U_{nit}–3

Variations

Exercise 3.1

Q.1 **Expressing as a ratio and as a fraction.**
Solution:

(i) Here   (GRW 2016, FSD 2017) **(A.B)**
a = Rs.750
b = Rs.1250
Now,

$$a:b \quad = 750:1250 \qquad (\div \, by \, 10)$$
$$= 75:125 \qquad (\div \, by \, 5)$$
$$= 15:25 \qquad (\div \, by \, 5)$$
$$\Rightarrow a:b = 3:5$$

$\therefore$ **The fractional form of this expression is** $\dfrac{3}{5}$

(ii) Here   **(A.B)**
a = 450 cm
b = 3m = 3 × 100cm = 300 cm
Now,

$$a:b \quad = 450:300 \qquad (\div \, by \, 10)$$
$$= 45:30 \qquad (\div \, by \, 5)$$
$$\Rightarrow a:b = 3:2 \qquad (\div \, by \, 15)$$

$\therefore$ **The fractional form of this expression is** $\dfrac{3}{2}$.

(iii) Here   **(A.B)**
$$a = 4kg = 4 \times 1000 \, gm = 4000 \, gm$$
$$b = 2kg.750 \, gm = [(2 \times 1000) + 750] \, gm$$
$$= 2750 \, gm$$
Now
$$a:b \quad = 4000:2750$$
$$= 400:275 \quad (\div \, by \, 10)$$
$$= 80:55 \qquad (\div \, by \, 5)$$

$$\Rightarrow a:b = 16:11 \qquad (\div \, by \, 5)$$

The fractional form of this expression is $\dfrac{16}{11}$

Unit–3   **Variations**

(iv) $a = 27$ min 30 sec $= (27 \times 60 + 30)$ sec
$$= (1620 + 30) \text{ sec}$$
$$= 1650 \text{ sec}$$
$$b = 1 \text{ hour} = 1 \times 60 \times 60 \text{ sec}$$
$$= 3600 \text{ sec}$$
$$\Rightarrow a : b = 1650 \text{ sec} : 3600 \text{ sec}$$
$$= 1650 : 3600$$
$$= 165 : 360 \quad (\div \text{ by } 10)$$
$$= 33 : 72 \quad (\div \text{ by } 5)$$
$$= 11 : 24 \quad (\div \text{ by } 3)$$

The fractional form of this expression is $\dfrac{11}{24}$

(v)   Here   **(A.B)**
(SWL 2015, BWP 2016, D.G.K 2014)
$$a = 75^o, b = 225^o$$
Now, $a : b = 75 : 225 \quad (\div 25)$
$$= 3 : 9 \quad (\div 3)$$
$$\Rightarrow a : b = 1 : 3$$
The fractional form of this expression is $\dfrac{1}{3}$.

Q.2   In a class of 60 students, 25 students are girls and remaining students are boys. Computer the ratio of
Given   (A.B + K.B)
Total students in the class = 60
Number of girls in the class = 25
Number of boys in the class = 60 – 25 = 35
Required
(i)   Ratio of boys to total students
(ii)   Ratio of boy to girls.
Solution:
(i)   **(A.B)**
     Now,
     Boys: Total students $= 35 : 60$
$$= 7 : 12 \quad (\div 5)$$
(ii)   Here,   **(A.B)**
     Boys: Girls $= 35 : 25$
$$= 7 : 5 \quad (\div \text{ by } 5)$$

Q.3   If $3(4x - 5y) = 2x - 7y$, find the ratios $x : y$.   (A.B)
Given
$$3(4x - 5y) = 2x - 7y$$
Required   (LHR 2015)
$x : y = ?$ **(MTN 2016)**
Solution:
     Here
$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = 15y - 7y$$
$$10x = 8y$$
$$\frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5}$$
$$\Rightarrow x : y = 4 : 5$$
Result:
$x : y = 4 : 5$

Q.4   Find the value of p, if the ratios $2p + 5 : 3p + 4$ and $3 : 4$ are equal.
Find value of 'p'   (A.B)
(GRW 2015, SWL 2016, RWP 2015, 17)
Solution:
     According to given condition.
$$2p + 5 : 3p + 4 = 3 : 4$$
$$\frac{2p + 5}{3p + 4} = \frac{3}{4}$$
By cross multiplication
$$4(2p + 5) = 3(3p + 4)$$
$$8p + 20 = 9p + 12$$
$$8p - 9p = 12 - 20$$
$$-P = -8$$
$$\Rightarrow P = 8$$
Result:
$p = 8$

Q.5   If the ratios $3x + 1 : 6 + 4x$ and $2 : 5$ are equal. Find the value of x.
Solution: (D.G.K 2015)   (A.B + K.B)
     Here
$$3x + 1 : 6 + 4x = 2 : 5$$
$$\Rightarrow \frac{3x + 1}{6 + 4x} = \frac{2}{5}$$
     By cross multiplication, we get

Unit–3   **Variations**

$5(3x+1)=2(6+4x)$

$15x+5=12+8x$

$15x-8x=12-5$

$7x=7$

$x=\dfrac{7}{7}$

$x=1$

Result

$x=1$

Q.6 **Two numbers are in the ratio $5:8$. If 9 is added to each number, we get a new ratio $8:11$. Find the numbers.** (FSD 2016) **(A.B + K.B)**

Solution:

Ratio between two numbers $=5:8$

Let required numbers are $5x, 8x$

According to given condition

$5x+9:8x+9=8:11$

$\dfrac{5x+9}{8x+9}=\dfrac{8}{11}$

By cross multiplication

$11(5x+9)=8(8x+9)$

$55x+99=64x+72$

$55x-64x=72-99$

$-9x=-27$

$x=\dfrac{-27}{-9}$

$x=3$

Now

$5x=5(3)=15$

And

$8x=8(3)=24$

Result:

∴ Required numbers are 15 and 24

Q.7 **If 10 is added in each number of the ratio $4:13$, we get a new ratio $1:2$. What are the numbers?**

(A.B + K.B)

Solution:

Ratio between two numbers = 4 : 13

Let, the two numbers be $4x$ & $13x$.

According to the given condition;

$4x+10:13x+10=1:2$

$\dfrac{4x+10}{13x+10}=\dfrac{1}{2}$

$2(4x+10)=1(13x+10)$

$8x+20=13x+10$

$20-10=13x-8x$

$10=5x$

$\dfrac{10}{5}=x$

$x=2$

$\Rightarrow\therefore 4x=4(2)=8$

$\Rightarrow 13x=13(2)=26$

Result:

∴ Required two numbers are 8 and 26.

Q.8 **Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250.**

Solution: **(A.B + K.B)**

Weight of mangoes = 5 kg

Cost of mangoes of 5kg = Rs.250

Now, weight of mangoes = 8kg

Here

weight : weight :: cost : cost

$5:8::250:x$

Product of extreme = product of means

$5x=250\times 8$

$5x=2000$

$x=\dfrac{2000}{5}$

$x=400$

Result:

∴ Cost of 8kg of mangoes is Rs. 400.

Q.9 **If $a:b=7:6$, find the value of $3a+5b:7b-5a$.**   **(A.B + K.B)**

Solution: (FSD 2015)

Here

$a:b=7:6$

$\Rightarrow \dfrac{a}{b}=\dfrac{7}{6}$

or $a=\dfrac{7}{6}b$

Consider

$3a+5b:7b-5a=3\left(\dfrac{7}{6}b\right)+5b:7b-5\left(\dfrac{7}{6}b\right)$

$=\dfrac{21b+30b}{6}:\dfrac{42b-35b}{6}$

Unit–3 **Variations**

$$= \frac{51b}{6} : \frac{7b}{6}$$

$$= 51\,b : 7b \qquad (\times\ by\ 6)$$

$$\Rightarrow 3a + 5b : 7b - 5a = 51 : 7 \qquad (\div\ by\ b)$$

Result:

$3a + 5b : 7b - 5a = 51 : 7$

Q.10 Complete the following:

(i) **(A.B + K.B)**

Given Data:

$$\frac{24}{7} = \frac{6}{x}$$

Required

$4x = ?$

Solution:

Consider,

$$\frac{24}{7} = \frac{6}{x}$$

By cross multiplication, we get;

$24(x) = 6(7)$

$24x = 6(7)$

$$\frac{24x}{6} = 7$$

$4x = 7$

Result:

$4x = 7$

(ii) **(A.B + K.B)**

Given Data:

$$\frac{5a}{3x} = \frac{15b}{y}$$

Required

$ay = ?$

Solution:

Consider,

$$\frac{5a}{3x} = \frac{15b}{y}$$

By cross multiplication, we get

$5a(y) = 15b(3x)$

$5ay = 45bx$

$$ay = \frac{45}{5}bx$$

$ay = 9bx$

Result

$ay = 9bx$

(iii) **(SWL 2014)** **(A.B + K.B)**

Given Data:

$$\frac{9pq}{2lm} = \frac{18p}{5m}$$

Required

$5q = ?$

Solution:

Consider,

$$\frac{9pq}{2lm} = \frac{18p}{5m}$$

By cross multiplication, we get

$9pq(5m) = 18p(2lm)$

$45mpq = 36mpl$

$$\frac{45mpq}{36mp} = l$$

$$\frac{5q}{4} = l$$

$5q = 4l$

Result:

$5q = 4l$

Q.11 Find x **in the following proportions.**

(i) **(A.B + K.B)**

(GRW 2014, SWL 2016, MTN 2017, RWP 2017)

Given Data:

$3x - 2 : 4 :: 2x + 3 : 7$ **Required**

$x = ?$

Solution:

$3x - 2 : 4 :: 2x + 3 : 7$

Product of extremes = product of means

$7(3x - 2) = 4(2x + 3)$

$21x - 14 = 8x + 12$

$21x - 8x = 12 + 14$

$13x = 26$

$$x = \frac{26}{13}$$

$x = 2$

Result:

$x = 2$

(ii) $\dfrac{3x-1}{7} : \dfrac{3}{5} :: \dfrac{2x}{3} : \dfrac{7}{5}$ **(A.B + K.B)**

Product of extremes = Product of means

$$\frac{\cancel{3}}{5}\left(\frac{2x}{\cancel{3}}\right) = \frac{\cancel{7}}{5}\left(\frac{3x-1}{\cancel{7}}\right)$$

Unit–3   **Variations**

$$\frac{2x}{\cancel{5}} = \frac{3x-1}{\cancel{5}}$$
$$2x = 3x - 1$$
$$1 = 3x - 2x$$
$$\Rightarrow x = 1$$

(iii) $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

                           (GRW 2014) **(A.B + K.B)**

Product of extremes = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right) = \left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$
$$(x-3)\left(\frac{2}{x+4}\right) = \frac{5}{3}$$

By cross multiplication
$$6(x-3) = 5(x+4)$$
$$6x - 5x = 20 + 18$$
$$x = 38$$

(iv)   Here   **(A.B + K.B)**

$$p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p+q} : (p-q)^2$$

$\therefore$ Product of extremes = Product of means

$$\left(p^2 + pq + q^2\right)(p-q)^2 = x\left(\frac{p^3 - q^3}{p+q}\right)$$
$$x\left(\frac{p^3 - q^3}{p+q}\right) = \left(p^2 + pq + q^2\right)(p-q)^2$$
$$x = \left(p^2 + pq + q^2\right)(p-q)^2 \times \frac{(p+q)}{p^3 - q^3}$$
$$= \frac{\left(p^2 + pq + q^2\right)(p-q)(p-q)(p+q)}{(p-q)\left(p^2 + pq + q^2\right)}$$
$$= (p-q)(p+q)$$
$$\Rightarrow x = p^2 - q^2$$

(v)   Here **(BWP 2014)**   **(A.B + K.B)**
$$8 - x : 11 - x :: 16 - x : 25 - x$$

$\therefore$ Product of extremes = Product of means
$$(8-x)(25-x) = (11-x)(16-x)$$
$$200 - 8x - 25x + \cancel{x^2} = 176 - 11x - 16x + \cancel{x^2}$$
$$200 - 33x = 176 - 27x$$
$$27x - 33x = 176 - 200$$
$$-6x = -24$$

$$x = \frac{-24}{-6}$$
$$\Rightarrow x = 4$$