Exercise 3.7

Q.1 **The surface area A of a cube varies directly as the square of the length l of an edge and $A = 27$ square units when $l = 3$ units. Find A when $l = 4$ units (ii) l when $A = 12$ sq. units.** **(A.B + K.B)**

Given

$A \propto l^2$

A = 27 square unit when l = 3 units

To find

A = ? when l = 4 units

l = ? when A = 12 square unit

Solution:

Here

$A \propto l^2$

$A = kl^2 \longrightarrow (i)$

Put A = 27, l = 3

$27 = k(3)^2$

$27 = 9k$

$3 = k$

$k = 3$

For value of A

Put k = 3, l = 4 in equation (i)

$A = 3(4)^2$

$A = 48$

For value of l

Put k = 3, A = 12 square unit in equation (i)

$12 = 3l^2$

$4 = l^2$

Or $l^2 = 4$

Taking square root on b/s

$l^2 = \pm 2$

$\Rightarrow l = 2$ (Length is always positive)

Result:

A = 48 square unit when l = 4 units

l = 2 unit when A = 12 square units

Q.2 **The surface area S of the sphere varies directly as the square of radius r, and $S = 16\pi$ when r = 2. Find r when $S = 36\pi$.** **(A.B K.B)**

Given

$S \propto r^2$

$S = 16\pi$ when r = 2

To Find

r = ? when s = 36π

Solution:

Here $S \propto r^2$

$S = kr^2 \longrightarrow (i)$

For value of k

Put S = 16π, r = 2 in equation (i)

$16\pi = k(2)^2$

$16\pi = 4k$

$4\pi = k$

Or $k = 4\pi$

$S = \pi r^2$

For value of r

Put k = 4π, S = 36π in equation (i)

$36\pi = 4\pi r^2$

$9 = r^2$

Taking square root on both sides

$r = \pm 3$

$\Rightarrow r = 3$ (Length is positive)

Result:

r = 3 when S = 36π

Q.3 **In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and $F = 32lb$ when S = 1.6 in. Find (i) S when F = 50lb (ii) F when S = 0.8 in** **(K.B +A.B)**

Given

$F \propto S$

F = 32 lb when S = 1.6 m.

To find

S = ? when F = 50 lb

F = ? when S = 0.8 in

Solution:

Here $F \propto S$

$F = kS \rightarrow (i)$

For value of k

Put F = 32 and S = 1.6 in equation (i)

32 = k(1.6)

20 = k Or $k = 20$

$F = 20S$

For value of S

Put k = 20 and F = 50

50 = 20S

$\frac{5}{2}$ =S

Or $S = \frac{5}{2}$

For value of F

Put k = 20 and S = 0.8 in equation (i)

F = 20(0.8)

F = 16

Result:

S = $\frac{5}{2}$ in when F = 50 lb

Q.4 F = 16 lb when S = 0.8 inThe intensity *I* of light from a given source varies inversely as the square of the distance *d* from it. If the intensity is 20 candlepower at a distance of 12ft. From the source, in *d* the intensity at a point 8ft. from the source. (K.B +A.B)

Given

$I \propto \frac{1}{d^2}$

I = 20 candle power when d = 12ft

To find

I = ? when d = 8ft

Solution:

Here

$I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2} \rightarrow (i)$

For value of k

Put I = 20 and d = 12 in equation (i)

$20 = \frac{k}{(12)^2}$

$20 \times 144 = k$

Or $2880 = k$

$I = \frac{2880}{d^2}$

For value of I

Put $k = 2880$ and $d = 8$ in equation (i)

$I = \frac{2880}{(8)^2}$

$I = \frac{2880}{64}$

I = 45

Result

I = 45 candle power when d = 8ft.

Q.5 The pressure *P* in a body of fluid varies directly as the depth *d*. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. **high is** 2.25 lb/sq**. in, how deep must the fluid be to exert a pressure of** 9 lb/sq**. in?**

(K.B +A.B)

Given

P ∝ d

P = 2.25 lb/sq when d = 5ft

To Find

d = ? when P = 9 lb/sq

Solution:

Here

P ∝ d

P = kd ⟶ (i)

For value of k

Put P = 2.25 and d = 5

2.25 = k(5)

Or k=0.45

$P = 0.45d$

For value of d

Put k = 0.45 and P = 9

9 = 0.45 d

$\frac{9}{0.45}$ = d

20 = d

Or d = 20

Result

d = 20ft when P = 9 lb/sq.

Q.6 **Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600workers for 18 days**

(K.B +A.B)

Given

$c \propto nd$

c = Rs. 286000 when $n = 800$ workers, $d = 13$ days

To find

c = ? when n = 600 workers, d = 18 days

Solution:

Here

$c \propto nd$

$c = knd \longrightarrow (i)$

For value of k

Put c = 286000, n = 800 and d = 13 in eq (i)

$286000 = k(800)(13)$

$\frac{286000}{800\times 13} = k$

$\Rightarrow \frac{55}{2} = k$

$c = \frac{55}{2}nd$

For value of c

Put $k = \frac{55}{2}$, n = 600 and 18 in equation (i)

$c = \frac{55}{2}\times 600\times 18$

$c = 297000$

Result:

c = Rs.29700 when n = 600 workers and $d = 18$ days

Q.7 **The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?**

(K.B +A.B)

Given

$c \propto d^4$ and $c \propto \frac{1}{l^2}$

c = 63 tons when d = 6 inches and l = 30 feet

To Find

l = ? when d = 4 inches c = 28 tons

Solution:

Here

$c \propto d^4$ and $c \propto \frac{1}{l^2}$

In joint variation:

$c \propto \frac{d^4}{l^2}$

$\Rightarrow c = k\frac{d^4}{l^2}$

For value of k

Put c = 63, d = 6 and l = 30 in equation (i)

$63 = k\frac{(6)^4}{(30)^2}$

$63 = k\frac{1296}{900}$

$\frac{63\times 900}{1296} = k$

$\frac{175}{4} = k$

Or $k = \frac{175}{4}$

$c = \frac{175d^4}{4e^2}$

For value of l

Put $k = \frac{175}{4}$, d = 4, c = 28 in equation (i)

$28 = \frac{175}{4}\frac{(4)^4}{l^2}$

$28 \times 4l^2 = 175 \times 256$

$l^2 = 400$

Taking positive square root on both sides

$\Rightarrow l = 20$

Result

l = 20 feet when d = 4 inches and $c = 28$ tons

Q.8 **The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of**

the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

(K.B +A.B)

Given

$T \propto wd$ and $T \propto \frac{1}{P}$

T = 25 sec when P = 4 hp, w = 500 lb

To Find

P = ? when c = 800 lb, d = 120 ft and T = 40 sec.

Solution:

Here

$T \propto wd$ and $T \propto \frac{1}{P}$

In joint variation:

$T \propto \frac{wd}{P}$

$T = \frac{kwd}{P} \longrightarrow (i)$

For value of k

Put $T = 25, P = 4, w = 500, d = 40$ in equation (i)

$25 = k\ \frac{500 \times 40}{40}$

$\frac{25}{5000} = k$

$\frac{1}{200} = k$

Or $k = \frac{1}{200}$

$T = \frac{wd}{200P}$

For value of P

Put $k = \frac{1}{200}$, w = 800, d = 120 and $T = 40$ in equation (i)

$40 = \frac{1}{200}\frac{800 \times 120}{P}$

$40P = 4 \times 120$

$P = 12$

Result:

P = 12 hp when w = 800 lb, d = 120 ft and $T = 40$ sec.

Q.9 The kinetic energy (*K.E.*) of a body varies jointly as the mass "*m*" of the body and the square of its velocity "*v*". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec. (K.B +A.B)

Given

$K.E \propto mv^2$

K.E = 4320 ft/lb when m = 45 lb and v = 24 ft/sec

To find

K.E = ? when m = 3000 lb, v = 44 ft/sec

Solution:

Here

$K.E \propto mv^2$

$K.E = kmv^2 \longrightarrow (i)$

For value of k

Put K.E = 4320, m = 45 and v = 24

$4320 = k(45)(24)^2$

$\frac{4320}{45 \times 576} = k$

$k = \frac{1}{6}$

$K.E = \frac{1}{6}mv^2$

For value of $K.E$

Put k = $\frac{1}{6}$, m = 3000, v = 44 in equation (i)

K.E = $\frac{1}{6} \times 3000 \times (44)^2$

K.E = 968000

Result:

K.E = 968000 *ft*/*lb* when m = 3000 *lb* and $v = 44\,ft/sec$