Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(LHR 2017, GRW 2017, SGD 2013, 17, SWL 2013, 15, RWP 2016, MTN 2017)

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$ **(A.B)**

Solution:

$39x^7y^3z = 3\times13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

H.C.F $=13\times x.x.x.x.x.y.y.y.z$

H.C.F = $13x^5y^3z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times3\times17\times x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17\times x.y.z.z$

H.C.F $=17xyz$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2+5x+6, x^2-4x-12$ **(LHR 2013, GRW 2015)**

Solution:

Factorizing x^2+5x+6

$= x^2+3x+2x+6$

$= x(x+3)+2(x+3)$

$= (x+3)(x+2)$

Factorizing $x^2-4x-12$

$= x^2-6x+2x-12$

$= x(x-6)+2(x-6)$

$= (x-6)(x+2)$

So,

H.C.F = $(x+2)$

(ii) $x^3-27, x^2+6x-27, 2x^2-18$ **(A.B)**

Solution:

Factorizing x^3-27

$=(x)^3-(3)^3$

$=(x-3)\left[(x)^2+(x)(3)+(3)^2\right]$ $\therefore a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(x-3)(x^2+3x+9)$

Factorizing $x^2+6x-27$

$=x^2+9x-3x-27$

$=x(x+9)-3(x+9)$

$=(x+9)(x-3)$

Factorizing $2x^2-18$

$=2(x^2-9)$

$=2\left[(x)^2-(3)^2\right]$

$=2(x-3)(x+3)$ $\therefore a^2-b^2=(a+b)(a-b)$

So,

H.C.F = $(x-3)$

(iii) $x^3-2x^2+x, x^2+2x-3, x^2+3x-4$

Factorizing x^3-2x^2+x

$=x(x^2-2x+1)$

$=x(x^2-x-x+1)$

$=x\left[x(x-1)-1(x-1)\right]$

$=x(x-1)(x-1)$

Factorizing x^2+2x-3

$=x^2+3x-x-3$

$=x(x+3)-1(x+3)$

$=(x+3)(x-1)$

Factorizing x^2+3x-4

$=x^2+4x-x-4$

$=x(x+4)-1(x+4)$

$=(x+4)(x-1)$

So, H.C.F $=(x-1)$

(iv) $18(x^3-9x^2+8x), 24(x^2-3x+2)$ **(A.B)**

Solution:

$18\left(x^3-9x^2+8x\right)$
$=2\times3^2\times x\left(x^2-9x+8\right)$
$=2\times3^2\times x\left(x^2-8x-x+8\right)$
$=2\times3^2\times x\left[x\left(x-8\right)-1\left(x-8\right)\right]$
$=2\times3^2\times x\left(x-8\right)\left(x-1\right)$

$24\left(x^2-3x+2\right)$
$=2^3\times3\left(x^2-3x+2\right)$
$=2^3\times3\left(x^2-2x-x+2\right)$
$=2^3\times3\left[x\left(x-2\right)-1\left(x-2\right)\right]$
$=2^3\times3\left(x-2\right)\left(x-1\right)$

So,
H.C.F = $2\times3\left(x-1\right)$
H.C.F = $6\left(x-1\right)$

(v) $36\left(3x^4+5x^3-2x^2\right),54\left(27x^4-x\right)$

Factorizing $36\left(3x^4+5x^3-2x^2\right)$
$=3\times3\times2\times2\times x^2\left(3x^2+5x-2\right)$
$=3\times3\times2\times2\times x^2\left(3x^2+6x-x-2\right)$
$=3\times3\times2\times2\times x^2\left[3x\left(x+2\right)-1\left(x+2\right)\right]$
$=3\times3\times2\times2\times x^2\left(x+2\right)\left(3x-1\right)$

Factorizing $54\left(27x^4-x\right)$
$=3\times3\times3\times2\times x\left(27x^3-1\right)$
$=3\times3\times3\times2\times x\left[\left(3x\right)^3-\left(1\right)^3\right]$
$=3\times3\times3\times2\times x\left(3x-1\right)\left[\left(3x\right)^2+\left(3x\right)\left(1\right)+\left(1\right)^2\right]$ $\quad\therefore a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)$
$=3\times3\times3\times2\times x\left(3x-1\right)\left(9x^2+3x+1\right)$

So,
H.C.F = $3\times3\times2\times x\left(3x-1\right)$
$=18x\left(3x-1\right)$

Q.3 Find the H.C.F. of the following by division method.

(i) $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$ **(A.B)**

Solution: $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

$$\begin{array}{r|l} & 1 \\ x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\ & \pm x^3 \pm x^2 \mp 10x \pm 8 \\ \hline & 2x^2-6x+4 \\ & 2\left(x^2-3x+2\right) \end{array}$$

$$\begin{array}{r|l} & x+4 \\ x^2-3x+2 & x^3+x^2-10x+8 \\ & \pm x^3 \mp 3x^2 \pm 2x \\ \hline & 4x^2-12x+8 \\ & \pm 4x^2 \mp 12x \pm 8 \\ \hline & \times \end{array}$$

H.C.F = $\left(x^2-3x+2\right)$

(ii) $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

Solution: $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

$$\begin{array}{r|l} & x+2 \\ 5x^3+3x^2-17x+6 & x^4+x^3-2x^2+x-3 \\ & \times 5 \\ \hline & 5x^4+5x^3-10x^2+5x-15 \\ & \pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x \\ \hline & 2x^3+7x^2-x-15 \\ & \times 5 \\ \hline & 10x^3+35x^2-5x-75 \\ & \pm 10x^3 \pm 6x^2 \mp 34x \pm 12 \\ \hline & 29x^2+29x-87 \\ & 29(x^2+x-3) \end{array}$$

$$\begin{array}{r|l} & 5x-2 \\ x^2+x-3 & 5x^3+3x^2-17x+6 \\ & \pm 5x^3 \pm 5x^2 \mp 15x \\ \hline & -2x^2-2x+6 \\ & \mp 2x^2 \mp 2x \pm 6 \\ \hline & \times \end{array}$$

H.C.F $=\left(x^2+x-3\right)$

(iii) $2x^5-4x^4-6x,x^5+x^4-3x^3-3x^2$ **(A.B)**

$$2x^5-4x^4-6x\,\overline{)\,x^5+x^4-3x^3-3x^2\,}\quad 1$$

$$\times 2$$

$$2x^5+2x^4-6x^3-6x^2$$

$$-2x^5 \mp 4x^4 \qquad \mp 6x$$

$$6x^4-6x^3-6x^2+6x$$

$$6\left(x^4-x^3-x^2+x\right)$$

$$x^4-x^3-x^2+x\,\overline{)\,2x^5-4x^2-6x\,}\quad 2x+2$$

$$\pm 25x^5 \pm 2x^2 \qquad \mp 2x^4 \mp 2x^3$$

$$2x^4+2x^3-6x^2-6x$$

$$\pm 24x^3 \mp 2x^2 \mp 2x^2 \mp 2x$$

$$4x^3-4x^2-8x$$

$$\left(x^3-x^2-2x\right)$$

$$x^3-x^2-2x\,\overline{)\,x^4-x^3-x^2+x\,}\quad x$$

$$\mp x^4 \mp x^3 \mp 2x^3$$

$$x^2+x$$

$$x^2+x\,\overline{)\,x^3-x^2-2x\,}\quad x-2$$

$$\pm x^3 \pm x^2$$

$$-2x^2-2x$$

$$\mp 2x^2 \mp 2x$$

$$\times$$

H.C.F $=x^2+x=x\left(x+1\right)$

Q.4 Find the L.C.M. of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$ **(A.B)**

Solution:

$39x^7y^3z = 3\times 13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

Common$=13x^5y^3z$

Uncommon$=3\times 7\times x^2y^3z^6$

$=21x^2y^3z^6$

L.C.M= common factors$\times$uncommon factors

$=13x^5y^3z\times 21x^2y^3z^6$

$=273x^7y^6z^7$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$ **(LHR 2015)**

Solution:

$102\,xy^2z = 2\times3\times17.x.y.y.z$

$85\,x^2yz = 5\times17\times x.x.y.z$

$187\,xyz^2 = 11\times17.x.y.z.z$

Common $=17xyz$

Uncommon $= 2\times3\times5\times11.xyz$

$= 330\,xyz$

L.C.M = common × uncommon

$=17xyz\times330xyz$

$=5610x^2y^2z^2$

Q.5 Find the L.C.M of the following by factorizing. **(A.B)**

(i) $x^2-25x+100$ **and** x^2-x-20

Solution:

Factorizing $x^2-25x+100$

$=x^2-20x-5x+100$

$=x(x-20)-5(x-20)$

$=(x-20)(x-5)$

Factorizing x^2-x-20

$=x^2-5x+4x-20$

$=x(x-5)+4(x-5)$

$=(x-5)(x+4)$

Common factors $=(x-5)$

Non common factors = $(x-20)(x+4)$

So, L.C.M = common factors × non common factors

L.C.M = $(x-5)(x+4)(x-20)$

(ii) $x^2+4x+4,\ x^2-4,\ 2x^2+x-6$

Solution:

Factorizing x^2+4x+4

$=x^2+2x+2x+4$

$=x(x+2)+2(x+2)$

$=(x+2)(x+2)$

Factorizing x^2-4

$=(x)^2-(2)^2$

$=(x-2)(x+2)$ $\quad\therefore\ a^2-b^2=(a+b)(a-b)$

Factorizing $2x^2+x-6$

$$= 2x^2 + 4x - 3x - 6$$
$$= 2x(x+2) - 3(x+2)$$
$$= (x+2)(2x-3)$$

Common factors $= (x+2)$

Non common factors = $(x-2)(2x-3)$

So, L.C.M = common factors × non common factors

L.C.M $= (x+2)(x+2)(x-2)(2x-3)$

$$= (x+2)^2(x-2)(2x-3)$$

(iii) $2(x^4 - y^4), 3(x^3 + 2x^2y - xy^2 - 2y^3)$

Factorizing $2(x^4 - y^4)$

$$= 2\left[(x^2)^2 - (y^2)^2\right]$$
$$= 2(x^2 + y^2)(x^2 - y^2) \qquad \therefore\ a^2 - b^2 = (a+b)(a-b)$$
$$= 2(x^2 + y^2)(x+y)(x-y)$$

Factorizing $3(x^3 + 2x^2y - xy^2 - 2y^3)$

$$= 3\left[x^2(x+2y) - y^2(x+2y)\right]$$
$$= 3(x+2y)(x^2 - y^2) \qquad \therefore\ a^2 - b^2 = (a+b)(a-b)$$
$$= 3(x+2y)(x+y)(x-y)$$

Common factors $= (x+y)(x-y)$

Non common factors = $2(x^2 - y^2)(x+2y)$

So, L.C.M = common factors × non common factors

L.C.M = $(x+y)(x-y)(x^2+y^2)(x+2y) \times 2 \times 3$

$$= 6(x^2 - y^2)(x^2 + y^2)(x+2y)$$
$$= 6(x+2y)(x^4 - y^4)$$

(iv) $4(x^4 - 1), 6(x^3 - x^2 - x + 1)$ **(A.B)**

Solution:

Factorizing $4(x^4 - 1)$

$$= 2 \times 2\left[(x^2)^2 - (1)^2\right] \qquad \therefore\ a^2 - b^2 = (a+b)(a-b)$$
$$= 2 \times 2(x^2 + 1)(x^2 - 1)$$
$$= 2 \times 2(x^2 + 1)(x+1)(x-1)$$

Factorizing $6(x^3 - x^2 - x + 1)$

$=2\times3\left[x^2(x-1)-1(x-1)\right]$

$=2\times3\left[(x-1)(x^2\text{-}1)\right]$

$=2\times3(x-1)(x-1)(x+1)$

Common factors $= 2(x+1)(x-1)$

Non common factors = $2\times3(x^2+1)(x-1)$

So, L.C.M = common factors × non common factors

$L.C.M = 2\times2\times3(x-1)(x+1)(x-1)(x^2+1)$

$=12(x-1)(x^4-1)$

Q.6 For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

$P(x)=x^2+x-(2k+2)$

Since $x+4$ is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

Hence $P(-4)=0$

$x^2+x-(2k+2)=0$

By putting the value of x

$(-4)^2+(-4)-(2k+2)=0$

$16-4-2k-2=0$

$-2k+10=0$

$2k=10$

$k=\frac{10^{5}}{2}$

$k=5$

Q.7 If $(x+3)(x-2)$ is the H.C.F. of $P(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2+7x-l)$, find k and l. **(A.B)**

Solution:

Since $(x–2)(x+3)$ is H.C.F of $p(x)$ and $q(x)$, then $(x–2)$ is factor of $(2x^2-3x+k)$ and $(x+3)$is factor of $(3x^2+7x-l)$.

Consider

$2x^2-3x+k$

Put $x–2 = 0$ or $x = 2$

$=2(2)^2-3(2)+k$

$=8-6+k$

$=2+k$

Remainder is equal to zero

$2+k=0$

$k = -2$

Now consider

$3x^2 + 7x - l$

Put $x + 3 = 0$ or $x = -3$

$= 3(-3)^2 + 7(-3) - l$

$= 27 - 21 - l$

$= 6 - l$

Remainder is equal to zero

$6 - l = 0$

$l = 6$

Q.8 The L.C.M. and H.C.F. of two polynomials $P(x)$ and $q(x)$ are $2(x^4 - 1)$ and $(x+1)(x^2+1)$ respectively. If $P(x) = x^3 + x^2 + x + 1$, find $q(x)$.

Solution:

We know that

$p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F.}$

$$q(x) = \frac{\text{L.C.M.} \times \text{H.C.F.}}{P(x)}$$

By putting the values

$$q(x) = \frac{2(x^4 - 1)(x+1)(x^2+1)}{x^3 + x^2 + x + 1}$$

$$q(x) = \frac{2(x^4 - 1)(x+1)(x^2+1)}{x^2(x+1) + 1(x+1)} = \frac{2(x^4 - 1)\cancel{(x+1)}\cancel{(x^2+1)}}{\cancel{(x+1)}\cancel{(x^2+1)}}$$

$$q(x) = 2(x^4 - 1)$$

Q.9 Let $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$ and $q(x) = 10x(x+3)(x-1)^2$, if the H.C.F. of $p(x), q(x)$ is $10(x+3)(x-1)$. Find their L.C.M.

Solution:

We know that

$$\text{L.C.M.} = \frac{p(x) \times q(x)}{\text{H.C.F.}}$$

By putting the values

$$\text{L.C.M.} = \frac{\cancel{10}(x^2 - 9)(x^2 - 3x + 2) \times 10x\cancel{(x+3)}(x-1)^{\cancel{2}}}{\cancel{10}\cancel{(x+3)}\cancel{(x-1)}}$$

$$\text{L.C.M.} = 10x(x^2 - 9)(x^2 - 3x + 2)(x - 1)$$

Or $\text{L.C.M} = 10x(x^2 - 9)(x^2 - 2x - x + 2)(x - 1)$

$$=10x\left(x^2-9\right)\left[x(x-2)-1(x-2)\right](x-1)$$

$$=10x\left(x^2-9\right)(x-1)^2(x-2)$$

Q.10 Let the product of L.C.M. and H.C.F. of two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.

Solution:

We know that

$$p(x)\times q(x)=\text{L.C.M.}\times\text{H.C.F.}$$

By putting the values

$$(x+3)(x-2)\left(x^2+kx+15\right)=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=\frac{(x+3)^{2}(x-2)(x+5)}{(x+3)(x-2)}$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+5x+3x+15$$

$$x^2+kx+15=x^2+8x+15$$

$$kx=x^2+8x+15-x^2-15$$

$$kx=8x$$

$$k=\frac{8x}{x}$$

$$k=8$$

Q.11 Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.

Solution:

Finding H.C.F of 176 and 128.

$$128\overline{)176}\;\;1$$
$$\underline{128}$$
$$48\overline{)128}\;\;2$$
$$\underline{-96}$$
$$32\overline{)48}\;\;1$$
$$\underline{-32}$$
$$16\overline{)32}\;\;2$$
$$\underline{-32}$$
$$0$$

Highest no. of children =16