Exercise 3.4

Q.1

(i) Given **(A.B)**

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

By applying componendo-dividedo property

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)}=\frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{8a}{10b}=\frac{8c}{10d}$$

(by using cancellation prop)

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow a:b=c:d$

Proved

(ii) Given **(A.B)**

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

By applying componendo-dividendo prop

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow a:b=c:d$

Proved

(iii) Given **(A.B)**

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By applying componendo-dividendo prop

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}=\frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2}=\frac{c^3}{d^3}$$

$$\frac{ac^2}{bd^2}=\frac{c^3}{d^3}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow a:b=c:\text{d}$

Hence Proved

(iv) Given **(A.B)**

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

By applying componendo-dividendo prop

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)}$$

$$=\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$

$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d}=\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$

$$\frac{2a^2c}{2b^2d}=\frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d}=\frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$ on both sides

$$\frac{a^2c}{b^2d}\times\frac{bd}{ac}=\frac{ac^2}{bd^2}\times\frac{bd}{ac}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(v) Given **(A.B)**

$Pa+pq : pq-qb = pc+qd : pc-qd$

To prove

$a:b=c:d$

Proof

Here

$Pa+pq : pq-qb = pc+qd : pc-qd$

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

By applying componendo-dividendo prop

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)}=\frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb}=\frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

$$\frac{2pa}{2qb}=\frac{2pc}{2qd}$$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

Multiply by $\frac{q}{p}$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(vi) Given **(A.B)**

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By applying componendo-dividendo prop

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+\cancel{c}+\cancel{d}+a+b-\cancel{c}-\cancel{d}}{\cancel{a}+\cancel{b}+c+d-\cancel{a}-\cancel{b}+c+d}=\frac{a-b+\cancel{c}-\cancel{d}+a-b-\cancel{c}+\cancel{d}}{\cancel{a}-\cancel{b}+c-d-\cancel{a}+\cancel{b}+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

By applying alternendo property

$$\frac{2a+2b}{2a-2b}=\frac{2c+2d}{2c-2d}$$

Again applying componendo-dividendo prop

$$\frac{(2a+2b)+(2a-2b)}{(2a+2b)-(2a-2b)}=\frac{(2c+2d)+(2c-2d)}{(2c+2d)-(2c-2d)}$$

$$\frac{2a+2\cancel{b}+2a-2\cancel{b}}{2\cancel{a}+2b-2\cancel{a}+2b}=\frac{2c+2\cancel{d}+2c-2\cancel{d}}{2\cancel{c}+2d-2\cancel{c}+2d}$$

$$\frac{4a}{4b}=\frac{4c}{4d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(vii) Given **(A.B)**

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

By applying cmponendo-dividendo prop

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}$$

$$=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d}$$

$$=\frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b+2c-3d-2a+3b+2c-3d}$$

$$\frac{4a+9b}{4c+9d}=\frac{4a-9b}{4c-9d}$$

By applying alternendo property

$$\frac{4a+9b}{4a-9b}=\frac{4c+9d}{4c-9d}$$

Again applying componendo dividendo prop

$$\frac{(4a+9b)+(4a-9b)}{(4a+9b)-(4a-9b)}=\frac{(4c+9d)+(4c-9d)}{(4c+9d)-(4c-9d)}$$

$$\frac{4a+9b+4a-9b}{4a+9b-4a+9b}=\frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$

$$\frac{8a}{18b}=\frac{8c}{18d}$$

Multiply by $\frac{18}{8}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(viii) Given **(A.B)**

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By applying componendo-dividendo prop

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)}=\frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$

$$\frac{a^2}{b^2}=\frac{ac}{bd}$$

(Multiply both sides by $\frac{b}{a}$)

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

Q.2 Using theorem of compoenendo-dividendo, find the value of

(i) Given **(A.B)**

$$x=\frac{4yz}{y+z}$$

Required

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=?$$

Solution:

Here

$$x=\frac{4yz}{y+z}$$

Dividing by 2y

$$\frac{x}{2y}=\frac{4yz}{2y(y+z)}$$

$$\frac{x}{2y}=\frac{2z}{y+z}$$

By applying componendo dividendo prop

$$\frac{x+2y}{x-2y}=\frac{2z+(y+z)}{2z-(y+z)}$$

$$=\frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y}=\frac{3z+y}{z-y}\to\text{(i)}$$

Again consider

$$x=\frac{4yz}{y+z}$$

Dividing by 2z

$$\frac{x}{2z}=\frac{4yz}{2z(y+z)}$$

$$=\frac{2y}{y+z}$$

By applying compoenendo dividendo prop

$$\frac{x+2z}{x-2z}=\frac{2y+(y+z)}{2y-(y+z)}$$

$$=\frac{2y+y+z}{2y-y-z}$$

$$=\frac{3y+z}{y-z}\to\text{(ii)}$$

Adding equation (i) and (ii)

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}+\frac{3y+z}{y-z}$$

$$=\frac{3z+y}{z-y}+\frac{3y+z}{-(z-y)}$$

$$=\frac{3z+y}{z-y}-\frac{3y+z}{z-y}$$

$$=\frac{3z+y-(3y+z)}{z-y}$$

$$=\frac{3z+y-3y-z}{z-y}$$

$$=\frac{2z-2y}{z-y}$$

$$=\frac{2(z-y)}{(z-y)}$$

$$\Rightarrow\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=2$$

(ii) Given: **(A.B)**

$$m=\frac{10np}{n+p}$$

Required

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=?$$

Solution:

Here

$$m=\frac{10np}{n+p}$$

Dividing by 5n

$$\frac{m}{5n}=\frac{10np}{(5n)(n+p)}$$

$$\frac{m}{5n}=\frac{2p}{n+p}$$

By applying componendo dividendo property

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n}\to\text{(i)}$$

Again consider

$$m=\frac{10np}{n+p}$$

Dividing by 5p

$$\frac{m}{5p}=\frac{10np}{5p(n+p)}$$

$$\frac{m}{5p}=\frac{2n}{n+p}$$

Applying componendo – devidendo prop

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p}\to\text{(ii)}$$

Adding (i) and (ii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{3n+p}{-(p-n)}$$
$$=\frac{3p+n}{p-n}-\frac{3n+p}{p-n}$$
$$=\frac{3p+n-(3n+p)}{p-n}$$
$$=\frac{3p+n-3n-p}{p-n}$$
$$=\frac{2p-2n}{p-n}$$
$$=\frac{2(p-n)}{p-n}$$
$$=2$$

Hence

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=2$$

(iii) Given **(A.B)**

$$x=\frac{12ab}{a-b}$$

Required

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=?$$

Solution:

Here

$$x=\frac{12ab}{a-b}$$

Dividing by $6a$

$$\frac{x}{6a}=\frac{2b}{a-b}$$

Applying componendo-devindendo prop

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$
$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$
$$\frac{x+6a}{x-6a}=\frac{b+a}{3b-a}$$

By invertendo theorem

$$\frac{x-6a}{x+6a}=\frac{3b-a}{a+b}\rightarrow(\text{i})$$

Again consider

$$x=\frac{12ab}{a-b}$$

Dividing by 6b

$$\frac{x}{6b}=\frac{12ab}{6b(a-b)}$$
$$\frac{x}{6b}=\frac{2a}{a-b}$$

Applying componendo-Dividendo prop

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$
$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$
$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b}\rightarrow(\text{ii})$$

Subtracting (i) and (ii)

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$
$$=\frac{3b-a-(3a-b)}{a+b}$$
$$=\frac{4b-4a}{a+b}$$
$$=\frac{4(b-a)}{a+b}$$

Hence

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{4(b-a)}{a+b}$$

(iv) Given **(A.B)**

$$x=\frac{3yz}{y-z}$$

Required

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}$$

Solution:

Here

$$x=\frac{3yz}{y-z}$$

Divided by 3y

$$\frac{x}{3y}=\frac{3yz}{3y(y-z)}$$

$$\frac{x}{3y}=\frac{z}{y-z}$$

By applying componendo dividendo prop

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$=\frac{z+y-z}{z-y+z}$$

$$=\frac{y}{2z-y}$$

Applying invertendo prop

$$\frac{x-3y}{x+3y}=\frac{2z-y}{y}\rightarrow\text{(i)}$$

Again consider

$$x=\frac{3yz}{y-z}$$

Divided by 3z

$$\frac{x}{3z}=\frac{3yz}{3z(y-z)}$$

$$=\frac{y}{y-z}$$

By applying componendo-dividendo prop

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$=\frac{y+y-z}{y-y+z}$$

$$=\frac{2y-z}{z}\rightarrow\text{(i)}$$

Sub.equation (i) and (ii)

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$=\frac{2z^2-yz-2y^2+yz}{yz}$$

$$=\frac{2z^2-2y^2}{yz}$$

$$\Rightarrow\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2(z^2-y^2)}{yz}$$

(v) Given **(A.B)**

$$s=\frac{6pq}{p-q}$$

To find value of

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=?$$

Solution:

Here

$$s=\frac{6pq}{p-q}$$

Divided by $3p$

$$\frac{s}{3p}=\frac{6pq}{3p(p-q)}$$

$$\frac{s}{3p}=\frac{2q}{p-q}$$

By applying componedo-dividendo prop (ii)

$$\frac{s-3p}{s+3p}=\frac{2q-(p-q)}{2q+(p-q)}$$

$$\frac{s-3p}{s+3p}=\frac{2q-p+q}{2q+p-q}$$

$$=\frac{3q-p}{p+q}\rightarrow\text{(i)}$$

Again consider

$$s=\frac{6pq}{p-q}$$

Divided by $3q$

$$\frac{s}{3q}=\frac{6pq}{3q(p-q)}$$

$$=\frac{2p}{p-q}$$

By applying componendo-dividendo prop

$$\frac{s+3q}{s-3q}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$=\frac{2p+p-q}{2p-p+q}$$

$$=\frac{3p-q}{p+q}\rightarrow\text{(ii)}$$

Adding equation (i) and (ii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$=\frac{3q-p+3p-q}{p+q}$$

$$=\frac{2p+2q}{p+q}$$

$$=2\frac{(p+q)}{p+q}$$

$$\Rightarrow \frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=2$$

(vi) **(FSD 2016)** **(A.B)**

Solution:

Here

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

By applying invertendo property

$$\frac{(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2}=\frac{13}{12}$$

By applying componendo-dividendo prop

$$\frac{\left[(x-2)^2+(x-4)^2\right]+\left[(x-2)^2-(x-4)^2\right]}{\left[(x-2)^2+(x-4)^2\right]-\left[(x-2)^2-(x-4)^2\right]}=\frac{13+12}{13-12}$$

$$\frac{(x-2)^2+\cancel{(x-4)^2}+(x-2)^2-\cancel{(x-4)^2}}{\cancel{(x-2)^2}+(x-4)^2-\cancel{(x-2)^2}+(x-4)^2}=\frac{25}{1}$$

$$\frac{2(x-2)^2}{2(x-4)^2}=25$$

$$\left(\frac{x-2}{x-4}\right)^2=25$$

Taking square root on both sides

$$\sqrt{\left(\frac{x-2}{x-4}\right)^2}=\sqrt{25}$$

$$\frac{x-2}{x-4}=\pm 5$$

Either

$\frac{x-2}{x-4}=5$	or	$\frac{x-2}{x-4}=-5$
$x-2=5(x-4)$		$x-2=-5(x-4)$
$x-2=5x-20$		$x-2=-5x+20$
$x-5x=-20+2$		$x+5x=20+2$
$-4x=-18$		$6x=22$
$x=\frac{18}{4}$		$x=\frac{11}{3}$
$x=\frac{9}{2}$		$x=\frac{11}{3}$

$\therefore$ **Solution Set =** $\left\{\frac{11}{3},\frac{9}{2}\right\}$

(vii) **(SGD 2015)** **(A.B)**

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

By applying conponendo-dividendo prop

$$\frac{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)+\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)-\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\cancel{\sqrt{x^2-2}}+\sqrt{x^2+2}-\cancel{\sqrt{x^2-2}}}{\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}-\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=3$$

$$\sqrt{\frac{x^2+2}{x^2-2}}=3$$

Squaring on both sides

$$\frac{x^2+2}{x^2-2}=9$$

$$x^2+2=9\left(x^2-2\right)$$

$$x^2+2=9x^2-18$$

$$x^2-9x^2=-18-2$$

$$-8x^2=-20$$

$$x^2=\frac{-20}{-8}$$

$$\Rightarrow x^2 = \frac{5}{2}$$

Taking square root on both sides

$$x = \pm\sqrt{\frac{5}{2}}$$

$\therefore$ **Solution Set**$=\left\{\pm\sqrt{\frac{5}{2}}\right\}$

(viii) **(A.B)**

Solution:

$$\frac{\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}=\frac{1}{3}$$

By applying Invertendo Property

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}=\frac{3}{1}$$

By applying componendo-devidendo prop

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}+\left(\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}\right)}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}-\left(\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}\right)}$$

$$=\frac{3+1}{3-1}$$

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}+\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}-\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}$$

$$=\frac{4}{2}$$

$$\frac{2\sqrt{x^2+8P^2}}{2\sqrt{x^2-P^2}}=2$$

Squaring on both sides

$$\left(\sqrt{\frac{x^2+8P^2}{x^2-P^2}}\right)^2=(2)^2$$

$$\frac{x^2+8P^2}{x^2-P^2}=4$$

$$x^2+8P^2=4x^2-4P^2$$

$$x^2-4x^2=-4P^2-8P^2$$

$$-3x^2=-12P^2$$

$$3x^2=12P^2$$

$$x^2=\frac{12P^2}{3}$$

$$x^2=4P^2$$

Taking square root on both sides

$$\sqrt{x^2}=\sqrt{4P^2}$$

$$x=\pm 2P$$

Either

$x=2P$ or $x=-2P$

$\therefore$ **Solution Set**$=\{2P,-2P\}$

(ix) $\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$ **(A.B)**

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

By applying invertendo property

$$\frac{(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3}=\frac{14}{13}$$

Now by applying componendo- dividendo property

$$\frac{\left[(x+5)^3+(x-3)^3\right]+\left[(x+5)^3-(x-3)^3\right]}{\left[(x+5)^3+(x-3)^3\right]-\left[(x+5)^3-(x-3)^3\right]}=\frac{14+13}{14-13}$$

$$\frac{(x+5)^3+\cancel{(x-3)^3}+(x+5)^3-\cancel{(x-3)^3}}{\cancel{(x+5)^3}+(x-3)^3-\cancel{(x+5)^3}+(x-3)^3}=\frac{27}{1}$$

$$\frac{2(x+5)^3}{2(x-3)^3}=27$$

$$\left(\frac{x+5}{x-3}\right)^3=27$$

Taking cube root on both sides

$$\sqrt[3]{\left(\frac{x+5}{x-3}\right)^3}=\sqrt[3]{27}$$

$$\frac{x+5}{x-3}=3$$

$$x+5=3(x-3)$$

$$x+5=3x-9$$

$$x-3x=-9-5$$

$$-2x=-14$$

$$x = \frac{-14}{-2}$$

$$x = 7$$

$\therefore$ **Solution Set** $= \{7\}$