Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

Solution:

$$\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$$

$$=\frac{x^2-3x+2x-6}{(x)^2-(3)^2}+\frac{x^2+6x-4x-24}{x^2-4x+3x-12} \qquad \therefore\ a^2-b^2=(a+b)(a-b)\ \textbf{(K.B)}$$

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x+3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$=\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}+\frac{(x+6)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$$

$$=\frac{(x+2)}{(x+3)}+\frac{(x+6)}{(x+3)}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{8+2x}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q.2 $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$ **(A.B)**

Solution:

$$\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{x^2+2x+1-(x^2+1-2x)}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1} \qquad \therefore\ a^2-b^2=(a+b)(a-b)\ \textbf{(K.B)}$$

$$= \left[\frac{\cancel{x^2} + 2x \cancel{+1} - \cancel{x^2} \cancel{-1} + 2x}{x^2 - 1} - \frac{4x}{x^2+1}\right] + \left[\frac{4x}{x^4-1}\right]$$

$$= \left[\frac{4x}{x^2-1} - \frac{4x}{x^2+1}\right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{4x(x^2+1) - 4x(x^2-1)}{(x^2-1)(x^2+1)}\right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{4x^3 + 4x - 4x^3 + 4x}{x^4-1}\right] + \frac{4x}{x^4-1}$$

$$= \frac{8x}{x^4-1} + \frac{4x}{x^4-1}$$

$$= \frac{8x+4x}{x^4-1}$$

$$= \frac{12x}{x^4-1}$$

Q.3 $\frac{1}{x^2-8x+15} + \frac{1}{x^2-4x+3} - \frac{2}{x^2-6x+5}$ **(A.B)**

Solution:

$$\frac{1}{x^2-8x+15} + \frac{1}{x^2-4x+3} - \frac{2}{x^2-6x+5}$$

$$= \frac{1}{x^2-3x-5x+15} + \frac{1}{x^2-3x-1x+3} - \frac{2}{x^2-5x-x+5}$$

$$= \frac{1}{x(x-3)-5(x-3)} + \frac{1}{x(x-3)-1(x-3)} - \frac{2}{x(x-5)-1(x-5)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-3)(x-1)} - \frac{2}{(x-5)(x-1)}$$

$$= \frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$= \frac{\cancel{x} - \cancel{1} + \cancel{x} - \cancel{5} - \cancel{2x} + \cancel{6}}{(x-3)(x-5)(x-1)}$$

$$= \frac{0}{(x-3)(x-5)(x-1)}$$

$$= 0$$

Q.4 $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$=\frac{(x+2)(x+3)}{(x)^2-(3)^2}+\frac{(x+2)\left[2(x^2-16)\right]}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)(\cancel{x+3})}{(x-3)(\cancel{x+3})}+\frac{(x+2)\left[2\{(x)^2-(4)^2\}\right]}{(x-4)[x(x-3)+2(x-3)]}$$ $\therefore\ a^2-b^2=(a+b)(a-b)$ **(K.B)**

$$=\frac{(x+2)}{(x-3)}+\frac{(\cancel{x+2})\left[2(x+4)(\cancel{x-4})\right]}{(\cancel{x-4})(x-3)(\cancel{x+2})}$$

$$=\frac{(x+2)}{(x-3)}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2}{x-3}+\frac{2x+8}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q.5 $\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$ **(A.B)**

Solution:

$$\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$ $\therefore\ a^2-b^2(a+b)(a-b)$ **(A.B)**

$$=\frac{(\cancel{x+3})}{(\cancel{x+3})(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{2(2x-3)+(2x+3)-4x\times 2}{2(2x-3)(2x+3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$=\frac{-2x-3}{2(2x-3)(2x+3)}$$

$$=\frac{-1(\cancel{2x+3})}{2(2x-3)(\cancel{2x+3})}$$

$$= \frac{-1}{2(2x-3)}$$

Q.6 $A - \frac{1}{A}$, where $A = \frac{a+1}{a-1}$

Solution:

Here $A = \frac{a+1}{a-1}$

$$A - \frac{1}{A} = \frac{a+1}{a-1} - \frac{1}{\frac{a+1}{a-1}} \quad \text{(by putting value of A)}$$

$$= \frac{a+1}{a-1} - \frac{a-1}{a+1}$$

$$= \frac{(a+1)^2 - (a-1)^2}{(a-1)(a+1)}$$

$$= \frac{a^2 + 2a + 1 - (a^2 - 2a + 1)}{a^2 - 1} \quad \therefore \ (a+b)^2 = a^2 + 2ab + b^2, \ (a-b)^2 = a^2 - 2ab + b^2$$

$$= \frac{\cancel{a^2} + 2a \cancel{+1} - \cancel{a^2} + 2a \cancel{-1}}{a^2 - 1}$$

$$= \frac{4a}{a^2 - 1}$$

Q.7 $\left[\frac{x-1}{x-2} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2}\right]$ **(A.B)**

Solution:

$$\left[\frac{x-1}{x-2} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2}\right]$$

$$= \left[\frac{x-1}{x-2} + \frac{2}{-x+2}\right] - \left[\frac{x+1}{x+2} + \frac{4}{-x^2+4}\right]$$

$$= \left[\frac{x-1}{x-2} - \frac{2}{x-2}\right] - \left[\frac{x+1}{x+2} + \frac{4}{-\left(x^2-4\right)}\right]$$

$$= \left[\frac{x-1}{x-2} - \frac{2}{x-2}\right] - \left[\frac{x+1}{x+2} - \frac{4}{x^2-4}\right]$$

$$= \left[\frac{x-1-2}{x-2}\right] - \left[\frac{x+1}{x+2} - \frac{4}{(x+2)(x-2)}\right]$$

$$=\frac{(x-3)}{(x-2)}-\left[\frac{(x+1)(x-2)-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\left[\frac{x^2-2x+x-2-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\left[\frac{x^2-x-2-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)+2(x-3)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$=\cancel{\frac{x-3}{x-2}}-\cancel{\frac{x-3}{x-2}}$$

$= 0$

Q.8 What rational number should be subtracted from **(A.B+K.B)**

$$\frac{2x^2+2x-7}{x^2+x-6}\text{ to get }\frac{x-1}{x-2}$$

Solution:

Let required rational number be $P(x)$

According to condition

$$\frac{2x^2+2x-7}{x^2+x-6}-P(x)=\frac{x-1}{x-2}$$

$$P(x)=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x(x+3)-2(x+3)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-(x^2+3x-x-3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$=\frac{x^2-4}{(x+3)(x-2)}$$

$$=\frac{x^2-2^2}{(x+3)(x-2)}$$ $\therefore\ a^2-b^2=(a+b)(a-b)$ **(K.B)**

$$=\frac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

$$=\frac{x+2}{x+3}$$

Q.9 $\dfrac{x^2+x-6}{x^2-x-6}\times\dfrac{x^2-4}{x^2-9}$

Solution:

$$\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\frac{x^2-2^2}{x^2-3^2}$$

$$=\frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)}\times\frac{(x-2)(x+2)}{(x-3)(x+3)}$$ $\therefore\ a^2-b^2=(a+b)(a-b)$ **(A.B+K.B)**

$$=\frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+2)}}\times\frac{(x-2)\cancel{(x+2)}}{(x-3)\cancel{(x+3)}}$$

$$=\frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

Solution:

$$\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{(x^2)-(2)^2}\times\frac{x^2+4x+2x+8}{x^2-x-x+1}$$

$$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\times\frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)}$$

$\therefore\ a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^2-b^2=(a+b)(a-b)$

$$=\frac{x^2+2x+4}{(x+2)}\times\frac{(x+4)(x+2)}{(x-1)(x-1)}$$

(A.B+K.B)

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Q.11 $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution:

$$\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$

$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$\therefore\ a^3-b^3=(a-b)(a^2+ab+b^2)$ **(K.B)**

$$=\frac{x(x-2)(x^2+2x+4)}{(2x-1)(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$=1$

Q.12 $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

Solution:

$$\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$$

$$=\frac{2y^2+8y-1y-4}{3y^2-12y-y+4}\div\frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}$$

$\therefore\ a^2-b^2=(a+b)(a-b)$ **(K.B)**

$$=\frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)}\div\frac{(2y-1)(2y+1)}{3y(2y+1)-1(2y+1)}$$

$$=\frac{(y+4)(2y-1)}{(3y-1)(y-4)}\div\frac{(2y-1)(2y+1)}{(3y-1)(2y+1)}$$

$$=\frac{(y+4)(2y-1)}{(3y-1)(y-4)}\times\frac{(3y-1)}{(2y-1)}$$

$$=\frac{y+4}{y-4}$$

Q.13 $\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$

Solution:

$$\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$$

$$=\left[\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$$

$$=\left[\frac{(x^4+2x^2y^2+y^4)-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{(x^2+2xy+y^2)-(x^2-2xy+y^2)}{x^2-y^2}\right]$$

$$\because (a+b)^2=a^2+2ab+b^2,\quad (a-b)^2=a^2-2ab+b^2$$

$$=\left[\frac{x^4+2x^2y^2+y^4-x^4+2x^2y^2-y^4}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{x^2+2xy+y^2-x^2+2xy-y^2}{x^2-y^2}\right]$$

$$=\left[\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{4xy}{x^2-y^2}\right]$$

$$=\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\times\frac{x^2-y^2}{4xy}$$

$$=\frac{4xy.xy}{(x^2-y^2)(x^2+y^2)}\times\frac{x^2-y^2}{4xy}$$

$$=\frac{xy}{x^2+y^2}$$