NUMERICAL PROBLEMS

(U.B+A.B)

10.1 The time period of a simple pendulum is 2s. What will be its length on Earth? What will be its length on the moon if $g_m = g_e / 6$? Where $g_e = 10ms^{-2}$. (FSD-G1)-2015

Solution:

Given Data:

Time period of simple pendulum = T = 2 sec.

Value of 'g' on Earth = $g_e = 10\ ms^{-2}$

Value of 'g' on Moon = $g_m = \frac{g_e}{6} = \frac{10}{6} = 1.6\,ms^{-2}$

To Find:

(i) Length of pendulum on earth = ℓ_e = ?

(ii) Length of pendulum on moon = ℓ_m = ?

Formula:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

(ii) For Moon

$$T^2 = \frac{4\pi^2 \ell_m}{g_m}$$

$$\ell_m = \frac{T^2 g_m}{4 \times \pi^2}$$

By putting the values, we have

$$\ell_m = \frac{(2)^2 \times 1.6}{4 \times (3.14)^2} = \frac{6.44}{39.44}$$

$$\boxed{\ell_m = 0.17\,m}$$

Calculation:

(i) For Earth

$$T = 2\pi\sqrt{\frac{\ell_e}{g_e}}$$

By taking square on both sides, we have

$$T^2 = 4\pi^2 \frac{\ell_e}{g_e}$$

or

$$\ell_e = \frac{T^2 \times g_e}{4\pi^2}$$

By putting the values, we have

$$\ell_e = \frac{(2)^2 \times 10}{4 \times (3.14)^2} = \frac{4 \times 10}{4 \times 9.86}$$

$$\boxed{\ell_e = 1.02\,m}$$

Result:

Hence, the length of pendulum on Earth and on Moon will be 1.02 m and 0.17 m respectively.

10.2 A pendulum of length 0.99 m is taken to the Moon by an astronaut. The period of the pendulum is 4.9s. What is the value of g on the surface of the moon? **(MTN-G2)-2015**

Solution:

Given Data:

Length of pendulum on Moon = ℓ_m = 0.99m

Time period of pendulum = T = 4. 9 s

To Find:

Value of g on moon = g = ?

Formula:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Calculation:

By using formula, we have

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$4.9\text{sec} = 2\times 3.14\sqrt{\frac{0.99}{g}}$$

Squaring

$$g = \frac{4\times(3.14)^2\times 0.99}{(4.9)^2}$$

$g = 1.63\ ms^{-2}$

Result:

Hence, the value of 'g' of the surface of Moon will be 1.6 ms^{-2}.

10.3 Find the time periods of a simple pendulum of 1 meter length, placed on Earth and on moon. The value of g on the surface of moon is 1/6th of its value on Earth. When g_e is 10ms^{-2}.

Solution:

Given Data:

Length of simple pendulum = ℓ = 1m

Value of 'g' on Earth = g_e = 10 ms^{-2}

Value of 'g' on Moon = g_m = 1.62 ms^{-2}

To Find:

Time period on earth = T_e = ?

Time period on moon = T_m = ?

Formula:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Calculation:

(i) For Earth:

$$T = 2\pi\sqrt{\frac{\ell_e}{g_e}}$$

$$T_e = 2(3.14)\sqrt{\frac{1}{10}}$$

$$T_e = (6.28)\sqrt{0.1}$$

T_e = (6.28) (0.316)

T_e = 1.985 sec.

T_e = 2sec. Ans

(ii) For Moon:

$$T_m = 2\pi\sqrt{\frac{\ell_m}{g_m}}$$

$$T_m = 2(3.14)\sqrt{\frac{1}{1.67}}$$

$$T = 2(3.14)\sqrt{0.6172}$$

T = 4.9 sec

Result:

Hence, the time period of simple pendulum on Earth and Moon will be 2 s and 4.9 s respectively.

10.4 A simple pendulum completes one vibration in two seconds. Calculate its length when g = 10.0 ms^{-2}

Solution:

Given Data:

Time period of second pendulum = T = 2sec

Gravitational acceleration = g = 10 ms^{-2}

To Find:

Length of simple pendulum = ℓ = ?

Calculation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Squaring on both sides

$$T^2 = 4\pi^2 \times \frac{\ell}{g}$$

$$\ell = \frac{T^2 g}{4\pi^2}$$

$$\ell = \frac{(2)^2 \times 10}{4 \times (3.14)^2}$$

$$\ell = \frac{\cancel{4} \times 10}{\cancel{4} \times 9.85}$$

ℓ = 1.02 m

Result:

Hence, the length of simple pendulum will be 1.02 m.

10.5 If 100 waves pass through a point of a medium in 20 seconds, what is the frequency and the time period of the wave? If its wavelength is 6cm, calculate the wave speed.

Solution:

Given Data:

No. of waves passed through a point = n = 100

Time taken = t = 20s

Wavelength = λ = 6cm = 0.06 m

To Find:

(i) Frequency of wave = f = ?

(ii) Time period of wave = T = ?

(iii) Speed of wave = v = ?

Formula:

(i) $f = n/t$ $\frac{\text{no. of waves passed}}{\text{Time taken}}$

(ii) Time period of wave $= T = \frac{1}{f}$

(iii) Speed of wave = v = fλ

Calculation:

(i) By using formula, we have

$$f = \frac{n}{t}$$

$$f = \frac{100}{20}$$

f = 5Hz

(ii) As, we know that

$$T = \frac{1}{f}$$

$$T = \frac{1}{5Hz}$$

T = 0.2 sec

(iii) By using wave equation, we have

$V = f\lambda$

$V = 5 \times 0.06$

$V = 0.3\ ms^{-1}$

Result:

Hence, the frequency, time period and speed of the wave will be 5 Hz, 0.2 s and 0.3 ms^{-1} respectively.

10.6 A wooden bar vibrating into the water surface in a ripple tank has frequency of 12Hz. The resulting wave has a wavelength of 3cm. What is the speed of the wave?

Solution:

Given Data:

Frequency of wooden bar = f = 12 Hz

Wavelength = λ = 3cm = 0.03 m

To Find:

Speed of wave = v = ?

Formula:

We know that

$V = \lambda f$

Calculations:

By using wave equation,

$v = f\lambda$

$v = (0.03)(12)$

$v = 0.36\ ms^{-1}$

Result:

Hence, the speed of wave will be 0.36 ms^{-1}.

10.7 A transverse wave produced on a spring has a frequency of 190 Hz and travels along the length of the spring of 90m, in 0.5s.

(a) What is the period of wave?
(b) What is the speed of the wave?
(c) What is the wavelength of the wave?

Solution:

Given Data:

Frequency of wave = f = 190 Hz

Distance travelled by wave = d = 90 m

Time taken = t = 0.5s

To Find:

(i) Time period of wave = T = ?

(ii) Speed of wave = V = ?

(iii) Wavelength = λ = ?

Formula:

(i) $T = \frac{1}{f}$

(ii) $v = \frac{d}{t}$

(iv) $\lambda = \frac{v}{f}$

Calculations:

(i) Time period:

By using formula, we have

$T = 1/f$

$T = 1/190$

$T = 0.005$

$T = 0.01s$

(ii) Speed of wave:

By using formula, we have

$V = d/t$

$V = 90/0.5$

$V = 180$ m/s

(iii) Wavelength:

By using wave equation, we have

$\lambda = v/f$

$\lambda = 180/190$

$\lambda = 0.95$ m

Result:

Hence, the time period, speed and wavelength of the wave will be 0.01 s, 180 ms^{-1} and 0.05 m respectively.

10.8 Water waves in a shallow dish are 6.0 cm long. At one point, the water moves up and down at a rate of 4.8 oscillations per second.
(a) What is the speed of the water waves?
(b) What is the period of the water waves?

Solution:
Given Data:
Length of dish = d = 6.0 cm = 0.06 m
Frequency of wave = f = 4.8 Hz
To Find:
(i) Speed of waves = ?
(ii) Time period of waves = ?
Formula:
(i) $v = \frac{d}{t}$
(ii) $T = \frac{1}{f}$

Calculations:
(i) Time period:
By using formula, we have
T = 1/f
T = 1/4.8
T = 0.21 s
(ii) Speed of waves:
By using formula, we have
V = d/t
V = 0.06/0.21
V = 0.29m/s

Result:

Hence, the speed and time period of water wave will be 0.29 ms^{-1} and 0.21 s respectively.

10.9 At one end of a ripple tank 80 cm across, 5 Hz vibrator produces waves whose wavelength is 40mm. Find the time the waves need to cross the tank.

Solution:
Given Data:
Distance travelled = d = 80 cm = 0.8 m
Frequency = f = 5Hz
Wavelength = λ = 40mm = 0.04 m
To Find:
Time taken by the wave = t = ?
Formula:
(i) $v = \frac{d}{t}$
Calculation:
Using wave equation
v = fλ
v = (5) (0.04) = 0.2 m/s
Know by using formula, we have
$v = \frac{d}{t}$
So, $t = \frac{d}{v}$
t = 0.8/0.02
t = 4s

Result:

Hence, time taken by the wave to cross the tang will be 4s.

10.10 What is the wavelength of the radio waves transmitted by an FM station at 90 MHz? Where 1M = 10^6, and speed of radio wave is 3 x $10^8 ms^{-1}$.

Solution:

Given Data:

Frequency of radio waves = f = 90 MHz

$f = 9\times10^7$ Hz

Speed of radio waves = $v = 3 \times 10^8 ms^{-1}$

To Find:

Wave length of the radio waves = λ = ?

Formula:

According to the wave equation

$v = f \lambda$

$\lambda = v/f$

Calculation:

By wave equation,

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3\times10^8}{9\times10^7}$$

$$\lambda = \frac{3\times10^{8-7}}{9.0}$$

$\lambda = 3.333m$

Result:

Hence, the wavelength of the radio waves transmitted by an FM station will be 3.33