Application of Laws of Logarithm in Numerical Calculations

So far we have applied laws of logarithm to simple type of products, quotients, powers or roots of numbers. We now extend their application to more difficult examples to verify their effectiveness in simplification.

Example 1: **(A.B+K.B+U.B)**

Show that $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}=\log 2$

Solution:

L.H.S. = $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}$

$\because \log_a\frac{m}{n}=\log_a m-\log_a n$

$= 7[\log 16-\log 15]+5[\log 25-\log 24]+3[\log 81-\log 80]$

$= 7\left[\log 2^4-\log(3\times 5)\right]+5\left[\log 5^2-\log\left(2^3\times 3\right)\right]+3\left[\log 3^4-\log\left(2^4\times 5\right)\right]$

$\because \log_a(mn)=\log_b m+\log_a n$

$= 7\left[4\log 2-(\log 3+\log 5)\right]+5\left[2\log 5-(3\log 2+\log 3)\right]+3\left[4\log 3-[4\log 2+\log 5]\right]$

$= 28\log 2-15\log 2-12\log 2-7\log 3-5\log 3+12\log 3-7\log 5+10\log 5-3\log 5$

$= (28-15-12)\log 2+(-7-5+12)\log 3+(-7+10-3)\log 5$

$= (1)\log 2+(0)\log 3+(0)\log 5$

$= \log 2+0+0$

$= \log 2 = \text{R.H.S}$

Example # 2 **(A.B)**

Evaluate: $\sqrt[3]{\frac{0.0792\times(18.99)^2}{(5.79)^4\times 0.9474}}$

Solution:

Let y $=\sqrt[3]{\frac{0.07921\times(18.99)^2}{(5.79)^4\times(0.9474)}}$

$=\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right]^{\frac{1}{3}}$

Taking log on both sides

$$\log y = \log\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right]^{\frac{1}{3}}$$

$$\because \log_a (m)^n = n\log_a m$$

$$= \frac{1}{3}\log\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times(0.9474)}\right]$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \frac{1}{3}\left[\log 0.07921\times(18.99)^2\right] - \log\left[(5.79)^4\times 0.9474\right]$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \frac{1}{3}\left[(\log 0.07921 + 2\log 18.99) - (4\log 5.79 + \log 0.9474)\right]$$

$$= \frac{1}{3}\left[\log 0.07921 + 2\log 18.99 - 4\log 5.79 - \log 0.9474\right]$$

$$= \frac{1}{3}\left[\bar{2}.8988 + 2(1.2786) - 4(0.7627) - \bar{1}.9765\right]$$

$$= \frac{1}{3}\left[\bar{2}.8988 + 2.5572 - 3.0508 - \bar{1}.9765\right]$$

$$= \frac{1}{3}\left[1.4660 - 3.0273\right] = \frac{1}{3}\left(\bar{2}.4287\right)$$

$$= \frac{1}{3}\left[\bar{3} + 1.4287\right]$$

$$= \bar{1} + 0.4762 = \bar{1}.4762$$

Or y = antilog $\bar{1}.4762$

y = 0.2993

Example # 3 **(A.B)**

Given $A = Aoe^{-kd}$ **If k = 2 what be the value of d to make** $A = \frac{Ao}{2}$?

$$A = A_o e^{-kd}$$

$$\frac{A_o}{2} = A_o e^{-kd}$$

$$\frac{\cancel{A_o}}{2\cancel{A_o}} = e^{-kd}$$

$$\frac{1}{2} = e^{-kd}$$

Substituting k = 2 and where e = 2.718

$$\frac{1}{2}=(2.718)^{-2d}$$

Taking Common log on both sides

$$\log 10\frac{1}{2}=\log 10(2.718)^{-2d}$$

$$\log_{10}1-\log_{10}2=-2d\log_{10}(2.718)$$

$$0-0.3010=-2d(0.4343)$$

$$-0.3010=-0.8686(d)$$

$$\frac{-0.3010}{-0.8686}=d$$

$$d=0.3465$$

Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64 **(A.B)**

Solution:

Suppose

$x=0.8176\times 13.64$

Taking log on both sides

$\log x=\log(0.8176\times 13.64)$

According to first law of logarithm

$\log x=\log 0.8176+\log 13.64$

$=\bar{1}.9125+1.1348$

$\log x=-1+0.9125+1.1348$

$\log x=1.0473$

Taking antilog on both sides

$x=\text{antilog}\,1.0473$

$x=11.15$

(ii) $(789.5)^{\frac{1}{8}}$

Solution:

Let $x=(789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x=\log(789.5)^{\frac{1}{8}}$

According to third law

$$\log x=\frac{1}{8}\log(789.5)$$

$$\log x=\frac{1}{8}(2.8974)$$

$$=\frac{2.8974}{8}$$

$\log x=0.3622$

Taking antilog on both sides

$x=\text{antilog}\,0.3622$

$x=2.302$

(iii) $\dfrac{0.678\times 9.01}{0.0234}$

Solution:

Suppose

$$x=\frac{0.678\times 9.01}{0.0234}$$

Taking log on both sides

$$\log x=\log\frac{0.678\times 9.01}{0.0234}$$

According to 1st and 2nd law of log

$\log x=\log 0.678+\log 9.01-\log 0.0234$

$\log x=\bar{1}.8312+0.9547-\bar{2}.3692$

$=-1+0.8312+0.9547-(-2+0.3692)$

$=2.4167$

Taking antilog on both sides

$x=\text{antilog}\,2.4167$

$x=261.0$

(iv) $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

Solution:

Let $x=\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

$=(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}$

Taking log on both sides

$\log x = \log\left[(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}\right]$

According to law of logarithm

$\log x = \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

According to third law of logarithm

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$= \frac{1}{5}0.4328 + \frac{1}{7}0.0931$

$= 0.0866 + 0.0133$

$= 0.0999$

Taking antilog on both sides

x = antilog 0.999

$x = 1.259$

(v) $\frac{1.23 \times 0.6975}{0.0075 \times 1278}$

Solution:

Suppose

$x = \frac{1.23 \times 0.6975}{0.0075 \times 1278}$

$\log x = \log \frac{1.23 \times 0.6975}{0.0075 \times 1278}$

$= \log(1.23 \times 0.6975) - \log(0.0075 \times 1278)$

$= \log 1.23 + \log 0.6975 - (\log 0.0075 + \log 1278)$

$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.8999 + (-1 + 0.8435) - (-3 + 0.8751) - 3.1065$

$= -1.0482$

$\log x = -2 + 2 - 1.0482$

$\log x = 0.9518 - 2 + 0.9518$

$\log x = \bar{2}.9518$

Taking antilog on both sides

x = antilog $\bar{2}.9518$

$= 0.08950$

(vi) $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Solution:

Let $x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

$x = \left[\frac{0.7214 \times 20.37}{60.8}\right]^{\frac{1}{3}}$

Taking log on both sides

$\log x = \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$

3rd of logarithm

$\log x = \frac{1}{3}\log\left[\frac{0.7214 \times 20.37}{60.8}\right]$

According to first and 2nd law

$\log x = \frac{1}{3}[\log 0.7214 + \log 37 - \log 60.8]$

$\log x = \frac{1}{3}\left[\bar{1}.8582 + 1.3089 - 1.7839\right]$

$= \frac{1}{3}[-1 + 0.8582 + 1.3089 - 1.7839]$

$= \frac{1}{3}(-0.6168)$

$= -0.2056$

$\log x$ is in negative, so

$\log x = -1 + 1 - 0.2056$

$= -1 + 0.79144$

$= \bar{1}.7944$

Taking antilog on both sides

x = antilog $\bar{1}.7944$

= 0.6229

(vii)

Solution: $\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Suppose: $x = \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$x = \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$

Taking on both sides

$\log x = \log \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$

According to 1st and 2nd law of log

$\log x = \log 83 + \log(92)^{\frac{1}{3}} - \log 127 - \log(246)^{\frac{1}{5}}$

According to third law of log

$\log x = \log 83 + \frac{1}{3}\log 92 - \log 27 - \frac{1}{5}\log 246$

$\log x = (1.9191) + \frac{1}{3}(1.9638) - (2.1038)$

$-\frac{1}{5}(2.3909)$

$= 1.9191 + 0.65460 - 2.1038 - 0.47818$

$= 1.9191 + 0.6546 - 2.1038 - 0.47818$

$= -0.0083$

$\log x$ is in negative, so

$\log x = -1 + 1 - 0.0083$

$= -1 + 0.9917$

$= \bar{1}.9917$

Taking antilog log on both sides

$x =$ antilog $\bar{1}.9917$

$x = 0.9811$

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Solution:

Suppose: $x = \frac{(438)^3\sqrt{0.056}}{(388)^4}$

$x = \frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$

$x = \frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$

Taking log on both sides

$\log x = \log\left(\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}\right)$

According to 1st and 2nd law

$\log x = \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$

According to third law

$\log x = 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(38)$

$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$

$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$

$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$

$= 7.9245 - 0.6259 - 10.3552$

$= -3.0566$

log is in negative, so

$\log x = -4 + 4 - 3.0566$

$= -4 + 0.9434 = \bar{4}.9434$

Taking antilog on both sides

$x =$ antilog $\bar{4}.9434$

$= 0.0008778$

Q.2 A gas is expanding according to the law $pv^n = C$.
Find C when p = 80, v = 3.1 and $n = \frac{5}{4}$.

Solution:

Given $pv^n = C$

Taking log on both sides

$\log(pv^n) = \log C$

Applying laws of logarithm

$\log C = \log P + n\log v$

Putting P=80, v=3.1 and $n = \frac{5}{4}$

$\log C = \log 80 + \frac{5}{4}\log 3.1$

$= 1.9031 + \frac{5}{4}(0.4914)$

$= 1.9031 + 0.6143$

$\log C = 2.5174$

Taking antilog both sides

C=Antilog (2.5174)

C=329.2

Q.3 The formula $p = 90\ (5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00? (A.B)

Solution:

Given that $p = 90(5)^{\frac{-q}{10}}$

Taking log on both sides

$log\ p = \log 90(5)^{\frac{-q}{10}}$

Applying laws of logarithm

log P = log 90 $-\frac{q}{10}$ log 5

Putting the value of P

log 18= log 90- $\frac{q}{10}$ log 5

1.2553=1.9542- $\frac{q}{10}$ × 0.6990

1.2553-1.9542 = $-\frac{q}{10}$ × 0.6990

-0.6989 × 10 = - q × 0.6990

-6.989=-q × 0.6996

6.989 = q × 0.6996

$\frac{6.989}{0.6990} = q$

q =10 approximately

Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, **find A, when** $\pi = \frac{22}{7}$ **and** $r = 15$ **. (A.B)**

Solution:

Given that $A = \pi r^2$

Taking log on both sides

log A=log πr^2

Putting $\pi = \frac{22}{7}$ and r =15

$\log A = \log \frac{22}{7}(15)^2$

Applying laws of logarithm

log A = log 22-log 7+2 log 15

= 1.3424-0.8451+2(1.1761)

= 0.4973+2.3522

log A = 2.8495

Taking antilog on both sides

A = antilog 2.8495

A = 707.1

Q.5 If $V = \frac{1}{3}\pi r^2 h$, **find V, when** $\pi = \frac{22}{7}$, $r = 2.5$ **and** h**=4.2. (A.B)**

Solution:

Given that $V = \frac{1}{3}\pi r^2 h$

Taking log on both sides

$\log V = \log \frac{1}{3}\pi r^2 h$

Putting the values

$\log V = \log \frac{1}{3} \times \frac{22}{7}(2.5)^2(4.2)$

Applying laws of logarithm

$= \log 1 - \log 3 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$

$= -0.4771 + 1.3424 - 0.8450 + 2(0.3979) + 0.6232$

$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$

log V=1.4394

Taking antilog on both sides

V=antilog 1.4394

V=27.50