Mathematics-9

Exercise - 3.3

First Law of Logarithm **(K.B+U.B)**

$$\log_a(mn)=\log_b m+\log_a n$$

Proof:

Let

$\log_a m = x$ and $\log_a n = y$

Writing in exponential form

$a^x = m \rightarrow (i)$ and $a^y = n \rightarrow (ii)$

Multiplying equation (i) and (ii)

$a^x \times a^y = mn$

$a^{x+y} = mn$

In logarithmic form

$\log_a(mn) = x + y$

Putting the values of x and y

$\log_a(mn) = \log_a m + \log_a n$

Hence proved

Note **(K.B+U.B)**

(i) $\log_a(mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a(m+n)$

(iii) $\log_a(mnp...) = \log_a m + \log_a n + \log_a p + ...$

Example # 1 **(A.B)**

Evaluate: 291.3 ×42.36

Solution:

Let $x = 291.3 \times 42.36$

Taking log on both sides

$\log x = \log(291.3 \times 42.36)$

$\log x = \log 291.3 + \log 42.36$

$\because \log_a mn = \log_a m + \log_a n$

$= 2.4643 + 1.6269$

$= 4.0912$

Taking antilog on both sides

antilog x = antilog 4.0912

$x = 12340$

$\because 291.3 \times 42.36 = 12340$

Second Law of Logarithm **(U.B+K.B)**

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof:

Let

$\log_a m = x$ and $\log_a n = y$

Writing in exponential form

$a^x = m \rightarrow (i)$ and $a^y = n \rightarrow (ii)$

Dividing equation (i) and (ii)

$\frac{a^x}{a^y} = \frac{m}{n}$

$\Rightarrow a^{x-y} = \frac{m}{n}$

In logarithmic form

$\log_a\left(\frac{m}{n}\right) = x - y$

Putting the values of x and y

$\log_a \frac{m}{n} = \log_a m - \log_a n$

Note **(U.B+K.B)**

(i) $\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m-n)$

(iii) $\log_a\left(\frac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n$

$\because (\log_a 1 = 0)$

Example # 1 **(A.B)**

Evaluate: $\frac{291.3}{42.36}$

Solution:

Let $x = \frac{291.3}{42.36}$

Taking log on both sides

$\log x = \log \frac{291.3}{42.36}$

$\log x = \log 291.3 - \log 42.36$

$\because \log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

$= 2.4643 - 1.6269$

$= 0.8374$

Taking antilog on both sides

antilog x = antilog 0.8374

$x = 6.877$

$\Rightarrow \frac{291.3}{42.36} = 6.877$

Third law of logarithm **(U.B+K.B)**

$\log_a (m)^n = n\log_a m$

Proof:

Let $\log_a m = x$

Writing in exponential form

$a^x = m$

Taking nth power on both sides

$(a^x)^n = m^n$

$a^{nx} = m^n$

In logarithmic form

$\log_a m^n = nx$

Putting the value of x

$\log_a m^n = n\log_a m$

Example # 1 **(A.B)**

Evaluate: $\sqrt[4]{(0.0163)^3}$

Solution:

Let $y = \sqrt[4]{(0.0163)^3}$

$= \left[(0.0163)^3\right]^{1/4} = (0.0163)^{\frac{3}{4}}$

Taking log on both sides

$\log y = \log(0.0163)^{\frac{3}{4}}$

$\log y = \frac{3}{4}(\log 0.0163)$

$= \frac{3}{4}(\bar{2}.2122)$

$= \frac{3}{4}(-2 + 0.2122)$

$= \frac{3}{4}(-1.7878)$

$= -1.3408$

$= -1.3408 + 2 - 2$

$= \bar{2}.6592$

Taking antilog on both sides

antilog y = antilog $\bar{2}.6592$

$y = 0.04562$

$\Rightarrow \sqrt[4]{(0.0163)^3} = 0.04562$

Fourth law of logarithm / Change of Base Formula **(U.B+K.B)**

$\log_a n = \log_b n \times \log_a b$ or $\frac{\log_b n}{\log_b a}$

Proof:

Let $\log_b n = x$.

Writing in exponential form

$n = b^x$

Taking log to the base a on both sides

$\log_a n = \log_a b^x$

Applying 3rd law of logarithm

$\log_a n = x\log_a b$

Putting the value of x

Thus $\log_a n = \log_b n \times \log_a b$

$\log_a n = \log_b n \times \frac{1}{\log_b a}$

$\log_a n = \frac{\log_b n}{\log_b a}$

Note **(K.B+U.B)**

- $\log_a n = \log_b n \times \log_a b$

  Putting n = a in the above result, we get

  $\log_a a = \log_b a \times \log_a b$

  $1 = \log_b a \times \log_a b$

  Or $\log_a b = \frac{1}{\log_b a}$

- $\log_{10} e = \log 2.718 = 0.4343$

- $\log_e 10 = \frac{1}{\log_{10} e} = \frac{1}{0.4343} = 2.3026$

Example: **(A.B)**

Calculate $\log_2 3 \times \log_3 8$

Solution:

$$\log_2 3 \times \log_2 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$

$$\because \log_a n = \frac{\log_b n}{\log_b a}$$

$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$

$$\because \log_a (m)^n = n \log_a m$$

$$= \frac{3\log 2}{\log 2}$$

$$\Rightarrow \log_2 3 \times \log_3 8 = 3$$

Exercise 3.3

Q.1 Write the following into sum or difference $\log(A \times B)$ **(A.B)**

(i) $\log(A \times B)$

Solution: $\log(A \times B)$

$$\log A \times B = \log A + \log B$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

(ii) $\log \frac{15.2}{30.5}$

Solution:

$$\log \frac{15.2}{30.5}$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log 15.2 - \log 30.5$$

(iii) $\log \frac{21 \times 5}{8}$

Solution:

$$\log \frac{21 \times 5}{8}$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log(21 \times 5) - \log 8$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \log 21 + \log 5 - \log 8$$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

Solution:

$$\log \sqrt[3]{\frac{7}{15}} = \log\left(\frac{7}{15}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\log\left(\frac{7}{15}\right)$$

$$\because \log_a (m)^n = n \log_a m$$

$$= \frac{1}{3}(\log 7 - \log 15)$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \frac{1}{3}\log 7 - \frac{1}{3}\log 15$$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Solution:

$$\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log 22^{\frac{1}{3}} - \log 5^3$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \frac{1}{3}\log 22 - 3\log 5$$

$$\because \log_a (m)^n = n \log_a m$$

(vi) $\log \frac{25 \times 97}{29}$

Solution:

$$\log \frac{25 \times 47}{29} = \log(25 \times 47) - \log 29$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log 25 + \log 47 - \log 29$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

Q.2 Express $\log x - 2\log x + 3\log(x+1) - \log(x^2 - 1)$ **as a single logarithm.**

Solution:

$$\log x-2\log x+3\log(x+1)-\log(x^2-1)$$

$$=\log x-\log x^2+\log(x+1)^3-\log(x^2-1)$$

$$\because \log_a(m)^n=n\log_a m$$

$$=\log\left(\frac{x}{x^2}\right)+\log\frac{(x+1)^3}{x^2-1}$$

$$\because \log_a\frac{m}{n}=\log_a m-\log_a n$$

$$=\log\left(\frac{x}{x^2}\times\frac{(x+1)^3}{x^2-1}\right)$$

$$\because \log_a(mn)=\log_b m+\log_a n$$

$$=\log\left(\frac{x(x+1)^3}{x^2(x^2-1)}\right)$$

$$=\log\frac{x(x+1)^2(x+1)}{x\times x(x-1)(x+1)}$$

$$=\log\frac{(x+1)^2}{x(x-1)}$$

Q.3 Write the following in the form of a single logarithm. (A.B)

(i) $\log 21+\log 5$

Solution:

$$\log 21+\log 5$$

$$\because \log_a(mn)=\log_b m+\log_a n$$

$$=\log(21\times 5)$$

(ii) $\log 25-2\log 3$

Solution: $\log 25-2\log 3$

$$=\log 25-2\log 3$$

$$=\log 25-\log 3^2$$

$$\because \log_a(m)^n=n\log_a m$$

$$=\log\frac{25}{3^2}\quad \because \log_a\frac{m}{n}=\log_a m-\log_a n$$

(iii) $2\log x-3\log y$

(BWP 2017, SWL 2018, 19, MTN 2019, D.G.K 2019)

Solution:

$$2\log x-3\log y$$

$$=\log x^2-\log y^3$$

$$=\log\frac{x^2}{y^3}\quad\because \log_a\frac{m}{n}=\log_a m-\log_a n$$

(iv) $\log 5+\log 6-\log 2$ **(FSD 2018)**

Solution:

$$\log 5+\log 6-\log 2$$

$$\because \log_a(mn)=\log_b m+\log_a n$$

$$=\log(5\times 6)-\log 2$$

$$\because \log_a\frac{m}{n}=\log_a m-\log_a n$$

$$=\log\frac{5\times 6}{2}\ \textbf{Ans}$$

Q.4 Calculate the following.

(i) $\log_3 2\times\log_2 81$

(LHR 2019, FSD 2017, 19)

Solution:

$$\log_3 2\times\log_2 81$$

$$=\frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$$

$$=\frac{\log 81}{\log 3}$$

$$=\frac{\log 3^4}{\log 3}$$

$$\because \log_a(m)^n=n\log_a m$$

$$=\frac{4\log 3}{\log 3}$$

$$=4\ \textbf{Ans}$$

(ii) $\log_5 3\times\log_3 25$

Solution:

$$\log_5 3\times\log_3 25$$

Applying 4^{th} law of logarithm

$=\frac{\log 3}{\log 5}\times\frac{\log 25}{\log 3}$

$=\frac{\log 25}{\log 5}$

$=\frac{\log 5^2}{\log 5}$

$\because \log_a (m)^n = n\log_a m$

$=\frac{2\log 5}{\log 5}$

$=2$ **Ans**

Q.5 If $\log 2 = 0.3010, \log 3 = 0.4771$ **and** $\log 5 = 0.6990$, **then find the values of the following. (A.B)**

(i) $\log 32$

$=\log 2^5$

$\because$ using 3^{rd} law of logarithm

$=5\log 2$

By putting the value of $\log 2$

$=5(0.3010)$

$=1.5050$ **Ans**

(ii) $\log 24$

Solution:

$\log 24$

$=\log(2^3\times 3)$

$=\log 2^3+\log 3$

$\because \log_a (m)^n = n\log_a m$

$=3\log 2+\log 3$

By putting the value

$=3(0.3010)+0.4771$

$=0.9030+0.4771$

$=1.3801$

(iii) $\log\sqrt{3\frac{1}{3}}$

Solution:

$\log\sqrt{3\frac{1}{3}}$

$=\log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$\because \log_a (m)^n = n\log_a m$

$=\frac{1}{2}\log\left[\frac{2\times 5}{3}\right]$

Using 1^{st} and 2^{nd} laws of logarithm

$=\frac{1}{2}(\log 2+\log 5-\log 3)$

By putting the values

$=\frac{1}{2}(0.3010+0.69900-0.4771)$

$=\frac{1}{2}(1-0.4771)$

$=\frac{1}{2}(0.5229)$

$=0.26145$

(iv) $\log\frac{8}{3}$

Solution:

$\log\frac{8}{3}$

$=\log\frac{2^3}{3}$

$\because \log_a \frac{m}{n} = \log_a m - \log_a n$

$=\log 2^3-\log 3$

$\because \log_a (m)^n = n\log_a m$

$=3\log 2-\log 3$

By putting the values

$=3(0.3010)-0.4771$

$=0.9030-0.4771$

$=0.4259$ **Ans**

(v) $\log 30$

Solution:

$\log 30$

$= \log(5 \times 2 \times 3)$

$\because$ using first law of logarithm

$= \log 5 + \log 2 + \log 3$

By putting the values

$= (0.6990) + (0.3010) + (0.4771)$

$= 1.4771$ **Ans**