Exercise 3.3

Q.1 Find a third proportional to

(i) 6, 12 **(A.B)**

(SGD 2017, RWP 2017, D.G.K 2016, 17)

Let, the third proportional = a

According to the given condition;

$6:12::12:a$

$\because$ Product of means = product of extremes

$12(12) = 6a$

$144 = 6a$

$\frac{144}{6} = a$

$\Rightarrow a = 24$

$\therefore$ **Third proportional is** 24

(ii) $a^3, 3a^2$ **(A.B)**

(MTN 2014, 16, RWP 2017, D.G.K 2014)

Let, third proportional = x

According to the given condition;

$a^3 : 3a^2 :: 3a^2 : x$

$\because$ Product of extremes = Product of means

$(a^3)x = (3a^2)(3a^2)$

$a^3x = 9a^4$

$x = \frac{9a^4}{a^3}$

$x = 9a$

$\therefore$ **Third proportional is** $9a$

(iii) $a^2 - b^2, a - b$ **(A.B)** **(LHR 2015, GRW 2014, 16, SWL 2016, BWP 2015)**

Let, third proportional = x

According to the given condition;

$a^2 - b^2 : a - b :: a - b : x$

$\because$ Product of extremes = product of means

$(a^2 - b^2)(x) = (a - b)^2$

$(a - b)(a + b)(x) = (a - b)^2$

$(a + b)x = a - b$

$x = \frac{a - b}{a + b}$

$\therefore$ **Third proportional is** $\frac{a - b}{a + b}$

(iv) $(x - y)^2, (x^3 - y^3)$ **(A.B)**

(FSD 2015, GRW 2016)

Let, third proportional = a

According to the given condition,

$(x - y)^2 : (x^3 - y^3) :: (x^3 - y^3) : a$

$\because$ Product of extreme = product of mean

$(x - y)^2 a = (x^3 - y^3)(x^3 - y^3)$

$a(x - y)(x - y)$

$= (x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2)$

$a = (x^2 + xy + y^2)^2$

$\therefore$ **Third proportional is** $(x^2 + xy + y^2)^2$

(v) $(x + y)^2, x^2 - xy - 2y^2$ **(A.B)**

Let third proportional = a

According to given condition

$(x + y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : a$

$\because$ Product of extremes = Product of means

$a(x + y)^2 = (x^2 - xy - 2y^2)(x^2 - xy - 2y^2)$

$a(x + y)^2 = (x^2 - xy - 2y^2)^2$

$a(x + y)^2 = (x^2 - 2xy + xy - 2y^2)^2$

$a(x + y)^2 = [x(x - 2y) + y(x - 2y)]^2$

$= [(x - 2y)(x + 5)]^2$

$a(x + y)^2 = (x - 2y)^2 (x + y)^2$

$a = (x - 2y)^2$

$\therefore$ **Third proportional** $= (x - 2y)^2$

(vi) $\frac{P^2 - q^2}{P^3 + q^3}, \frac{P - q}{P^2 - Pq + q^2}$ **(A.B)**

Let, the third proportional = a

According to the given condition;

$$\frac{p^2-q^2}{p^3+q^3}:\left(\frac{p-q}{p^2-pq+q^2}\right)::\left(\frac{p-q}{P^2-pq+q^2}\right):a$$

∵ Product of extremes = Product of means

$$a\left(\frac{p^2-q^2}{p^3+q^3}\right)=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)$$

$$a=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p^3+q^3}{p^2-q^2}\right)$$

$$a=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{(p+q)(p^2-pq+q^2)}{(p-q)(p+q)}\right)$$

$$a=\frac{p-q}{p^2-pq+q^2}$$

∴ **Third proportional is** $\frac{p-q}{p^2-pq+q^2}$

Q.2 Find a fourth proportional to

(i) $5,8,15$ **(A.B)**

(LHR 2014, GRW 2017, BWP 2016)

Let, the fourth proportional = x

According to the given condition:

$5:8::15:x$

∵ Product of mean = product of extreme

$$(5)x=8(15)$$

$$5x=120$$

$$x=\frac{120}{5}$$

$$x=24$$

∴ **Fourth proportional is** 24

(ii) $4x^4,2x^3,18x^5$ **(A.B)**

(SWL 2015, BWP 2016)

Let, the fourth proportional = a

According to the given condition:

$4x^4:2x^3::18x^5:a$

∵ Product of means = product of extremes

$$(18x^5)(2x^3)=(4x^4)(a)$$

$$\frac{36(x^8)}{4x^4}=a$$

$$\frac{18x^{8-4}}{2}=a \qquad 9x^4=a$$

$$\Rightarrow \quad a=9x^4$$

∴ **Fourth proportional is** $9x^4$

(iii) $15a^5b^6,10a^2b^5,21a^3b^3$ **(A.B)**

(FSD 2017, D.G.K 2015, 17)

Let, the third proportional = x

According to the given condition;

$15a^5b^6:10a^2b^5::21a^3b^3:x$

∵ Product of extremes = product of means

$$(15a^5b^6)(x)=(10a^2b^5)(21a^3b^3)$$

$$x=\frac{10\times21\times a^5\times b^8}{15a^5b^6}$$

$$x=\frac{2\times21\times b^2}{3}$$

$$x=2\times7\times b^2$$

$$x=14b^2$$

∴ **Fourth proportional is** $14b^2$

(iv) $x^2-11x+24,x-3,5x^4-40x^3$ **(A.B)**

Let, Fourth proportional is a

According to the given condition;

$x^2-11x+24:(x-3)::5x^4-40x^3:a$

∵ Product of extremes = Product of means

$$(x^2-11x+24)(a)=(x-3)(5x^4-40x^3)$$

$$(x^2-8x-3x+24)a=(x-3)(5x^4-40x^3)$$

$$[x(x-8)-3(x-8)]a=5x^3(x-3)(x-8)$$

$$(x-3)(x-8)a=5x^3(x-3)(x-8)$$

$$a=5x^3$$

∴ **Fourth proportional is** $5x^3$

(v) $p^3+q^3,p^2-q^2,p^2-pq+q^2$

Let, Fourth proportional = a

According to given condition:

$p^3+q^3:p^2-q^2::p^2-pq+q^2:a$

∵ Product of extremes = Product of means

$$(p^3+q^3)a=(p^2-q^2)(p^2-pq+q^2)$$

$$(p+q)(p^2-pq+q^2)a$$

$$=(p+q)(p-q)(p^2-pq+q^2)$$

$$a=p-q$$

∴ **Fourth proportional** = $p-q$

(vi) $(p^2-q^2)(p^2+pq+q^2),p^3+q^3,p^3-q^3$ **(A.B)**

Let, the fourth proportional = x

According to the given condition;

$(p^2-q^2)(p^2+pq+q^2):p^3+q^3::p^3-q^3:x$

∵ Product of extremes = Product of means

$$(p^2-q^2)(p^2+pq+q^2)(x)=(p^3+q^3)(p^3-q^3)$$

$$(p+q)(p-q)(p^2+pq+q^2)(x)$$

$= (p+q)(p^2 - pq + q^2)(p-q)(p^2 + pq + q^2)$

$x = p^2 - pq + q^2$

$\therefore$ **Fourth proportional is** $p^2 - pq + q^2$

Q.3 Find a mean proportional between

(i) 20, 45 **(A.B)**

(LHR 2016, GRW 2014, D.G.K 2016)

Let, the mean proportional = x

According to the given condition;

$20 : x :: x : 45$

$\because$ Product of means = Product of extremes

$(x)(x) = (20)(45)$

$x^2 = 900$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{900}$

$x = \pm 30$

$\therefore$ **The mean proportional is** ± 30

(ii) $20x^3y^3, 5x^7y$ **(A.B)**

Let, mean proportional = a

According to given condition

$20x^3y^3 : a :: a : 5x^7y$

$\because$ Product of extremes = Product of means

$(20x^3y^5)(5x^7y) = a.a$

$100x^{10}y^6 = a^2$

$a^2 = 10x^{10}y^6$

Taking Sq. root on both sides

$\sqrt{a^2} = \sqrt{100x^{10}y^6}$

$\Rightarrow a = \pm 10x^5y^3$

$\therefore$ **Mean proportional** = $\pm 10x^5y^3$

(iii) $15p^4qv^3,\ 135q^5r^7$ **(A.B)**

Let, the mean proportional = a

According to the given condition,

$15p^4qv^3 : a :: a : 135q^5r^7$

$\because$ Product of means = Product of extremes

$a^2 = (15p^4qr^3)(135q^5r^7)$

$a^2 = 2025p^4q^6r^{10}$

Taking square root on both sides,

$\sqrt{a^2} = \sqrt{2025p^4q^6r^{10}}$

$a = \pm 45p^2q^3r^5$

$\therefore$ **The mean proportional is** $\pm 45p^2q^3r^5$

(iv) $x^2 - y^2 = \dfrac{x-y}{x+y}$ **(GRW 2015)** **(A.B)**

Let, the mean proportional = a

According to given condition;

$x^2 - y^2 : a :: a : \dfrac{x-y}{x+y}$

$\because$ Product of means = Product of extremes

$a^2 = (x^2 - y^2)\left(\dfrac{x-y}{x+y}\right)$

$a^2 = (x-y)(x+y)\left(\dfrac{x-y}{x+y}\right)$

$a^2 = (x-y)^2$

Taking square root on both sides;

$\sqrt{a^2} = \sqrt{(x-y)^2}$

$a = \pm(x-y)$

$\therefore$ **The mean proportional is** $\pm(x-y)$

Q.4 Find the value of the letter involved in the following continued proportions.

(i) 5, p, 45 **(A.B)**

According to given condition

$5 : p :: p : 45$

$\because$ Product of means = product of extremes

$p^2 = 5 \times 45$

$p^2 = 225$

Talking square root

$p = \pm 15$

Result

$p = \pm 15$

(ii) 8, x, 18 **(A.B)**

According to condition

$8 : x :: x : 18$

$\because$ Product of means = Product of extremes

$x^2 = 8 \times 18$

$x^2 = 144$

Taking square root on both sides

$x = \pm\sqrt{144}$

$\Rightarrow x = \pm 12$

Result:

$x = \pm 12$

(iii) $12, 3p-6, 27$ **(A.B)**

According to given condition

$12:3p-6::3p-6:27$

$\because$ Product of extremes = Product of means

$12 \times 27 = (3p-6)(3p-6)$

$324 = (3p-6)^2$

$(3p-6)^2 = 324$

Taking Sq. root on both sides

$3p-6 = \pm 18$

Either

$3p-6=-18$ or $3p-6=18$

$3p=6-18$ $3p=6+18$

$3p=-12$ $3p=24$

$p=\frac{-12}{3}$ $p=\frac{24}{3}$

$p=-4$ $p=8$

Result:

$p=-4,8$

(iv) $7, m-3, 28$ According to given condition: **(A.B)**

(GRWP 2015, 17, FSD 2017, MTN 2017, BWP 2015)

$7:m-3::m-3:28$

$\because$ Product of means = Product of extremes

$(m-3)^2 = 7 \times 28$

$(m-3)^2 = 196$

Taking square root on both sides

$m-3 = \pm 14$

Either

$m-3=-14$ or $m-3=14$

$m=-14+3$ $m=14+3$

$\Rightarrow m=-11$ $m=17$

Result:

$m=-11,17$