Exercise 1.4

Q.1 Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}-2\\3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}-2 & -1\\1 & 3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix}$ **(K.B)**

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix}1 & 2\\0 & -1\\-1 & -2\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix}3 & 2 & 1\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 2\\-2 & 3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A=\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}$, $B=\begin{bmatrix}6\\5\end{bmatrix}$ **find**

(i) AB **(GRW 2018, MTN 2017, 18) (A.B)**

Solution:

$$AB=\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}\begin{bmatrix}6\\5\end{bmatrix}$$
$$=\begin{bmatrix}(3\times6)+(0\times5)\\(-1\times6)+(2\times5)\end{bmatrix}$$
$$=\begin{bmatrix}18+0\\-6+10\end{bmatrix}$$
$$=\begin{bmatrix}18\\4\end{bmatrix}$$

(ii) *BA* **(if possible)** **(K.B)**

Solution:

BA is not possible because number of columns of B are not equal to number of rows of *A*.

Q.3 Find the following products

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ **(A.B)**

(GRW 2019, SGD 2016, MTN 2017)

Solution:

$$\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}=\begin{bmatrix}(1\times4)+(2\times0)\end{bmatrix}$$
$$=\begin{bmatrix}4+0\end{bmatrix}$$
$$=\begin{bmatrix}4\end{bmatrix}$$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$ **(A.B)**

(GRW 2017, FSD 2017, 18)

Solution:

$$\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}=\begin{bmatrix}(1\times5)+(2\times-4)\end{bmatrix}$$

$= [5+(-8)]$
$= [5-8]$
$= [-3]$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ **(LHR 2017)** **(A.B)**

Solution:

$$\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \left[(-3\times 4)+(0\times 0)\right]$$
$$= [-12+0]$$
$$= [-12]$$

(iv) $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}6 & +0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = [6\times 4+(-0)(0)]$$
$$= [24-0]$$
$$= [24]$$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$$
$$= \begin{bmatrix}1\times4+2\times0 & 1\times5+2\times(-4)\\ -3\times4+0\times0 & -3(5)+0\times(-4)\\ 6(4)+(-1)(0) & 6(5)+(-1)(-4)\end{bmatrix}$$
$$= \begin{bmatrix}4+0 & 5-8\\ -12+0 & -15-0\\ 24-0 & 30+4\end{bmatrix}$$
$$= \begin{bmatrix}4 & -3\\ -12 & -15\\ 24 & 34\end{bmatrix}$$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$ **(LHR 2019)** **(A.B)**

Solution:

$$\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$$
$$= \begin{bmatrix}2\times2+(3\times3) & (2\times-1)+(3\times0)\\ (1\times2)+(1\times3) & (1\times-1)+(1\times0)\\ (0\times2)+(-2\times3) & (0\times-1)+(-2\times0)\end{bmatrix}$$
$$= \begin{bmatrix}4+9 & -2+0\\ 2+3 & -1+0\\ 0-6 & 0+0\end{bmatrix}$$
$$= \begin{bmatrix}13 & -2\\ 5 & -1\\ -6 & 0\end{bmatrix}$$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$ **(A.B)**

(LHR 2015, RWP 2015, MTN 2016)

Solution: $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$$= \begin{bmatrix}(1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1)\\ (4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1)\end{bmatrix}$$
$$= \begin{bmatrix}1+6+(-3) & 2+8+3\\ 4+15+(-6) & 8+20+6\end{bmatrix}$$
$$= \begin{bmatrix}7-3 & 13\\ 19-6 & 34\end{bmatrix}$$
$$= \begin{bmatrix}4 & 13\\ 13 & 34\end{bmatrix}$$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$ **(A.B)**

(RWP 2015, GRW 2016)

Solution:

$$\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}(1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6)\\(3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6)\\(-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6)\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 2+10 & 3+12\\3+16 & 6+20 & 9+24\\-1+4 & -2+5 & -3+6\end{bmatrix}$$

$$=\begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$ **(A.B)**

(LHR 2015, SGD 2018)

Solution:

$$\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$$

$$=\begin{bmatrix}(8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4)\\(6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4)\end{bmatrix}$$

$$=\begin{bmatrix}16+(-20) & \frac{-40}{2}+20\\12+(-16) & \frac{-30}{2}+16\end{bmatrix}$$

$$=\begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times0)+(2\times0) & (-1\times0)+(2\times0)\\(1\times0)+(3\times0) & (1\times0)+(3\times0)\end{bmatrix}$$

$$=\begin{bmatrix}0+0 & 0+0\\0+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Q.5 **Let** $A=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$ **verify whether**

(FSD 2015, MTN 2015, D.G.K 2017)

(i) $AB = BA$ **(K.B) (A.B) (U.B)**

Verification:

L.H.S. =AB

$$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\(2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}$$

R.H.S. = BA $=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\-3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\-7 & -9\end{bmatrix}$$

Since $\text{L.H.S} \neq \text{R.H.S}$

$AB \neq BA$

(ii) $A(BC)=(AB)C$ **(A.B)**

Verification:

L.H.S.=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S.=(AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S. = R.H.S.

A (BC) = (AB) C

Hence proved

(iii) $A(B+C)=AB+AC$ **(A.B)**

Verification:

L.H.S.=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S.=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}$$

$$+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & 4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since L.H.S.=R.H.S.

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC **(A.B)**

Verification:

L.H.S.=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S.=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S. = R.H.S.

A (B-C) =AB-AC

Hence proved

Q.6 For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$ **(A.B)**

Verification:

L.H.S. = $(AB)^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

R.H.S.=B^tA^t

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S. = R.H.S.

$(AB)^t = B^tA^t$

Hence proved

L.H.S. = R.H.S.

(ii) $(BC)^t = C^tB^t$ **(A.B)**

Verification:

L.H.S. = $(BC)^t$

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,x\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Now

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$=\begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

R.H.S. = C^tB^t

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$\text{R.H.S.} = C^tB^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-15)\\ 6+(-18) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-15\\ 6-18 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

Hence proved

L.H.S.=R.H.S.