Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations by: **(A.B + U.B)**

(i) The matrix inversion method

(ii) The Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

(FSD 2018, SGD 2018, BWP 2018)

By matrices inversion method

$$\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\6\end{bmatrix}$$

Let $AX = B$

Where $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}4\\6\end{bmatrix}$

$$A=\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$$

$$|A|=(2)(2)-(-2)(3)$$

$$|A|=4+6$$

$$|A|=10 \neq 0$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}2\times4+2\times6\\-3\times4+2\times6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}8+12\\-12+12\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}20\\0\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{20}{10}\\\frac{0}{10}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}$$

$x=2, y=0$

Solution: Set $=\{(2, 0)\}$

By Cramer's rule

$$A=\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$$

$$=(2)(2)-(-2)(3)$$

$$=4-(-6)$$

$$=4+6$$

$$=10$$

$$|A_x|=\begin{vmatrix}4 & -2\\6 & 2\end{vmatrix}$$

$$=(4)(2)-(-2)(6)$$

$$=8+12$$

$$=20$$

$$|A_y|=\begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}$$

$$=(2)(6)-(4)(3)$$

$$=12-12$$

$$=0$$

$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{20}{10}$$
$$x=2$$
$$y=\frac{|A_y|}{|A|}$$
$$y=\frac{0}{10}$$
$$y=0$$

Solution: Set = {(2, 0)}

(ii) $2x+y=3$ **(A.B)**
$6x+5y=1$

(GRW 2018, D.G.K 2018)

Matrices inversion method

$$\begin{bmatrix}2 & 1\\ 6 & 5\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 1\end{bmatrix}$$

Let $AX = B$

where $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\end{bmatrix}$ $B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$A=\begin{bmatrix}2 & 1\\ 6 & 5\end{bmatrix}$$
$$|A|=\begin{vmatrix}2 & 1\\ 6 & 5\end{vmatrix}$$
$$=(2)(5)-(1)(6)$$
$$=10-6$$
$$=4 \quad \text{as} \quad |A|\neq 0$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}5 & -1\\ -6 & 2\end{bmatrix}$$
$$AX=B$$
$$X=A^{-1}B$$
$$X=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5 & -1\\ -6 & 2\end{bmatrix}\begin{bmatrix}3\\ 1\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5\times 3+(-1\times 1)\\ -6\times 3+2\times 1\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}15+(-1)\\ -18+2\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}14\\ -16\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\overset{7}{\cancel{14}}\times\frac{1}{\underset{2}{\cancel{4}}}\\ -\overset{4}{\cancel{16}}\times\frac{1}{\underset{1}{\cancel{4}}}\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{7}{2}\\ -4\end{bmatrix}$$
$$x=\frac{7}{2},\ y=-4$$

$\therefore$ **Solution Set** = $\left\{\left(\frac{7}{2},-4\right)\right\}$

By Cramer's Rule

$$A=\begin{bmatrix}2 & 1\\ 6 & 5\end{bmatrix}\ B=\begin{bmatrix}3\\ 1\end{bmatrix}$$
$$|A|=\begin{vmatrix}2 & 1\\ 6 & 5\end{vmatrix}$$
$$=(2)(5)-(1)(6)$$
$$=10-6$$
$$=4 \quad \text{as} \quad |A|\neq 0$$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}3 & 1\\ 1 & 5\end{vmatrix}$$
$$=(3)(5)-(1)(1)$$
$$=15-1$$
$$=14$$
$$|A_y|=\begin{vmatrix}2 & 3\\ 6 & 1\end{vmatrix}$$
$$=(2)(1)-(3)(6)$$
$$=2-18$$
$$=-16$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{14}{4}$$

$$x = \frac{7}{2}$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{16}{4}$$

$$y = -4$$

Solution Set $= \left\{\left(\frac{7}{2}, -4\right)\right\}$

(iii) $4x + 2y = 8$ **(FSD 2019)** **(A.B)**

$3x - y = -1$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10 \quad \text{as} \quad |A| \neq 0$$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

As we know that

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -8+2 \\ -24+(-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

$$x = \frac{3}{5}, y = \frac{14}{5}$$

Solution Set $= \left\{\left(\frac{3}{5}, \frac{14}{5}\right)\right\}$

By Cramer's rule

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10 \quad \text{as} \quad |A| \neq 0$$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$$

$$= (8)(-1) - (2)(-1)$$

$$= -8 - (-2)$$

$$= -6$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{6}{10}$$

$$x = \frac{3}{5}$$

$$|A_y| = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

$$= (4)(-1) - (8)(3)$$

$$= -4 - 24$$

$$= -28$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{-28}{-10}$$

$$y = \frac{14}{5}$$

Solution Set $= \left\{ \left(\frac{3}{5}, \frac{14}{5} \right) \right\}$

(iv) $3x - 2y = -6$ **(A.B)**

$5x - 2y = -10$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Let AX = B

Where,

$$A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 - (-10)$$

$$= -6 + 10$$

$= 4$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 \times -6 + 2 \times -10 \\ -5 \times -6 + 3 \times -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 + (-20) \\ 30 + (-30) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$x = -2, y = 0$

Solution: Set $= \{(-2, 0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 - (-10)$$

$$= -6 + 10$$

$= 4$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}$$

$$= (-6)(-2) - (-2)(-10)$$

$$= +12 - (+20)$$

$$= 12 - 20$$

$$= -8$$

$$|A_y| = \begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$$

$$= (3)(-10) - (-6)(5)$$

$= -30-(-30)$
$= -30+30$
$= 0$

$x = \frac{|A_x|}{|A|}$

$x = \frac{-8}{4}$

$x = -2$

$y = \frac{|A_y|}{|A|}$

$= \frac{0}{4}$

$y = 0$

Solution Set $= \{(-2,0)\}$

(v) $3x-2y=4$ **(A.B)**
$-6x+4y=7$

(GRW 2015, SGD 2015, SWL 2018)

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Here $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$

$= (3)(4)-(-2)(-6)$
$= 12-(+12)$
$= 12-12$
$= 0$

Solution is not possible because A is singular matrix.

(vi) $4x+y=9$ **(A.B)**
$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let AX = B

Where, $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1)-(1)(-3)$
$= -4+3$
$= -1$ as $|A| \neq 0$

Solution is possible because $|A|$ is non singular

$AdjA = \begin{vmatrix} -1 & -1 \\ 3 & 4 \end{vmatrix}$

As we know that

$X = A^{-1}B$

$X = \frac{1}{|A|} \times AdjA \times B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} -9+5 \\ 27+(-20) \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-4}{-1} \\ \frac{7}{-1} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

$x = 4, y = -7$

Solution Set= $\{(4,-7)\}$

By Cramer's rule

$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}, B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1)-(1)(-3)$
$= -4-(-3)$
$= -4+3$
$= -1$ as $|A| \neq 0$

Solution is possible because A is non-singular matrix

$$|A_x| = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}$$

$$=(9)(-1)-(1)(-3)$$

$$=-9-(-5)$$

$$=-9+5$$

$$=-4$$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-4}{-1}$$

$$x=4$$

$$|A_y| = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}$$

$$=(4)(-5)-(9)(-3)$$

$$=-20-(-27)$$

$$=-20+27$$

$$=7$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{7}{-1}$$

$$y=-7$$

Solution Set$=\{(4,-7)\}$

(vii) $2x-2y=4$ **(A.B)**

$-5x-2y=-10$ **(LHR 2019)**

By Matrices Inversion Method

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$=(2)(-2)-(-2)(-5)$$

$$=-4-(+10)$$

$$=-4-10$$

$=-14$ as $|A|\neq 0$

Solution is possible because A is non-singular matrix

$$AdjA=\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

As we know that

$$X=A^{-1}B$$

$$X=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} -8+(-20) \\ 20+(-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$x=2, y=0$$

Solution Set$=\{(2,0)\}$

By Cramer's rule

$$A=\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B=\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$=(2)(-2)-(-2)(-5)$$

$$=-4-(+10)$$

$$=-4-10$$

$=-14$ as $|A|\neq 0$

Solution is possible because A is non-singular matrix

$$|A_x| = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$= (4)(-2) - (-2)(-10)$$

$$= -8 - (+20)$$

$$= -8 - 20$$

$$= -28$$

$$y = \frac{|A_y|}{|A|} = ?$$

$$|A_y| = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$= (2)(-10) - (4)(-5)$$

$$= -20 - (-20)$$

$$= -20 + 20$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-28}{-14}$$

$$x = 2$$

Solution Set $= \{(2,0)\}$

(viii) $3x - 4y = 4$ **(A.B)**

$x + 2y = 8$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2) - (-4)(1)$$

$$= 6 - (-4)$$

$$|A| = 6 + 4$$

$= 10$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

As we know that

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2\times4+4\times8 \\ -1\times+3\times8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$x = 4, y = 2$

Solution Set $= \{(4,2)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2) - (-4)(1)$$

$$= 6 - (-4)$$

$$= 6 + 4$$

$= 10$ as $|A| \neq 0$

Solution is possible because a is non-singular matrix

$$|A_x| = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$$= (4)(2) - (-4)(8)$$

$$= 8 - (-32)$$

$$= 8 + 32$$

$$= 40$$

$$|A_y| = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$$= (3)(8) - (4)(1)$$

$$= 24 - 4$$

$$= 20$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{40}{10}$$

$$x = 4$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{20}{10}$$

$$y = 2$$

Solution Set $= \{(4, 2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle. (A.B+U.B+K.B)

Solution:

Let width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y = 4x$$

$$-4x + y = 0 \quad \rightarrow \ldots\text{(i)}$$

According to 2nd condition

2(length + Width)=Perimeter

$$2(y + x) = 150$$

$$y + x = \frac{\cancel{150}^{75}}{\cancel{2}}$$

$$x + y = 75 \quad \rightarrow \ldots\text{(ii)}$$

$$-4x + y = 0$$

$$x + y = 75$$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$X = A^{-1}B$$

(By matrix inversion method)

Let $A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$= -\frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} 0 - 75 \\ 0 - 300 \end{bmatrix}$$

$$= \frac{1}{-5}\begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$$x = 15, y = 60$$

Result:

Width of rectangle = x = 15cm

Length of rectangle = y = 60cm

By Cramer's rule

$$A=\begin{bmatrix}-4 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}0\\ 75\end{bmatrix}$$

$$|A|=\begin{vmatrix}-4 & 1\\ 1 & 1\end{vmatrix}$$

$$=(-4)(1)-(1)(1)$$

$$=-4-1$$

$$=-5$$

$$|A_x|=\begin{vmatrix}0 & 1\\ 75 & 1\end{vmatrix}$$

$$=(0)(1)-(1)(75)$$

$$=0-75$$

$$=-75$$

$$|A_y|=\begin{vmatrix}-4 & 0\\ 1 & 75\end{vmatrix}$$

$$=(-4)(75)-(0)(1)$$

$$=-300+0$$

$$=-300$$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-75}{-5}$$

$$x=15$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{-300}{-5}$$

$$y=60$$

Result:

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm. (K.B)

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y-x=3.5$$

$$-x+y=3.5 \quad \rightarrow(\text{i})$$

According to 2nd condition

$$2(L+B)=P$$

$$2(y+x)=67$$

$$x+y=\frac{67}{2}$$

$$x+y=33.5 \quad \rightarrow(\text{ii})$$

Changing eq (i) and (ii) into matrix form

$$\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$$

(By matrix inversion method)

Let $A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$

$$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$$

$$=(-1)(1)-(1)(1)$$

$$=-1-1$$

$$=-2$$

$$AdjA=\begin{bmatrix}1 & -1\\ -1 & -1\end{bmatrix}$$

As we know that

$$X=A^{-1}B$$

$$X=\frac{1}{|A|}\times AdjA\times B$$

Putting the values

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}3.5\\ +33.5\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}1(3.5)+(-1)(33.5)\\ -1(3.5)+(-1)(33.5)\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}3.5-33.5\\ -3.5-33.5\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}-30\\ -37\end{bmatrix}$$

$$= \begin{bmatrix} \frac{-30}{-2} \\ \frac{-37}{-2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 18.5 \end{bmatrix}$$

$\Rightarrow x = 15, \; y = 18.5$

By Cramer's rule

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$|A_x| = \begin{vmatrix} 3.5 & 1 \\ 33.5 & 1 \end{vmatrix}$$

$$= (3.5)(1) - (1)(33.5)$$

$$= 3.5 - 33.5$$

$$= -30$$

$$|A_y| = \begin{vmatrix} -1 & 3.5 \\ 1 & 33.5 \end{vmatrix}$$

$$= (-1)(33.5) - (3.5)(1)$$

$$= -33.5 - 3.5$$

$$= -37$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-30}{-2}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{-37}{-2}$$

$$y = \frac{37}{2} = 18.5$$

Result:

Width of rectangle =x=15cm

Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles Δ is $16°$ less than the sum of two equal angles. Find three angles of the triangle. (K.B)

Solution:

Let each equal angles are x and third angle is y

According to condition $y = 2x - 16$

$2x - y = 16 \quad$ (i)

As we know that

$x + x + y = 180$

$2x + y = 180 \quad$ (ii)

$2x - y = 16$

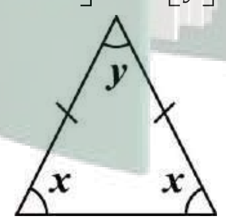
$2x + y = 180$

Changing into matrix form

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

Let $AX = B$

Where, $A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$



Using Matrix Inversion Method

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 2 \times 1 - (-1) \times 2$$

$$= 2 + 2$$

$= 4 \neq 0$ (None singular)

A^{-1} exist

$$AdjA=\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}$$

$$X=A^{-1}B$$

Or $X=\dfrac{1}{|A|}\times AdjA\times B$

Putting the values

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\ -2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}16+180\\ -32+360\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}196\\ 328\end{bmatrix}$$

$$=\begin{bmatrix}\dfrac{196}{4}\\ \dfrac{328}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}49\\ 82\end{bmatrix}$$

$x=49$ and $y=82$

Cramer Rule

$$A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix},X=\begin{bmatrix}x\\ y\end{bmatrix},B=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$$

$$=(2)(1)-(-1)(2)$$

$$=2-(-2)$$

$$=2+2$$

$$=4$$

$$|A_x|=\begin{bmatrix}16 & -1\\ 180 & 1\end{bmatrix}$$

$$=(16)(1)-(-1)(180)$$

$$=16+180$$

$$=196$$

$$|A_y|=\begin{bmatrix}2 & 16\\ 2 & 180\end{bmatrix}$$

$$=(2)(180)-(16)(2)$$

$$=360-32$$

$$=328$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{196}{4}$$

$$x=49$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{328}{4}$$

$$y=82$$

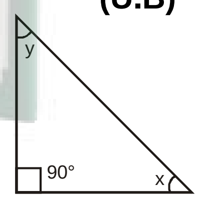
1^{st} angle = x = 49°

2^{nd} angle= x = 49°

3^{rd} angle =y= 82°

Q.5 One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle. (U.B)

Solution:



Let one acute angle = x

And other acute angle = y

According to given condition

$$x=2y+12$$

$$x-2y=12 \quad \rightarrow(\text{i})$$

As we know

$$x+y=90 \quad \rightarrow(\text{ii})$$

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix}1 & -2\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}12\\ 90\end{bmatrix}$$

Let $AX=B$

Where, $A=\begin{bmatrix}1 & -2\\ 1 & 1\end{bmatrix}X=\begin{bmatrix}x\\ y\end{bmatrix}B=\begin{bmatrix}12\\ 90\end{bmatrix}$

Using Matrix Inversion Method

$$|A|=\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}$$

$=(1)(1)-(-2)(1)$

$=1-(-2)$

$=3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$$

As we know that

$X=A^{-1}B$ *or*

$$X=\frac{1}{|A|}\times AdjA\times B$$

Putting the values

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}\begin{bmatrix}12\\ 90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}12+180\\ -12+90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}192\\ 78\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{192}{3}\\ \frac{78}{3}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}64\\ 26\end{bmatrix}$$

$x=64$ and $y=26$

Then

1st angle = x = 64°

2nd angle = y = 26°

By Cramer's rule

$$A=\begin{bmatrix}1 & -2\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}12\\ 90\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}$$

$=(1)(1)-(-2)(1)$

$=1-(-2)$

$=1+2$

$=3$

$$|A_x|=\begin{vmatrix}12 & -2\\ 90 & 1\end{vmatrix}$$

$=(12)(1)-(-2)(90)$

$=12+180$

$=192$

$$|A_y|=\begin{vmatrix}1 & 12\\ 1 & 90\end{vmatrix}$$

$=(90)-(12)$

$=90-12$

$=78$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{192}{3}$$

$x=64$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{78}{3}$$

$y=26$

Result:

1st angle = x = 64°

2nd angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

(K.B) (U.B)

Solution:

Suppose speed of 1st car = x

Suppose speed of 2nd car = y

According to 1st condition

$$x-y=6 \quad \rightarrow(\text{i})$$

According to 2nd condition

Total distance = 600 km

Left distance = 123 km

Covered distance = 600-123 = 477 km

Total time = $4\frac{1}{2}$ hours = $\frac{9}{2}$ hours

$$\text{Total Speed}=\frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{{}^{53}\cancel{477} \times 2}{\cancel{9}}$$

$$x+y=106 \quad \rightarrow(\text{ii})$$

$x - y = 6$

$x + y = 106$

By matrices inversion method

Changing eq (*i*) and (*ii*) into matrix form

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}6\\106\end{bmatrix}$$

Let $AX = B$, where

$$A=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}6\\106\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2 \quad \text{as} \quad |A|\neq 0$$

Solution is possible because a is non-singular matrix

$$AdjA=\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}$$

$X = A^{-1}B$ or

$$X=\frac{1}{|A|}\times AdjA\times B$$

Putting the values

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}6\\106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}6+106.\\-6+106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}112\\100\end{bmatrix}$$

$$=\begin{bmatrix}\frac{112}{2}\\\frac{100}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}56\\50\end{bmatrix}$$

By comparing

$x=56, y=50$

Result:

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's Rule

$$A=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}6\\106\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$= 1 - (-1)$

$= 1 + 1$

$= 2$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$= (6)(1) - (-1)(106)$

$= 6 - (-106)$

$= 6 + 106$

$= 112$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix}$$

$= (106)(1) - (6)(1)$

$= 106 - 6$

$= 100$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{112}{2}$$

$x = 56$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{100}{2}$$

$y = 50$

Result:

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h