)^2$$

$$= 3\left[2x - 3\right]^2$$

$$= 3(2x-3)(2x-3)$$

(d)   **Factorization of the Expression of the Type** $a^2 - b^2$

Example   (K.B)

Factorize: (i) $4x^2 - (2y - z)^2$

Solution:

(i) $4x^2 - (2y-z)^2 = (2x)^2 - (2y-z)^2$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= \left[2x - (2y-z)\right]\left[2x + (2y-z)\right]$$

$$= (2x - 2y + z)(2x + 2y - z)$$

(ii)   Factorize: $6x^4 - 96$

Solution:

$$6x^4 - 96 = 6\left(x^4 - 16\right)$$

$$= 6\left[(x^2)^2 - 4^2\right]$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= 6\left[x^2 - 4\right]\left[x^2 + 4\right]$$

$$= 6\left[x^2 - 2^2\right]\left[x^2 + 4\right]$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= 6(x-2)(x+2)\left(x^2 + 4\right)$$

(e)   **Factorization of the Expression of the Types** $a^2 \pm 2ab + b^2 - c^2$

We know that   (K.B)

$$a^2 \pm 2ab + b^2 - c^2$$

$$= (a \pm b)^2 - c^2 = (a \pm b - c)(a \pm b + c)$$

Example   (RWP 2016)   (A.B)

Factorize:

(i) $x^2 + 6x + 9 - 4y^2$

(ii) $1 + 2ab - a^2 - b^2$

Solution:

(i) $x^2 + 6x + 9 - 4y^2 = x^2 + 2(x)(3) + 3^2 - 4y^2$

$$= (x+3)^2 - (2y)^2$$

$$\because a^2 + 2ab + b^2 = (a+b)^2$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= \left[x + 3 - 2y\right]\left[x + 3 + 2y\right]$$

(ii) $1 + 2ab - a^2 - b^2 = 1 - \left(a^2 - 2ab + b^2\right)$

(GRW 2016, RWP 2017)

$$= (1)^2 - (a-b)^2$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= \left[1 - (a-b)\right]\left[1 + (a-b)\right]$$

$$= \left[1 - a + b\right]\left[1 + a - b\right]$$

Exercise 5.1

Q.1   **Factorize:**   (K.B)

(i) $2abc - 4abx + 2abd$

Solution:

$$2abc - 4abx + 2abd$$

$$= 2ab(c - 2x + d)$$

(ii) $9xy - 12x^2y + 18y^2$

Solution:

$$9xy - 12x^2y + 18y^2$$

$$= 3y(3x - 4x^2 + 6y)$$

(iii) $-3x^2y - 3x + 9xy^2$

Solution:

$$-3x^2y - 3x + 9xy^2$$

$$= -3x(xy + 1 - 3y^2)$$

(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

Solution:

$$5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$$

$$= 5abc(bc^2 - 2ab^2 - 4a^2c)$$

Unit – 5   **Factorization**

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

Solution:

$$3x^3y(x-3y)-7x^2y^2(x-3y)$$
$$=(x-3y)(3x^3y-7x^2y^2)$$
$$=(x-3y)\,x^2y(3x-7y)$$
$$=x^2y(x-3y)(3x-7y)$$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

Solution:

$$2xy^3(x^2+5)+8xy^2(x^2+5)$$
$$=(x^2+5)(2xy^3+8xy^2)$$
$$=(x^2+5)2xy^2(y+4)$$
$$=2xy^2(x^2+5)(y+4)$$

Q.2   Factorize   **(K.B)**

(i) $5ax-3ay-5bx+3by$

Solution:

$$5ax-3ay-5bx+3by$$
$$=5ax-5bx-3ay+3by$$
$$=5x(a-b)-3y(a-b)$$
$$=(a-b)(5x-3y)$$

(ii) $3xy+2y-12x-8$

Solution:

$$3xy+2y-12x-8$$
$$=3xy-12x+2y-8$$
$$=3x(y-4)+2(y-4)$$
$$=(y-4)(3x+2)$$

(iii) $x^3+3xy^2-2x^2y-6y^3$

Solution:

$$x^3+3xy^2-2x^2y-6y^3$$
$$=x^3-2x^2y+3xy^2-6y^3$$
$$=x^2(x-2y)+3y^2(x-2y)$$
$$=(x-2y)(x^2+3y^2)$$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

Solution:

$$(x^2-y^2)z+(y^2-z^2)x$$

$$=x^2z-y^2z+xy^2-xz^2$$
$$=x^2z+xy^2-xz^2-y^2z$$
$$=x^2z+xy^2-y^2z-xz^2$$
$$=x(xz+y^2)-z(xz+y^2)$$
$$=(xz+y^2)(x-z)$$

Q.3   Factorize

(i) $144a^2+24a+1$

Solution: **(K.B)**

$$144a^2+24a+1$$
$$=(12a)^2+2(12a)(1)+(1)^2$$
$$\because\ a^2+2ab+b^2=(a+b)^2$$
$$=(12a+1)^2$$

(ii) $\dfrac{a^2}{b^2}-2+\dfrac{b^2}{a^2}$

                                (FSD 2014, MTN 2016, D.G.K 2017)

Solution:

$$\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$$

$$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2$$

$$\because\ a^2-2ab+b^2=(a-b)^2$$

$$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

(iii) $(x+y)^2-14z(x+y)+49z^2$

Solution:

$$(x+y)^2-14z(x+y)+49z^2$$
$$\because\ a^2-2ab+b^2=(a-b)^2$$
$$=(x+y)^2-2(x+y)(7z)+(7z)^2$$
$$=(x+y-7z)^2$$

(iv) $12x^2-36x+27$

                                (SWL 2017, BWP 2016, FSD 2016)

Solution:

$$12x^2-36x+27$$
$$=3(4x^2-12x+9)$$

Unit – 5   **Factorization**

$$\because a^2 - 2ab + b^2 = (a - b)^2$$

$$= 3\left[(2x)^2 - 2(2x)(3) + (3)^2\right]$$

$$= 3(2x - 3)^2$$

Q.4   Factorize

(i) $3x^2 - 75y^2$ **(K.B+U.B)**

(LHR 2017, GRW 2014, BWP 2014)

Solution:

$$3x^2 - 75y^2$$

$$= 3\left(x^2 - 25y^2\right)$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= 3\left[(x)^2 - (5y)^2\right]$$

$$= 3(x + 5y)(x - 5y)$$

(ii) $x(x - 1) - y(y - 1)$ **(SGD 2015)**

Solution:

$$x(x - 1) - y(y - 1)$$

$$= x^2 - x - y^2 + y$$

$$= x^2 - y^2 - x + y$$

$$= \left(x^2 - y^2\right) - (x - y)$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= \left[(x + y)(x - y)\right] - (x - y)$$

$$= (x - y)(x + y - 1)$$

(iii) $128am^2 - 242an^2$

(MTN 2017, BWP 2014, D.G.K 2014)

Solution:

$$128am^2 - 242an^2$$

$$= 2a\left(64m^2 - 121n^2\right)$$

$$= 2a\left[(8m)^2 - (11n)^2\right]$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= 2a(8m + 11n)(8m - 11n)$$

(iv) $3x - 243x^3$ **(MTN 2017, FSD 2017)**

Solution:

$$3x - 243x^3$$

$$= 3x\left(1 - 81x^2\right)$$

$$= 3x\left[(1)^2 - (9x)^2\right]$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= 3x(1 + 9x)(1 - 9x)$$

Q.5   Factorize

(i) $x^2 - y^2 - 6y - 9$

Solution:

$$x^2 - y^2 - 6y - 9$$

$$= x^2 - \left[y^2 + 6y + 9\right]$$

$$= x^2 - [(y)^2 + 2(y)(3) + (3)^2]$$

$$\because (a + b)^2 = a^2 + 2ab + b^2$$

$$= x^2 - (y + 3)^2$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= (x + y + 3)\left[x - (y + 3)\right]$$

$$= (x + y + 3)(x - y - 3)$$

(ii) $x^2 - a^2 + 2a - 1$ **(GRW 2016)**

Solution:

$$x^2 - a^2 + 2a - 1$$

$$= x^2 - \left[a^2 - 2a + 1\right]$$

$$\because a^2 - 2ab + b^2 = (a - b)^2$$

$$= x^2 - (a - 1)^2$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= \left[x + (a - 1)\right]\left[x - (a - 1)\right]$$

$$= (x + a - 1)(x - a + 1)$$

(iii) $4x^2 - y^2 - 2y - 1$

Solution:

$$4x^2 - y^2 - 2y - 1$$

$$= 4x^2 - (y^2 + 2y + 1)$$

$$= 4x^2 - \left[(y)^2 + 2(y)(1) + (1)^2\right]$$

$$\because (a + b)^2 = a^2 + 2ab + b^2$$

Unit – 5   **Factorization**

$= 4x^2 - (y+1)^2$

$= (2x)^2 - (y+1)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= \left[2x + (y+1)\right]\left[2x - (y+1)\right]$

$= (2x+y+1)(2x-y-1)$

(iv) $x^2 - y^2 - 4x - 2y + 3$ **(LHR 2016)**

Solution:

$\quad x^2 - y^2 - 4x - 2y + 3$

$= x^2 - 4x + 4 - y^2 - 2y - 1$

$= \left(x^2 - 4x + 4\right) - \left(y^2 + 2y + 1\right)$

$= \left[(x)^2 - 2(x)(2) + (2)^2\right]$

$\quad - \left[(y)^2 + 2(y)(1) + (1)^2\right]$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$\because a^2 + 2ab + b^2 = (a+b)^2$

$= (x-2)^2 - (y+1)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= (x-2+y+1)\left[x-2-(y+1)\right]$

$= (x-2+y+1)(x-2-y-1)$

$= (x+y-2+1)(x-y-2-1)$

$= (x+y-1)(x-y-3)$

(v) $25x^2 - 10x + 1 - 36z^2$ **(GRW 2016)**

Solution:

$\quad 25x^2 - 10x + 1 - 36z^2$

$= (5x)^2 - 2(5x)(1) + (1)^2] - 36z^2$

$\quad \because a^2 - 2ab + b^2 = (a-b)^2$

$= (5x-1)^2 - (6z)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= \left[(5x-1) + 6Z\right]\left[(5x-1) - 6Z\right]$

$= (5x-1+6z)(5x-1-6z)$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solution:

$\quad x^2 - y^2 - 4xz + 4z^2$

$= x^2 - 4xz + 4z^2 - y^2$

$= \left[(x)^2 - 2(x)(2z) + (2z)^2\right] - y^2$

$\quad \because a^2 - 2ab + b^2 = (a-b)^2$

$= (x-2z)^2 - (y)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= (x-2z+y)(x-2z-y)$

$= (x+y-2z)(x-y-2z)$