Mathematics-9

Exercise - 3.2

Logarithm of a Real Number

(K.B+U.B)

If $a^x = y$ they x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

The relation $a^x = y$ and $\log_a y = x$ are equivalent.

Thus

$$\boxed{a^x = y \Leftrightarrow \log_a y = x}$$

Note

(U.B+K.B)

- $a^n = y$ and $\log_a y = x$ are respectively exponential and logarithmic forms of the same solution.
  $3^2 = 9$ is equivalent to $\log_3 9 = 2$ and
  $2^{-1} = \frac{1}{2}$ is equivalent to $\log_2 \left[\frac{1}{2}\right] = -1$
- Logarithm of a negative number is not defined at this stage.
- Idea of logarithm was given by a Muslim mathematician Abu Muhammad Musa Al Khwarizmi.
- Logarithmic table with base e was prepared was John Napier.
- Logarithmic table with base 10 was prepared was Professor Henry Briggs.
- Anti-logarithm table was prepared by Jobst Burgi in 1620 A.D.
- Napier's logarithm is also known as **Natural logarithm**.
- e is an irrational number whose approx value is 2.718.

Example # 3

(A.B)

Find $\log_4 2$, i.e., find log of 2 to the base 4

Solution:

Let $\log 4^2 = x$

Then its exponential form is $4^x = 2$

i.e. $2^{2x} = 2^1$

$2x = 1 \quad \because$ bases are same

$\Rightarrow x = \frac{1}{2}$

$\therefore \log_4 2 = \frac{1}{2}$

Deductions from Definition of logarithm

(K.B)

(i) Since $a^0 = 1 \quad \Rightarrow \log_a 1 = 0$

Logarithm of unity to any base is 0.

(ii) Since $a^1 = a \quad \Rightarrow \log_a a = 1$

Logarithm of a number with itself as base is 1.

Common Logarithm

(K.B)

In daily life we use decimal system (means system of base 10) so in numerical calculations, the base of logarithm is taken as 10. This logarithm is called common logarithm or Briggesian logarithm in honour of Henry Briggs, an English mathematician and astronomer, who developed them.

Characteristic

(K.B)

Integral part of the logarithm of a number is called characteristic. It may be positive or negative.

For example:

Characteristic of 99.6 is 1.

How to find Characteristic

(U.B)

It is always one less than the number of digits in the integral part of the number.

Or

When a number b is written in the scientific notation, i.e. in the form $b = a \times 10^n$ where $1 \le a < 10$, the power of 10 i.e. n will give the characteristic of log b.

Examples:

Number	Scientific Notation	Characteristic of the logarithm
1.02	1.02×10^o	0
1662.4	1.6624×10^3	3
0.872	8.72 × 10-1	–1
0.00345	3.45 ×10-3	–3

Note **(K.B+U.B)**

- Characteristic of the logarithm of a number less then 1, is always negative and one more then the number of zeros immediately after the decimal paint of the number.
- Instead of –3 or –1 characteristic is written as $\bar{3}, \bar{2}$ or $\bar{1}$ ($\bar{3}$ is read as bar 3) to avoid the mantissa becoming negative. i.e. $\bar{2}.3748$ does not mean -2.3748. In $\bar{2}.3748$,2 is negative but .3748 is positive where as in -2.3748 both 2 and .3748 are negative.

Mantissa **(K.B)**

Fractional part of the logarithm of a number is called mantissa. It may be positive or negative.

For example:

If $\log x = 2.3451$, then mantissa is 0.3451

Finding the Mantissa of the Logarithm of a Number **(K.B)**

Mantissa is found by making use of logarithm tables. These tables have been constructed to obtain the logarithms up to 7 decimal places. A four-figure logarithmic table provides sufficient accuracy.

A logarithmic table is divided into 3 parts.

(a) The first part of the table is the extreme left column headed b7 blank square. This column Contain numbers from 10 to 99 corresponding to the first two digits of the number whose logarithm is required.

(b) The second part of the table consists of 10 columns headed by 0,1,2,…..,9; These headings correspond to third from the left of the number. The numbers under these columns record mantissa of the logarithms with decimal point omitted for simplicity.

(c) The third part of the table further consists of small columns known as mean differences columns heard by 1,2,3…..9 these heading correspond to the fourth digit from the left of the number. The readings of these columns are added to the mantissa recorded in second part (b) above.

Example # 1 **(A.B)**

Find the mantissa of the logarithms of 43.254.

Solutions:

Rounding off 43.254 we consider only the four significant digits 4325.

(i) We first locate the row corresponding to 43 in the log tables and

(ii) Proceed horizontally tell we reach the column corresponding to 2. The number at the intersection is 6355.

(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.

(iv) Adding the two numbers 6355 and 5 we get. 0.6360 as the mantissa of the logarithm of 43.25.

Example # 2 **(A.B)**

Find the mantissa of the logarithm of 0.002347

Solution:

Here, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.002347 as 0.3705.

Note **(K.B+U.B)**

The logarithms of number having the same sequence of significant digits have the same mantissa e.g., the mantissa of log of number 0.002347 and 0.2347 is 0.3705.

Example # 3 **(A.B)**

Find (i) log 278.23 (ii) log 0.07058

Solution:

(i) 278.23 can be round off as 278.2
The characteristic is 2
The mantissa using log tables, is 0.4443
Thus, Log 278.23 = 2.4443

(ii) The characteristic of log 0.07058 is $\bar{2}$
Using log tables the mantissa is 0.8487
So that
log 0.07058 = $\bar{2}.8487$

Antilogarithm **(K.B)**

(GRW 2018, MTN 2015)

The number whose logarithm is given is called antilogarithm.

If $\log_a y = x$, then y is the antilogarithm of x, or y = antilog x.

Example **(A.B)**

Find the number whose logarithms are (i) 1.3247 (ii) $\bar{2}.1324$

Solution:

(i) **1.32147**

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column correspond to 4. The number at the intersection of this row and the mean difference column corresponding to 7 is 3 adding 2109 and 3 we get 2112.

Since the characteristic is 1 (it is increased by 1), there for the decimal point is fixed after two digits from left to right in 2112.

Hence

antilog of 1.3247 = 21.12

(ii) **$\bar{2}$.1324**

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristics is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point

Hence

antilog of $\bar{2}.1324$ = 0.01356

Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) $\log 232.92$ **(K.B)**

Solution:

$Ch = 2$

Mantissa = 0.3672

$\because \log 232.92 = 2.3672$

(ii) $\log 29.326$ **(K.B)**

Solution:

$Ch = 1$

Mantissa = 0.4672

$\Rightarrow \log 29.326 = 1.4672$

(iii) $\log 0.00032$ **(K.B)**

Solution:

$Ch = \bar{4}$

Mantissa = 0.5051

$\Rightarrow \log 0.00032 = \bar{4}.5051$

(iv) $\log 0.3206$ **(K.B)**

Solution:

$Ch = \bar{1}$

Mantissa = 0.5059

$\Rightarrow \log 0.3206 = \bar{1}.5059$

Q.2 If log $31.09 = 1.4926$, find the value of the following. **(K.B)**

(i) $\log 3.109$

(ii) $\log 310.9$

(iii) $\log 0.003109$

(iv) $\log 0.3109$

Solution:

If $\log\ 31.09 = 1.4926$

Then, mantissa = 0.4926

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$

(ii) $\log 310.9$

Characteristics = 2

Mantissa =0.4926

$\log 310.9 = 2.4926$

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$

Q.3 Find the numbers whose common logarithms are **(K.B)**

(i) 3.5621

Solution:

Let required number = x

Then, $\log x = 3.5621$

Taking antilog on both sides

antilog $\log x$ = antilog 3.5621

$\Rightarrow x = 3649.0$

Hence, required number = 3649

(ii) $\bar{1}.7427$

Solution:

Let required number = x

Then, $\log x = \bar{1}.7427$

Taking antilog on both sides

antilog $\log x$ = antilog $\bar{1}.7427$

$\Rightarrow x = 0.5530$

Hence, required number = 0.5530

Q.4 What replacement for the unknown in each of the following will make the true statements? **(U.B+K.B+A.B)**

(i) $\log_3 81 = L$

Solution:

$\log_3 81 = L$

Writing in exponential form.

$3^L = 81$

$3^L = 3^4$

$\because$ Bases are equal so

$L = 4$

(ii) $\log_a 6 = 0.5$ **(GRW 2017)**

Solution:

$\log_a 6 = 0.5$

In exponential form

$a^{0.5}=6$

$a^{\frac{1}{2}}=6$

$\sqrt{a}=6$

Taking square on both sides

$\sqrt{(a)}^{2}=(6)^{2}$

$a=36$

(iii) $\log_5 n=2$

Write in exponential form

$5^2=n$

$25=n$

Or $n=25$

(iv) $10^P=40$

Solution:

$10^P=40$

Changing into logarithmic form

$P=\log_{10}40$

$=\log 40$

$\Rightarrow P=1.6021$

Q.5 Evaluate.

(i) $\log_2\frac{1}{128}$ **(U.B)**

(LHR 2017, MTN 2017, SWL 2019)

Solution:

Suppose $\log_2\frac{1}{128}=x$

Writing in exponential form.

$2^x=\frac{1}{128}$

$2^x=\frac{1}{2^7}$

$2^x=2^{-7}$

$\because$ Bases are equal so

$x=-7$

(ii) log 512 to the base $2\sqrt{2}$ **(U.B)**

(GRW 2016, SWL 2013, MTN 2015, RWP 2016, D.G.K 2018)

Solution:

$\log_{2\sqrt{2}}512=x$

Writing in exponential form

$\left(2\sqrt{2}\right)^x=512$

$\left(2^1.2^{\frac{1}{2}}\right)^x=2^9$

$\left(2^{\frac{3}{2}}\right)^x=2^9$

$2^{\frac{3}{2}x}=2^9$

$\because$ Bases are equal so

$\frac{3}{2}x=9$

$x=\frac{9\times 2}{3}$

$x=\frac{18^{6}}{3}$

$x=6$

Q.6 Find the value of x from the following statements.

(i) $\log_2 x=5$ **(A.B)**

(GRW 2021, MTN 2018, 21, BWP 2019, SGD 2021)

Solution:

$\log_2 x=5$

Write in exponential form.

$2^5=x$

$32=x$

$x=32$

(ii) $\log_{81}9=x$ **(A.B)**

(LHR 2016, GRW 2013, FSD 2015, 17, SWL 2017, MTM 2013, D.G.K 2017)

Solution:

$\log_{81}9=x$

Writing in the exponential form.

$81^x=9$

$\left(9^2\right)^x=9$

$9^{2x}=9$

$\because$ Bases are equal so

$2x=1$

$x=\frac{1}{2}$

(iii) $\log_{64} 8=\frac{x}{2}$ **(A.B)**

(LHR 2013, GRW 2013, 17, SWL 2016, BWP 2018, SGD 2013, 14, 21, MTN 2019, FSD 2015, RWP 2013, 16)

Solution:

$\log_{64} 8=\frac{x}{2}$

Writing in exponential form.

$64^{\frac{x}{2}}=8$

$\left(8^{2}\right)^{\frac{x}{2}}=8$

$8^{x}=8$

$\because$ Bases are equal so

$x=1$

(iv) $\log_{x} 64=2$ **(A.B)**

(LHR 2018, 21, FSD 2021, RWP 2019, MTN 2018, 21, SWL 2021)

Solution:

$\log_{x} 64=2$

Writing in exponential form

$x^{2}=64$

$x^{2}=8^{2}$

$\because$ Exponents are equal so

$x=8$

(v) $\log_{3} x=4$ **(A.B)**

Solution:

$\log_{3} x=4$

Writing in exponential form

$3^{4}=x$

$81=x$

Or $x=81$

Laws of Logarithm **(K.B+U.B)**

(MTN 2018, RWP 2017)

(i) $\log_{a}(mn)=\log_{a} m+\log_{a} n$

(ii) $\log_{a}\left(\frac{m}{n}\right)=\log_{a} m-\log_{a} n$

(iii) $\log_{a} m^{n}=n\log_{a} m$

(iv) $\log_{a} n=\log_{b} n\times\log_{a} b$