



Mathematics-9

Unit 1 – 1.5

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Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ (A.B)

Solution:

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

Then,

$$\begin{aligned} |A| &= \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} \\ &= (-1)(0) - (2)(1) \\ &= 0 - 2 \\ &= -2 \end{aligned}$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$ (A.B)

(BWP 2018, D.G.K 2018)

Solution:

$$B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$$

Then,

$$\begin{aligned} |B| &= \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} \\ &= (1)(-2) - (2)(3) \\ &= -2 - 6 \\ &= -8 \end{aligned}$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ (BWP 2014, 16) (A.B)

Solution:

$$C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$$

Then,

$$\begin{aligned} |C| &= \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} \\ &= (3)(2) - (3)(2) \end{aligned}$$

$$= 6 - 6$$

$$= 0$$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ (D.G.K 2018) (A.B)

Solution:

$$D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Then,

$$\begin{aligned} |D| &= \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} \\ &= (3)(4) - (2)(1) \\ &= 12 - 2 \\ &= 10 \end{aligned}$$

Q.2 Find which of the following matrices are singular or non singular?

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ (A.B)

Solution:

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

Then,

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} \\ |A| &= (3)(4) - (2)(6) \\ |A| &= 12 - 12 \\ |A| &= 0 \end{aligned}$$

It is a singular matrix.

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (A.B)

Solution:

$$B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Then,

$$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$$

$$|B| = (4)(2) - (3)(1)$$

$$|B| = 8 - 3$$

$$|B| = 5$$

It is non singular matrix.

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ (SGD 2018) (A.B)

Solution:

$$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

Then,

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$$

$$|C| = (7)(5) - (3)(-9)$$

$$|C| = 35 + 27$$

$$|C| = 62$$

It is not equal to zero so

It is non singular matrix.

(iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$ (A.B)

Solution:

$$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

Then,

$$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$$

$$|D| = (5)(4) - (-2)(-10)$$

$$|D| = 20 - 20$$

$$|D| = 0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ (GRW 2018) (A.B)

Solution:

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

Then,

$$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$$

$$|A| = (-1)(0) - (2)(3)$$

$$|A| = 0 - 6$$

$$|A| = -6 \neq 0 \text{ (Non Singular)}$$

A^{-1} exists

$$AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

Putting the values

$$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & -1 \times \frac{1}{-6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ (A.B)

(GRW 2018, FSD 2018, SGD 2018)

Solution:

$$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

Then,

$$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|B| = (-1)(-5) - (-3)(2)$$

$$|B| = -5 + 6$$

$$|B| = 1 \neq 0 \text{ (Non Singular)}$$

B^{-1} exists

$$AdjB = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times \text{Adj}B$$

Putting the values

$$B^{-1} = \frac{1}{1} \times \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{1} & \frac{-2}{1} \\ \frac{3}{1} & \frac{1}{1} \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

(iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$ (A.B)

Solution:

To write in determinant form

$$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$$

$$|C| = (-2)(-9) - (3)(6)$$

$$|C| = 18 - 18$$

$$|C| = 0 \quad \text{Singular}$$

C^{-1} does not exist.

(iv) $D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$ (A.B)

Solution:

To write in determinant form

$$D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$$

$$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \times 2 - \frac{3}{4} \times 1$$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$$|D| = \frac{1}{4} \neq 0 \quad (\text{Non Singular})$$

D^{-1} exists

$$\text{Adj}D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \times \text{Adj}D$$

By putting the values

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \div \frac{1}{4} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \times \frac{4}{1} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 \times 2 & -\frac{3}{4} \times 4 \\ -1 \times 4 & \frac{1}{2} \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q.4 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then

verify that

(i) $A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$ (A.B)

Verification:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\det A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$= 1 \times 6 - 2 \times 4$$

$$= 6 - 8$$

$$= -2$$

$$A(\text{Adj}A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & (-2)+2 \\ 24-24 & -8+6 \end{bmatrix}$$

$$A(\text{Adj}A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{--- (i)}$$

$$(\text{Adj}A)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} (6) \times (1) + (-2) \times 4 & (6) \times 2 + (-2) \times 6 \\ (-4) \times 1 + (1) \times 4 & (-4) \times 2 + (1) \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{--- (ii)}$$

$$(\det A)I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 & 0 \times 2 \\ -2 \times 0 & 1 \times -2 \end{bmatrix}$$

$$(\det A)I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{--- (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

$$A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$$

$$\text{(ii) } BB^{-1} = I = B^{-1}B \quad \text{(A.B)}$$

Solution: (U.B)

$$B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$$

$$= -6 - (-2)$$

$$= -6 + 2$$

$$= -4 \neq 0 \text{ (Non singular)}$$

B^{-1} exists.

$$\text{Adj}B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}B$$

$$B^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

Now

$$BB^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{-4} & 0 \\ 0 & \frac{4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1}B = I$$

From (i) and (ii)

$$BB^{-1} = I = B^{-1}B$$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$ **(U.B)**

Solution:

$$\begin{aligned} & \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 21+(-20) & -15+15 \\ 28+(-28) & -20+21 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Since, $AA^{-1} = I$, given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$ **(A.B + U.B)**

Solution:

$$\begin{aligned} & \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -3+4 & 2+(-2) \\ -6+6 & 4+(-3) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Since, $AA^{-1} = I$, given matrices are multiplicative inverse of each other.

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$ **(A.B + U.B)**

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

L.H.S. = $(AB)^{-1}$

$$\begin{aligned} AB &= \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \times (-4) + 0(1) & 4 \times (-2) + 0(-1) \\ -1 \times (-4) + 2(1) & -1 \times (-2) + 2(-1) \end{bmatrix} \\ &= \begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2+(-2) \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

To write in determinant form

$$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$|AB| = 0 - (-48)$$

$$|AB| = 48$$

To write in Adj (AB)

$$\text{Adj}(AB) = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times \text{Adj}AB$$

$$= \frac{1}{48} \times \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix} \rightarrow (i)$$

R.H. S = $B^{-1}A^{-1}$

To write in determinant form

$$|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$$|B| = 4 - (-2)$$

$$|B| = 4 + 2$$

$$|B| = 6$$

To write in Adj B

$$\text{Adj}B = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times \text{Adj}B$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$|A| = 8 - (-0)$$

$$|A| = 8$$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S.

$$B^{-1}A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix} \rightarrow (ii)$$

From (i) and (ii)

L.H.S. = R.H.S.

Hence proved